Secure Attribute-Based Systems

Matthew Pirretti, Patrick Traynor, and Patrick McDaniel
SIIS Laboratory, CSE, Pennsylvania State University
University Park, PA, USA
pirretti, traynor, mcdaniel@cse.psu.edu

Brent Waters
SRI International
Menlo Park, CA, USA
bwaters@cs.sri.com

ABSTRACT
Attributes define, classify, or annotate the datum to which they are assigned. However, traditional attribute architectures and cryptosystems are ill-equipped to provide security in the face of diverse access requirements and environments. In this paper, we introduce a novel secure information management architecture based on emerging attribute-based encryption (ABE) primitives. A policy system that meets the needs of complex policies is defined and illustrated. Based on the needs of those policies, we propose cryptographic optimizations that vastly improve enforcement efficiency. We further explore the use of such policies in two example applications: a HIPAA compliant distributed file system and a social network. A performance analysis of our ABE system and example applications demonstrates the ability to reduce cryptographic costs by as much as 98% over previously proposed constructions. Through this, we demonstrate that our attribute system is an efficient solution for securely managing information in large, loosely-coupled, distributed systems.

Categories and Subject Descriptors: K.6 Management of Computing and Information Systems: Security and Protection

General Terms: Security, Performance

Keywords: Attribute-based encryption, secure systems, applied cryptography

1. INTRODUCTION
Attributes define, classify, or annotate the datum to which they are assigned. The semantics of an attribute

"Attribute-based systems have enormous potential for providing data security in distributed environments. Peer-to-peer systems are an example of one such beneficiary: individuals may publish documents that implicitly target those users who are assigned the appropriate attributes. Moreover, such publishing can be completely transparent to the peer-to-peer system. For example, a user Bob looking for employment in the field of secure systems engineering could place a copy of his résumé in publicly accessible web space encrypted with the attributes “secure systems engineering” and “human resources manager”. Only potential employers satisfying these attributes would be able to decrypt this information and contact Bob.

In this paper, we develop and evaluate a secure attribute system built on attribute-based encryption (ABE). A descriptive policy system is defined that predicates access on logical expressions over attributes. We show how these policies can be realized through applications of novel ABE
constructions. We also demonstrate their semantic depth through their use in two applications: a HIPAA compliant distributed file system and a social network.

We have developed an extensive ABE implementation tailored for the rapid creation of attribute systems- the first known implementation and characterization of such cryptographic constructions. In an effort to aid development and subsequent system use, we perform an in-depth empirical analysis of the input parameter space. Our implementation includes several novel optimizations to the original ABE cryptosystem described by Sahai and Waters [26]. The major operations of the system, including system initialization, key generation, and the encryption and decryption of objects are benchmarked. We then measure the cost of implementing complex attribute policies. Whereas past work has suggested that these constructions were too expensive for use in real systems [25], this analysis shows that such policies are not only feasible, but can also be highly efficient. For instance, we demonstrate that the cost of key generation and encryption can be reduced by more than 80% and 98%, respectively, by using constructions secure in the random oracle model.

The remainder of this paper is organized as follows: Section 2 presents an overview of the cryptographic mechanisms supporting ABE; Section 3 compares ABE systems to PKI systems; Section 4 introduces a descriptive policy system for use in ABE-based systems; Section 5 offers sample policies for two example applications; Section 6 gives the results of our performance analysis; Section 7 explores relevant related work; Section 8 offers concluding remarks.

2. ATTRIBUTE-BASED ENCRYPTION

We now give an overview of Attribute-Based Encryption (ABE) algorithms. The Sahai-Waters [26] (ABE) cryptosystem as implemented in this paper is specifically detailed. We focus our efforts on providing the description of the scheme and intuition for its construction. For the proof of security see Sahai and Waters [26].

Attribute-Based Encryption can be viewed as a generalization of Identity-Based Encryption (IBE) [5,9,30]. In IBE a user’s identity is a string such as “bobsmith@yahoo.com”. A party in the system can encrypt a message to this particular user with only the knowledge of the recipient’s identity and the system’s public parameters. In particular the encryption algorithm does not need to have access to a separate public key certificate of the recipient.

In Attribute-Based Encryption a user’s identity is composed of a set, $S$, of strings which serve as descriptive attributes of the user. For example, a user’s identity could consist of attributes describing their university, department, and job function. A party in the system can then specify another set of attributes $S'$ such that a receiver can only decrypt a message if his identity $S$ has at least $k$ attributes in common with the set $S'$, where $k$ is a parameter set by the system. Like traditional Identity-Based Encryption, a party in an Attribute-Based Encryption system only needs to know the receiver’s description in order to determine their public key. However, the expressiveness of an ABE system is potentially much more powerful. For example, there could be several different recipients that are able to decrypt a message encrypted for a set $S'$.

ABE-based systems can also leverage “threshold constructions” where a user with identity $S$ will be able to decrypt a message if it has at least $k$ attributes that overlap with a set $S'$ chosen by the encryptor. Although there could in theory exist even more expressive ABE systems, the threshold constructions described and illustrated in the following sections are sufficiently semantically deep that we can define complex and precise encryption policies.

2.1 ABE Algorithms

We now informally specify a threshold Attribute-Based Encryption system as a collection of four algorithms:

- **Setup($k$):** The Setup algorithm is run by an authority in order to create a new ABE system. Setup takes as input a threshold value, $k$ and outputs a master key MK and a set of public parameters PK.

- **Key-Gen($S$, MK):** The authority executes the Key-Gen algorithm for the purpose of generating a new secret key SK. The algorithm takes as input the user’s identity, $S$, as a set of strings representing a user’s attributes and the master-key MK and outputs $S$’s secret key SK.

- **Encrypt($M$, $S'$, PK):** The Encrypt algorithm is run by a user to encrypt a message $M$, with a target set $S'$, and the public parameters. It outputs a ciphertext, $C$.

- **Decrypt($C$, $S'$, $S$, SK):** The Decrypt algorithm is run by a user with identity $S$ and secret key SK to attempt to decrypt a ciphertext $C$ that has been encrypted with $S'$. If the set overlap $|S \cap S'|$ is greater than or equal to $k$ the algorithm will output the decrypted message $M$.

2.2 ABE Constructions

We have investigated the use of two separate ABE constructions: the Sahai-Waters construction and a variant of the Sahai-Waters construction that we refer to as the random oracle construction. The Sahai-Waters construction is from the Sahai-Waters Large Universe system as described in Section 6 of [26]. A formal explanation of both constructions can be found in Appendix B.

Both constructions use elliptic curves to perform pairing-based cryptography. Bilinear maps (pairings) $e: G_1 \times G_2 \rightarrow G_T$ upon elements of an elliptic curve are the basis for pairing-based cryptosystems. In the Sahai-Waters construction, decryption is possible by performing pairings between $k$ components of a ciphertext and $k$ private key components. We refer the reader to the IBE paper by Boneh-Franklin [5] for more details on pairing-based cryptosystems.

2.2.1 Sahai-Waters Construction

We note two observations about the Sahai-Waters construction, which have led us to design the random oracle construction. Both observations pertain to the use of following function used in both the Key-Gen and Encrypt algorithms:

$$T(i) = g^i \prod_{j=1}^{n+1} t_j^{i,j,n(i)},$$

where $N$ is the set $\{1, \ldots, n+1\}$.

Our first observation is that because of $T$, the Setup algorithm must take as input a ciphertext size $n$ in addition to the threshold value $k$. Without the techniques proposed in
Section 4.3, this construction mandates that each ciphertext must contain exactly $n$ attributes and that the threshold must be a fixed value $k$ for all ciphertexts.

Our second observation is based upon our experience in building an implementation of the Sahai-Waters construction. We have found $T$ requires a great deal of computational effort. It is easily seen that the number of exponentiations required to solve $T$ is equal to $n + 1$.

Because of these two observations we have proposed the following modification to the Sahai-Waters construction.

2.2.2 Random Oracle Construction

We drastically reduce computational overhead in the key generation and encryption algorithms by replacing $T$ with a hash function used as a random oracle [4]. A simple argument shows that the random oracle can be “programmed” such that the security proof of Sahai and Watters still holds. We refer the reader to the literature [4,8] for further discussion on the random oracle model.

Implementing $T$ as a random oracle has the following characteristics. First, ciphertexts can contain a variable number of attributes, rather than be required to contain $n$. Second, the $n + 1$ exponentiations needed to solve $T$ in the Sahai-Waters construction have been replaced with a single cryptographic hash.

However, using model that requires the random oracle heuristic results in a slightly weaker security model; the use of random oracles makes the security of the cryptosystem dependent upon the security of the hash function used to compute $T$. In Section 6, we experimentally compare implementations of the original Sahai and Waters construction with our variant.

2.2.3 Decryption Optimization

Under both constructions, the dominant operations are pairings followed by exponentiations. Decryption as described by Sahai and Waters has the following form [26]:

$$M = E' \prod_{i \in S} \left( \frac{e(d_i, E_i)}{e(D_i, E')} \right)^{\Delta_i, s(0)}$$

where $e$ denotes a pairing operation. In the equation above, there are $2k$ pairings and $k$ exponentiations. Decryption can be optimized to reduce the number of bilinear map operations by bringing the Lagrange coefficients in:

$$M = \frac{E' \prod_{i \in S} e(d_i, E_i)}{e(D_i, E')}$$

This optimization reduces the number of bilinear map operations from $2k$ to $k + 1$ at the expense of increasing the number of exponentiations from $k$ to $2k$. Because bilinear map operations are more computationally intensive than exponentiations, this optimization increases the overall speed of decryption.

3. ATTRIBUTE KEY INFRASTRUCTURE

ABE systems do not vouch for identity in the traditional sense, as users are represented by the summation of their attributes. Accordingly, as identities are no longer necessarily unique, there is no need to validate bindings between keys and users. This alleviates many of the managerial problems found in traditional PKI systems [12] (e.g., user name collisions, per-user revocation). However, new challenges arise.

We briefly consider these issues, as well as their similarity to traditional PKI systems. Note that this work is not intended to solve the problems of PKIs, but to apply available approaches where possible and invent others where needed.

The process by which users are certified in an ABE system is analogous to certification in a PKI. Similar to a traditional PKI, a user presents the authority with a set of credentials that prove their right to fulfill an attribute. Instead of mapping a user to an identity, certification establishes that the user fulfills the semantic of the attribute. Such semantics are specific to the supported community (e.g. job function in a business system, clubs belonged to in a social network). This process is repeated for all attributes appropriate to each user. Key distribution is significantly simplified in such a system, as public keys are simply the combination of the cryptosystem’s public parameters and attribute names.

The revocation process is significantly different in an ABE system as attributes, not users or keys, are revoked. In fact, there is no way to revoke a user, save revoking all of his attributes. Like traditional PKI systems, revocation can impact all users who either have or use an attribute. Unlike traditional PKI systems, however, the compromise of a particular attribute may not mandate its revocation. As Section 4 details, it is the specific application of multiple attributes that defines policy. The compromise of any single attribute may therefore be a necessary but not sufficient condition for its revocation. Consequently, it may be desirable to revoke all, a subset or none of the compromised user attributes. Explored in depth in Section 6.3, we consider both online and offline revocation approaches.

A superficial reading of the above issues may lead one to falsely conclude that ABE systems must be online. The creation of keys, certification of users, and adding attributes are largely isomorphic to certification issuance operations present in current PKI. Revocation can also be handled offline (however, online approaches such as OCSP [24] are likely to be desirable in some environments). Hence, ABE systems can operate entirely offline in largely the same that current PKI systems do.

4. ATTRIBUTE POLICY

We now informally define an expository system for describing encryption policies in attribute-based systems. An attribute policy (or just policy throughout) is a specification of cryptographic operations carried out on a plaintext in the attribute-based system. Hence, through encryption, a party is able to embed expressive policies into objects themselves, allowing for the decentralized enforcement of such policies. Note that the following policy description is not particular to the specific constructions of our implementation, and is appropriate for defining policy in any ABE system that supports threshold constructions.

4.1 Definition

There are two components central to the definition of policies: attributes and objects. An attribute consists of a uniquely identifying string, $Name$, and its hash, $H(Name)$. The semantics of the $Name$ identifier are irrelevant to the policy itself and will be driven by the application it supports (see Section 5 for examples). The hash is necessary for the ABE construction (see Appendix B), but plays no role in the formulation of policy. We broadly refer to all encrypted or recovered data as objects. For example, objects
in a distributed file system would be the files that it stores. Conversely, objects in a social network may include a mix of personal communiques (e.g. emails, instant messages, etc.), profile information, and pictures. We refer to the universe of all attributes as the set \( A = \{ a_1, a_2, \ldots, a_4 \} \), the set of objects \( O = \{ o_1, o_2, \ldots, o_3 \} \). Where meaning is obvious or differentiation unnecessary, subscripts may be omitted.

The attribute policy is a specification of the attributes and threshold used to encrypt an object. For example, consider a policy \( P \) that mandates encryption using a single attribute \( a \) under a threshold of 1. We denote this policy:

\[
P = T_1(a)
\]  

(4)

Of course, policies are of most interest when they are applied to objects. The application of the example attribute policy on an object \( o \) is denoted:

\[
E(o, P) \quad \text{or equivalently} \quad E(o, T_1(a))
\]  

(5)

which states that \( o \) has been encrypted under attribute \( a \) using a 1-out-of-1 threshold encryption function. This object can therefore only be decrypted by a user possessing this attribute. Now consider the application of a similar policy \( P' \) on \( o \) that encrypts using 2-out-of-3 threshold using attributes \( a_1, a_2, \) and \( a_3 \):

\[
E(o, P') \quad \text{or equivalently} \quad E(o, T_2(a_1, a_2, a_3))
\]  

(6)

Now consider the most general case. An arbitrary policy \( P'' \) is defined:

\[
P'' = T_k(S) \quad | \quad S \subseteq A, \ S \neq \emptyset, \ 1 \leq k \leq |S|
\]  

(7)

which states that the following must be true for any legal policy, \( a \) the set of attributes must be a nonempty subset of \( A \), and \( b \) the threshold must at least 1 and no more than the total number of attributes.

Note that policies can be arbitrarily nested. Users can build complex expressions of attributes, thresholds, and logical operators. For example one may wish to combine \( P \) and \( P' \) over \( o \) above to achieve,

\[
E(E(o, P'), P) \quad \text{or equivalently} \quad E(E(o, T_2(a_1, a_2, a_3)), T_1(a))
\]  

(8)

which states that one must decrypt under policy \( P \) first, then \( P' \) to recover \( o \). Explored more fully in the following section, logical conjunction and disjunction are expressible through attribute policy. For example, if \( P_1 \) and \( P_2 \) are policies, then:

\[
P_k = (P_1) \land (P_2) \quad \quad P_l = (P_1) \lor (P_2)
\]  

(9)

The semantics of these policies are straightforward. A logical ‘and’ policy states that one must be able to decrypt both under both policies to extract the plaintext. The logical ‘or’ policy requires that one must be able decrypt under either or both of the policies to obtain the plaintext. Such constructions are examined in greater detail in Section 5.

The remainder of this paper explores how we build constructions meeting the semantics of these policies and how they can be applied to build novel and interesting applications.

### 4.2 Implementing Policy

Implementing singular attribute or threshold policies is straightforward using ABE constructions. An example of a threshold policy is depicted in Figure 1. This policy states that decryption is possible if the party performing decryption possess at least three of the following attributes: \( a_1, a_2, a_3, a_4 \). This is illustrated by the requisite attributes being fed to the threshold operator \( T_3 \). The output of the threshold primitive is the desired policy \( P_1 \), which can then be used to define an encryption operation.

We refer to a policy where \( k \) out of \( k \) attributes are required to decrypt an object as an **and** policy, and 1 in \( k \) attributes as **or** logic policy. These policies can be easily implemented using the thresholding primitive, where the threshold is \( k \) in the case of **and** logic and 1 in **or** logic. Figure 2 illustrates an **and** policy. The policy \( P_2 \) requires that the party performing decryption must possess all four of the following attributes: \( a_1, a_2, a_3, a_4 \). \( P_2 \) is implemented by giving the threshold operator \( T_4 \) the four required attributes. An **or** policy is trivially similar, and is thus not illustrated.

Expressing policy becomes somewhat more complex when the input policies are not subsets of \( S \), i.e., not expressions over atomic policies. Consider the case of an **or** policy spanning three (possibly complex) policies \( P_1, P_2, \) and \( P_3 \). In this case, one need only encrypt each of the input objects under each policy and concatenate them together; anyone able to decrypt at least one of these objects should be able to recover the underlying object. Denoting concatenation as ‘\( \cdot \)’, the ciphertext of an object \( o_i \) encrypted under a policy \( P_1 \lor P_2 \lor P_3 \) would be \( E(o_i, P_1) \cdot E(o_i, P_2) \cdot E(o_i, P_3) \).

Now consider the case of an **and** policy spanning three
(possibly complex) policies $P_1$, $P_2$, and $P_3$. One cannot simply use a threshold as above because the input policies do not reflect a threshold over atomic attributes. Hence, another construction must be used. Observe that we can achieve and semantics by sequentially encrypting the object with each policy. Thus the policy $P_1 \land P_2 \land P_3$ would be $E(E(E(a_1, P_1), P_2), P_3)$. This satisfies the policy semantics because only principals which possess the underlying attributes satisfying all policies can recover the plaintext.

Conjunction and disjunction constructions can be nested arbitrarily. In Figures 3 and 4 we illustrate policies that use both and logic and or logic. Specifically, in the and-or policy, any party performing decryption must possess either the attributes $a_1$ and $a_2$, or the attributes $a_3$ and $a_4$. The or-and policy requires the decrypting party possess $a_1$ or $a_2$ in addition to attribute $a_3$.

Observe that the conjunction constructor has a weaker security model than the original ABE constructions, where the base objects are encrypted under attributes. Whereas ABE encryption prevents any collusion attack, it is possible for adversaries to collude to recover the plaintext in this construction. To illustrate, in the or-and example\(^1\) in Figure 4, two colluding parties satisfying $P_7$ and $P_8$ independently can recover the plaintext. The first adversary need decrypt the outer encryption using its $a_3$ assignment, then pass the inner $E(a_4, T_1(A_1))$ ciphertext to the second adversary who can then decrypt using $a_4$ to recover the original plaintext $a_1$. Work to improve this aspect of the associated cryptographic constructions is currently under way. Until then, our constructions are no weaker than standard cryptographic methods e.g. public key cryptosystems.

### 4.3 Extending the Flexibility of ABE

ABE natively supports a $k$-of-$n$ threshold primitive. However the cryptographic constructions discussed in Appendix B mandate that $k$ be a fixed constant across all ciphertext objects created by a given attribute system. Further, for the implementation without random oracles, the number of attributes in each ciphertext $n$, must also be fixed. These requirements greatly limit the liberty at which principals can craft policies; each policy must be created with a single type of threshold primitive. Thus, if $k = 4$, $n = 4$, then all policies would have to be written using $T_4$ threshold operators and each ciphertext would have to contain exactly $4$ attributes. As a result the policy $P_9 = T_2(a_1, a_2, a_3)$ could not be implemented.

Because we are interested in enabling the creation of highly expressive policies, we discuss three separate approaches which circumvent the fixed $n, k$ requirement for constructions without random oracles\(^2\). The first two approaches were initially introduced by Sahai and Waters [20]. To better understand the difference between these three solutions it is helpful to introduce the following notation. Let $(k, n)$ denote a valid pairing of $k, n$ for a particular system.

The first solution is to provide all principals in a system with "default attributes", which act as placeholders and are devoid of semantic meaning (i.e. they are given to all users regardless of their attributes). The purpose of these attributes is to enable objects to contain any threshold operator, $T_k$, such that $1 \leq k' \leq k$. To attain this end, each object must contain the maximum number of possible attributes $n$. The default attributes can then pad for any of the required $n$ attributes. This scheme extends a system where only $(k, n)$ is valid to a system where any of the following are valid: $(1, n - (k - 1)), (2, n - (k - 2)), \ldots, (k, n)$. Given $k, n = 10$, this method allows a single cryptosystem to express the ten and policies between $(1, 1)$ and $(10, 10)$; however, this example cryptosystem could not express any system in which $k, n$ were not equal.

The second solution entails creating $n$ separate cryptosystems, each with a different value of $k$, enabling policy to use any of the following: $(1, n), (2, n), \ldots, (n, n)$. Similar to the previous approach, this scheme extends policy expressive-

1This policy expression could be optimized to reduce the required number of encryptions. Specifically, $(a_1 \lor a_2) \land a_3 \equiv (a_1 \land a_3) \lor (a_2 \land a_3)$. This optimization would require two encryptions with $T_2$, as opposed to four encryptions with $T_1$. Such logic expression reductions have been thoroughly studied by other works [17] and are therefore not the focus of this work.

2While this discussion focuses on circumventing the fixed $n, k$ in the constructions without random oracles, our approaches can be extended to constructions with random oracles, which is only limited by having a fixed $k$.
5. APPLICATION OF POLICY

The threshold, conjunction and disjunction constructions discussed in Section 4 result in an expressive policy system. In this section we illustrate the use of policy in two separate applications: HIPAA compliant distributed storage systems and social networks.

5.1 Distributed File Systems

A content-addressable file system enables users to locate files based on attributes or keywords describing their contents. Accordingly, data becomes searchable in a more meaningful fashion than the traditional approach of specifying file paths. To date, most work on content-addressable file systems has focused on automatically generating descriptions of a file’s contents [6, 14, 16]. The use of ABE strengthens the security properties of such systems. Because the access control policy of every object is embedded within it, the enforcement of policy becomes an inseparable characteristic of the data itself. This is in direct contrast to most currently available systems, which rely directly upon a trusted host to mediate access and administer policy. As file systems become more distributed in nature and rely upon domains of varying trust to control access, traditional approaches no longer provide adequate guarantees.

Example systems include large multisite research efforts such as the Human Genome Project, which was formed in order to map the sequence of chemical building blocks composing the human genome. While this multinational research effort requires a total of only 3 gigabytes of space to store the genome itself, the space estimated for additional annotations will sufficiently dwarf the initial sequence data in the system [2]. As this and other research projects begin to require petabytes of storage, the ability to securely store such information across multiple sites becomes increasingly critical.

5.1.1 Policy for HIPAA Compliant Medical Systems

We use a Health Insurance Portability and Accountability Act (HIPAA) compliant medical system as an example of a loosely-coupled, content-addressable file system with strict security requirements. HIPAA was designed to clearly enumerate the security requirements and provide information flow control for electronically stored medical information such that patient privacy is maintained [32]. While the system below is by far not a comprehensive example of HIPAA requirements, it demonstrates the ease with which a fully compliant system could be constructed using ABE.

In this example, a patient’s medical information is composed of several fields. Patients define the following privacy policies to best protect each component of their medical information: currently used medications \(P_{i,Med} \), medical history \(P_{i,Hist} \), contact information \(P_{i,CI} \), and insurance information \(P_{i,Ins} \). A patient’s policy describes the attributes that must be possessed by medical personnel in order to access their medical information. As illustrated in Figure 6, these attributes describe various job functions of medical personnel as well as the health insurance plans they accept.

A patient, Robert Oppenheimer, supplements his limited insurance coverage through the ACME Corporation with a “Medicare D” prescription plan. His policy therefore stipulates that only doctors \(Dr \) and nurses \(RN\) supporting his insurance plan can view his full medical history, contact information, and a listing of his current medications. Oppenheimer’s policy also allows a pharmacist \(Rx\) in his plan to view the medications he is currently taking, so that he/she can ensure no conflicts between prescriptions exist. Further, a pharmacist is allowed access to contact information to notify him when prescriptions have been filled. Oppenheimer’s policy also restricts the access of billing personnel \(Bill\) to his insurance and contact information such that charges can be filed with his insurance providers without danger of exposing private information. Lastly, without revealing his contact information, Oppenheimer allows the list of...
of medications he is currently using to be made available to pharmaceutical representatives (Rep) analyzing the combination of drugs with which their products are prescribed in concert. As described above, Oppenheimer’s policies are represented as follows:

\[
P_{O,Hist} = T_1(Dr, Rn) \land T_1(ACME, MedicareD) \\
P_{O,CI} = T_1(Dr, Rn, Bill, Rx) \\
P_{O,Ins} = (T_1(Rep)) \\
P_{O,Med} = (T_1(Dr, Rn, Rx) \land T_1(ACME, MedicareD)) \lor T_1(Rep)
\]

From the above policies, only Dr. Crusher and Nurse Whitman can access his medical history. Dr. Crusher, Nurse Whitman, Billing Secretary Arnold, and Pharmacist Fleming can access his contact information. Billing Specialist Arnold can also access Oppenheimer’s insurance information. Dr. Crusher, Nurse Whitman, Pharmacist Fleming and Pharmaceutical Representative Miller are able to determine Oppenheimer’s current regime of medication.

### 5.2 Social Networks and Online Communities

Social networks, such as orkut, Facebook and Friendster [1], are online applications which enable users to find other users with similar interests. To use these applications, users must reveal large quantities of personal information (e.g., name, age, address, personal interests, sexuality, etc.) into the public domain. Groups of people sharing similar attributes and friends are then automatically linked to each other. Currently, such systems provide only weak privacy guarantees; network membership allows access to the wealth of user information. Accordingly, user data can readily be mined and abused by undesirable parties.

ABE-based systems are well suited to provide user controlled-privacy, as users in these communities are already characterized by their attributes. In Friendster, for example, a user with the attribute “Anon U. Alumnus” is automatically enrolled in a group of the same name. Accordingly, the creation of “white-lists” for communication immediately becomes possible without requiring enumeration of all user identities. Constructing a social network using ABE also provides scalability. Current social networks require a trusted central server to store all profile information and enforce policy. Because ABE-based systems do not require a trusted storage system, profile information could be stored on untrusted servers, significantly decreasing the traffic and storage requirements incurred by a system. Further, in an ABE-based system, objects are embedded with policy, enabling distributed enforcement.

#### 5.2.1 Policy in a Social Network

We now demonstrate policy in a social network through an application where the principals are users of an online dating service. Each user dictates their own policy in order to restrict access to their personal information.

Figure 7 illustrates a sample network. A principal’s policy can be viewed as a description of attributes they find desirable in other principals. Possession of the attributes described in the policy is therefore a prerequisite to being able to access another principal’s personal information. We begin with a relatively simple policy. Van Buren is only interested in meeting women with black hair, medium wealth, and medium weight. His policy is represented as:

\[
P_V = T_4(Female \land BlackHair \land MedWealth) \\
\lor MedWeight).
\]

Of the above principals, only Jones can access Van Buren’s profile. This policy, which is equivalent to policy \( P_6 \) as depicted in Figure 2, can be expressed using a single threshold primitive \( T_1 \). It is therefore possible to directly implement \( P_V \) with a single ABE encryption. Accordingly, data encrypted under this policy will be resistant to collusion.

Grant’s policy, whereby only blonde or red haired women can access his profile information, is represented as:

\[
P_G = T_1(Female) \land (T_1(Blonde) \lor T_1(Blond)).
\]

As such, only Anderson or Kidman can access his information. Notice that Grant’s policy is equivalent to policy \( P_6 \) in Figure 4 and therefore cannot be implemented using a single threshold operator. Accordingly, \( P_G \) is less resistant to collusion than \( P_V \).
Lastly, Anderson is interested in hearing from men who possess at least two of the following attributes: red hair, medium weight, overweight, or medium wealth. Her policy can be represented as:

$$P_A = T_1(\text{Male}) \land T_2(\text{Red, MedWeight, Overweight, MedWealth})$$

Given Anderson’s policy Coolidge and Grant can access her information. Notice, however, that a principal’s policy is not necessarily symmetric. For instance, of these two, only Grant has a policy that would allow Anderson to contact him.

