Medical Imaging Analysis

B-Spline Snake: A Flexible Tool for Parametric Contour Detection
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Introduction

This paper is an improvement over the Snake curve proposed by Kass et. al.[2]. Snake curve is described as an energy minimizing spline guided by external and internal energies. The external energies are defined by the given image and the internal energy is determined by the shape of the curve. As recognized in the literatures, this type of Snake has the following problems:

1) Slow convergence speed because of the large number of coefficients to optimize.

2) difficulty in determining the weights associated with the smoothness constraints;

3) description of the curve by a finite set of disconnected points;

4) high-order derivatives on the discrete curve may not be accurate in noisy environments.
The problems in 1) and 3) can be solved by using a parametric B-Spline representation of the curve [3]. Such formulation of an active contour allows local control, compact representation and implicit smoothness constraints.

The main contributions of this paper are 1) they proved of Snake is indeed a cubic B-Spline and 2) they took advantage of the continuity property of B-Spline and make the smoothness constraint implicit. The authors spent a whole page to prove that the optimal Snake obtained through the formulation in equation (1) is a cubic spline.

\[
S^*(u) = \arg \min_s \sum V(k, s(k)) + \lambda \int_{-\infty}^{\infty} (s''(u))^2 \, du \tag{1}
\]

However, there is an easier way to prove it. If we take the Euler-Lagrange equation of the 2\textsuperscript{nd} term of equation (1), the result is \(S^{(4)} = 0\). That is the fourth derivative of the curve is zero. This implies that a spline with 2\textsuperscript{nd} derivative smoothness constraint like the 2\textsuperscript{nd} term in equation (1) is a cubic spline. Furthermore, from the well-known property of B-Spline, that B-spline can represent any spline with the same degree, therefore we can conclude that \(S(u)\) is a cubic B-spline. By combining this result with the first term in equation (1), we can further conclude that Snake, which is the optimal solution in equation (1), is indeed a cubic B-Spline.

In the \textit{B-Snake} formulation proposed by Menet et. al., the authors still retain the smoothness constraint explicitly. That means they did not take advantage of the continuity property of B-Spline. Therefore the only advantage of B-Snake formulation over the original Snake is that less number of coefficients to be optimized.

The other significant contribution of the paper proposed by Brigger et. al. is that they make the smoothness constraint implicit by introducing a smoothness parameter \(h\).
The parameter $h$ controls the number of sampled points from the curve. The sampled points will then be used to optimize the control points of the curve. The larger $h$ is, the more smooth the curve will be. Figure 1 shows different number of sampling corresponds to different number of $h$. The authors of this paper showed that $h$ has same effect as the parameter $\lambda$ in equation (1).

![Figure 1: The implicit smoothness parameter $h$.](image)

When $h$ is equal to 1, the updating position of a control point depends on derivative of the energy function sampled from solely one point from the curve (red point from the left most figure in figure 1). With larger number of $h$, the position of a control point depends on the weighted average of the derivative of the energy function sampled from $h$ local points from the curve.

**User interface:**

The program is implemented in C++ using OpenCV. Users can click on the image to define the initial contour. The points users click will define the node points on the curve. The control points of the curve are then calculated based on the given node points. Users can drag a control/node point to change the shape of the initial contour.

After defining the initial contour, users have to press key ‘g’ to calculate the gradient magnitude of the image. Before calculating the gradient, the image is smoothed.
with a guassian kernel. The gradient magnitude will be shown on another window. User can then press key ‘o’ to start the optimization. Table 1 shows other functionalities of this program.

<table>
<thead>
<tr>
<th>Key</th>
<th>Functionality</th>
</tr>
</thead>
<tbody>
<tr>
<td>X</td>
<td>Show/hide control and node points on the curve.</td>
</tr>
<tr>
<td>Z</td>
<td>Show/hide sampled points on the curve. Red dots are the node points.</td>
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<tr>
<td>;</td>
<td>Decrease the value of h</td>
</tr>
<tr>
<td>‘</td>
<td>Increase the value of h</td>
</tr>
<tr>
<td>C</td>
<td>Clear current curve.</td>
</tr>
<tr>
<td>W</td>
<td>Save the current curve.</td>
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<tr>
<td>L</td>
<td>Load the previously saved curve.</td>
</tr>
<tr>
<td>I</td>
<td>Enable/disable node points of the curve.</td>
</tr>
<tr>
<td>O</td>
<td>Start the optimization process.</td>
</tr>
<tr>
<td>Esc</td>
<td>Stop optimization process or exit the program.</td>
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</tbody>
</table>

**Results**

Figure 2 shows the initial contour selected by user and the optimization result. The result shows that B-spline snake is able to locate the edge correctly. Figure 3 demonstrates the results of Snake and B-Spline snake. Figure 3(a) and 3(c) show the similar initial contours for Snake and B-Spline snake respectively and Figure 3(b) and 3(d) show the results of snake and B-Spline snake respectively. The results show that B-Spline snake was able to fit the sharp corner better than snake. Snake was able to reach the same result only when we completely disregard the smoothness constraint. However, for B-spline snake formulation, even though the smoothness constraint is defined implicitly, it was able to fit the sharp corner correctly. This demonstrates the flexibility of B-spline snake formulation.
Figure 2: Initial contour and the B-Spline Snake.

Figure 3: Comparison between Snake and B-spline snake.
The results in Figure 4 demonstrate that B-spline snake is insensitive to noise. The image in Figure 4(b) was applied with large amount of salt and pepper noise. Both optimizations in Figure 4 begin with same initial contour. The result shows that B-spline snake was able to converge correctly and obtained similar result to the one without noise.

![Figure 4](image)

(a) original image  (b) image with salt & pepper noise added

Figure 4.

Figure 5 shows the result of applying optimization with different smoothness parameter $h$. The result in (b) shows promising result with $h=4$. However, the result in (c) is not as good. The problem is because that the optimization of each control point is solely based on ONE point on the curve. This will often result in the situation that the optimization is stuck in local minimum. As the paper suggested, the choice of $h$ is usually greater than 1. However, when $h$ is set to be too large, this would result redundant calculations in optimization.
Figure 5. The initial contour and the result after optimization on a brain MRI image.

Figure 6 shows the ultrasound image and the result after optimization. Although ultrasound images are usually noisy, the edge of the chamber in Figure 6 is quite clear. That is why the result is acceptable. However, Figure 7(b) demonstrates the impact of noisy data on the optimization. The result is unacceptable compared to the result selected by an expert in Figure 7(c). The disappointed result in 7(b) is due to the highly noisy data, which is very common for ultrasound imaging. In this project, isotropic filtering (Gaussian kernel) is applied to the image before calculating the external energy function. This filtering removes noisy in the image and also the already blurred artery edge. One way to improve it, as proposed by Tauber et.al. in [4], is applying anisotropic filtering on the image first before calculating the external energy function, which is the gradient magnitude.

Figure 6. Ultrasound image of heart.
Figure 8. Ultrasound image of artery.

(a) Ultrasound of artery  (b) optimization result  (c) selection by an expert

Figure 9 demonstrates the result on detection of corpus callosum of brain. It shows reasonable result in (c) except for the front end of callosum (indicated by arrow). This error is due to the relatively low contrast in the region.

(a) MRI of callosum  (b) initial contour  (c) optimization result

Figure 9. MRI image of callosum,
References:


