COT5405: ANALYSIS OF ALGORITHMS

Homework # 3

Due date: Mar 9, 2005, Thursday (beginning of the class)

Your solutions should be concise, but complete, and typed or handwritten clearly. Feel free to consult textbooks, journal and conference papers and also each other, but write the solutions yourself and cite your sources. Answer **only five** of the following six questions. Each problem is worth 20 pts.

1. You are given an unlimited number of each of n different types of boxes. The dimensions of box type i are (x_i, y_i, z_i). In nesting boxes inside one another, you can place box A inside box B if and only if the dimensions of A are *strictly smaller* than the dimensions of B. Design and analyze an algorithm to determine the largest number of boxes that can be nested inside one another. (Hint: construct a directed graph first)

2. Suppose that a graph G has a minimum spanning tree MST already computed.
   (a) Design and analyze an algorithm to update the MST when an edge is added to G.
   (b) Design and analyze an algorithm to update the MST when an edge is deleted from G.

3. Recall the reweighting step of Johnson's algorithm discussed in class. Professor Shortcut claims that there is a simpler way to reweight the edges: Letting $w^* = min_{(u,v)\in E}\{w(u,v)\}$, just define $\hat{w}(u,v) = w(u,v) - w^*$ for all edges $(u,v) \in E$. What is wrong with the professor's method of reweighting?

4. Let $G = (V, E)$ be a flow network with source s, and sink t, and integer capacities. Suppose that we are given a maximum flow in G.
   (a) Suppose that the capacity of a single edge $(u,v) \in E$ is increased by 1. Give an $O(V+E)$-time algorithm to update the maximum flow.
   (b) Suppose that the capacity of a single edge $(u,v) \in E$ is decreased by 1. Give an $O(V+E)$-time algorithm to update the maximum flow.

5. Given an undirected graph G, design and analyze an algorithm that finds a cycle in the graph that visits each edge exactly once, or says that it can't be done.

6. A person want to fly from city A to city B in the shortest possible time. Flight time includes the waiting time for the connecting flights. S/he turns to a traveling agent who knows all the departure and arrival times of all the flights. Give an algorithm that will allow the agent to choose an optimal route. (Hint: Rather than modifying Dijkstra's algorithm, modify the data.)