6. SYSTEM EVALUATION

The policies discussed in the previous section illustrate the potential expressibility of ABE-based systems. In this section, we characterize the performance of systems providing such functionality. We begin by exploring the cost of the base cryptographic constructions. We then determine the cost of implementing a selection of the previously defined policies. We finish by comparing the performance of an ABE-based system to a comparable system implemented with RSA cryptographic primitives.

As demonstrated by numerous others (e.g. [10]), the selection of cryptographic parameters can have a drastic impact on system performance. In this section, we characterize the parameter space by profiling the performance of attribute systems under different input parameters. Such analysis is necessary to optimize the system for a particular application or environment. All experiments were carried out on a 2.0 GHz Apple Xserve G5 with 4GB memory running Mac OS X Server 10.3.9. All disk operations were performed on a 1.82TB RAID 5 disk array. All results are calculated from an average of 500 iterations of the measured operation.

We have implemented an ABE library upon which secure attribute systems can be constructed. This C library contains approximately 5,200 lines of code and has been tested on Solaris, OS X and Linux platforms. To our knowledge, we are the first to implement, measure, and characterize the theoretical mechanisms of attribute-based encryption. Accordingly, we explore a wide range of potential inputs and settings system architects should consider when designing new secure attribute-based environments. For instance, systems using our API can choose between the two most studied elliptic curve groups providing bilinear maps: supersingular elliptic curves (SS), which enable fast cryptographic pairing operations [22], and MNT elliptic curves, which are used to obtain small ciphertext sizes [23]. We use the Pairing-Based Crypto library [19] for the underlying implementation of these groups and OpenSSL [3] for providing a supporting Key Encapsulation Mechanism (KEM) [31].

The following analysis measures the four central functions of the attribute system as defined in Section 2: Setup System, Key Generation, Encryption, and Decrypt-
Table 1: Base cryptographic operations for the major attribute functions.

<table>
<thead>
<tr>
<th>operation</th>
<th>Random Oracle</th>
<th>No Random Oracle</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>hashes</td>
<td>expos.</td>
</tr>
<tr>
<td>System_Setup (x attributes)</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>Key_Generation (y attributes)</td>
<td>$x$</td>
<td>$3x$</td>
</tr>
<tr>
<td>Encryption (threshold $k$)</td>
<td>$y$</td>
<td>$2 + y$</td>
</tr>
<tr>
<td>Decryption</td>
<td>$2k$</td>
<td>$k + 1$</td>
</tr>
</tbody>
</table>

Table 2: Table of R-Square values (No Random Oracles)

<table>
<thead>
<tr>
<th># Attributes</th>
<th>Data Length</th>
<th>Curve Type</th>
<th>Rand Init</th>
</tr>
</thead>
<tbody>
<tr>
<td>Initialize_Randomness</td>
<td>$2.083E-5$</td>
<td>$1.043E-5$</td>
<td>$1.440E-6$</td>
</tr>
<tr>
<td>System_Setup</td>
<td>$0.8052$</td>
<td>$1.321E-4$</td>
<td>$0.0138$</td>
</tr>
<tr>
<td>New_Attribute</td>
<td>$0.0442$</td>
<td>$1.199E-4$</td>
<td>$6.959E-4$</td>
</tr>
<tr>
<td>Key_Generation</td>
<td>$0.0247$</td>
<td>$2.480E-4$</td>
<td>$0.0509$</td>
</tr>
<tr>
<td>Encryption</td>
<td>$0.7134$</td>
<td>$2.120E-4$</td>
<td>$0.0692$</td>
</tr>
<tr>
<td>Decryption</td>
<td>$0.5355$</td>
<td>$1.733E-4$</td>
<td>$0.2222$</td>
</tr>
</tbody>
</table>

Table 3: Table of R-Square values (Random Oracles)

