COT5520: COMPUTATIONAL GEOMETRY

Homework # 4

Due date: Dec 4, 2008, Thursday (beginning of the class)

Your solutions should be concise, but complete, and typed (or handwritten clearly). Feel free to consult textbooks, journal and conference papers and also each other, but write the solutions yourself and cite your sources. Answer **only five** of the following six questions. Each problem is worth 20 pts.

1. Let P be a finite set of points in the plane and H the dual set of lines. Translate the following statements about P to statements about H.

   (i) No three points of P lie on a common line.

   (ii) The point $p \in P$ is a vertex of the convex hull.

   (iii) P has a subset of k points in convex position.

   (iv) P is contained in a strip bounded by two parallel lines at unit distance from each other.

2. A k-*coloring* of a line arrangement is a map χ from the set of faces to $\{1, 2, \ldots, k\}$ such that $\chi(f) \neq \chi(g)$ if $f \neq g$ share a common edge.

   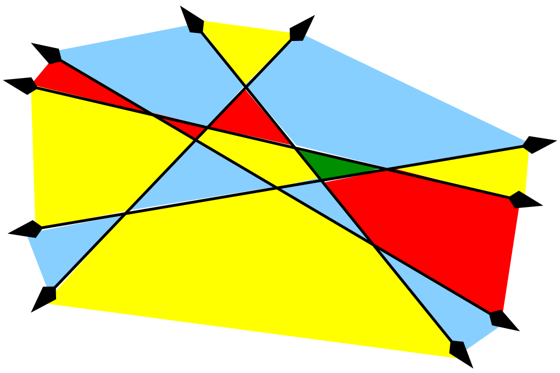

   Figure 1: A 4-coloring of an arrangement of 5 lines.

   (i) What is the smallest k such that every line arrangement has a k-coloring?

   (ii) Which line arrangements are 2-colorable?

   (iii) Draw a 2-colorable line arrangement for which the ratio of the number of faces of one color over the number of faces of the other color is as large as you can manage. What ratio do you get?

3. Let H be a set of n lines in general position. Consider the graph whose nodes are the vertices and whose arcs are the bounded edges in the arrangement $\mathcal{A}(H)$. Describe an algorithm that takes time $O(n^2)$ and space $O(n)$ to compute the x-monotone path with the largest number of arcs.

4. Let S be a set of n line segments in the plane. A *stabber* is a line h that intersects all segments of S.

   (i) Give an $O(n^2)$ algorithm to decide whether or not S has a stabber.

   (ii) Now assume that all line segments are vertical. Give a randomized algorithm with $O(n)$ expected running time that decides whether S has a stabber.

5. Let $\mathbb{I}^3 = [0,1] \times [0,1] \times [0,1]$ be the unit cube in $\mathbb{R}^3$ and consider a triangulation K of $\mathbb{I}^3$ whose only vertices are the 8 corner points of the cube.

   (a) Show that every such K has at most 6 tetrahedra.

   (b) Show that every such K has at least 5 tetrahedra.

   (c) Two triangulations K_1 and K_2 are *isomorphic* if $\exists$ a bijection $\beta : \mathrm{Vertices}(K_1) \to \mathrm{Vertices}(K_2)$ such that $\mathrm{ConvHull}(T) \in K_1$ iff $\mathrm{ConvHull}(\beta(T)) \in K_2$. Enumerate all pairwise non-isomorphic triangulations of unit cube (with no Steiner points).

6. Let A, B, C, and D be four non-overlapping disks such that A and B, B and C, C and D, and D and A are tangent to each other. Prove that these four tangency points are co-circular.