<table>
<thead>
<tr>
<th># Attributes</th>
<th>Data Length</th>
<th>Curve Type</th>
<th>Rand Init</th>
</tr>
</thead>
<tbody>
<tr>
<td>Initialize_Randomness</td>
<td>$1.167E-5$</td>
<td>$1.254E-5$</td>
<td>$3.363E-7$</td>
</tr>
<tr>
<td>System_Setup</td>
<td>$0.7908$</td>
<td>$8.915E-7$</td>
<td>$0.0176$</td>
</tr>
<tr>
<td>New_Attribute</td>
<td>$0.0394$</td>
<td>$1.014E-4$</td>
<td>$8.334E-5$</td>
</tr>
<tr>
<td>Key_Generation</td>
<td>$0.9999$</td>
<td>$9.792E-8$</td>
<td>$1.551E-4$</td>
</tr>
<tr>
<td>Encryption</td>
<td>$0.4781$</td>
<td>$3.167E-7$</td>
<td>$0.1993$</td>
</tr>
<tr>
<td>Decryption</td>
<td>$0.5608$</td>
<td>$4.543E-9$</td>
<td>$0.2041$</td>
</tr>
</tbody>
</table>

6.1 Experimental Results

The first set of experiments measure the degree to which different system parameters affect performance: we vary the number of attributes, length of data, elliptic curve and initialization of randomness parameters. We then perform an R-squared or coefficient of determination analysis over the measured results. This technique identifies the portion of observed variance in one variable that is directly attributable to a second. On a scale from zero to one, numbers closer to one represent a significant correlation between variables. For precision, we also include measurements for two additional subfunctions: Initialize_Randomness preloads random bytes from the local entropy pool and New_Attribute allocates a new attribute to a principal. The results of these tests reveal that the number of attributes followed by the elliptic curve used are the dominant factors, as shown in Tables 2 and 3. Lastly, to characterize the growth of execution time against the number of attributes, we run a regression analysis for the worst case and present our findings in the standard linear form, i.e., $y = mx + b$.

Figure 8 shows the cost of System_Setup as a function of the number of attributes. Systems using a SS curve without random oracles average 0.366 seconds ($\sigma = 0.049$) and 2.141 seconds ($\sigma = 0.133$) for 1 and 32 attributes, respectively. A system using the MNT elliptic curve without random oracles averages between 0.737 seconds ($\sigma = 0.202$) and 1.699 seconds ($\sigma = 0.284$) for the same range. Execution time for both curves scales linearly in the number of attributes (SS w/o random oracles: $y = 0.572x + 0.3126; r^2 = 0.9999$). Random oracles have no role in system setup, and hence have no bearing on performance. System setup therefore poses no significant computational burden in real systems. Figure 9 illustrates the cost of key generation, which is consistently cheaper for MNT curves. For a system using 32 attributes without random oracles, MNT curves require an average of 12.355 seconds ($\sigma = 0.035$) to generate a user key, compared to 25.05 seconds ($\sigma = 0.052$) for SS curves. Random oracle constructions are significantly faster - systems built on SS and MNT curves perform similarly at all numbers of attributes, e.g., at 5.051 ($\sigma = 0.017$) and 4.927 ($\sigma = 0.017$) seconds, respectively for 32 attributes. Execution time scales linearly for both curves with and without the use of random oracles (SS w/o random oracles: $y = 0.8003x - 2.37; r^2 = 0.9584$). Note that key generation for each user is performed infrequently (likely once). If the user community is fairly static, such costs will be amortized by operations on data. Conversely, in environments where users may join frequently, it behooves the administrator to select parameter choices that minimize these costs, e.g., MNT elliptic curves using random oracles.

As shown in Figure 10 for both SS and MNT elliptic curves, the construction without random oracles requires an average of 11.213 ($\sigma = 0.031$) and 3.946 ($\sigma = 0.017$)
seconds to encrypt data using 32 attributes. Systems implementing the construction with random oracles experience dramatically improved encryption performance, i.e., 1.207 \((\sigma = 0.009)\) and 0.204 \((\sigma = 0.006)\) seconds, respectively. Here, MNT elliptic curves are approximately 65% to 85% faster than their SS counter-parts (in constructions with and without random oracles, respectively). Systems using MNT curves with random oracles are in fact 98% faster those using SS curves without random oracles. Both systems scale linearly in the number of attributes with and without random oracles (SS w/o random oracles: \(y = 0.3590x - 1.148; r^2 = 0.9487\)). Conversely, as illustrated in Figure 11, a system of 32 attributes with and without random oracles exhibits a decryption time of 1.452 \((\sigma = 0.009)\) and 1.348 \((\sigma = 0.044)\) for an SS construction and 7.341 \((\sigma = 0.029)\) and 5.342 \((\sigma = 0.841)\) seconds in MNT, respectively. Execution time for both systems scales linearly in the number of attributes with and without random oracles (MNT w/o random oracles: \(y = 0.2298x - 0.103; r^2 = 0.9999\)). Note the systems using SS curves experience approximately 80% faster performance than their MNT counterparts.

Lastly, we compare the performance of ABE against traditional cryptographic techniques. From OpenSSL’s benchmarking tool [3], the platform used for ABE benchmarking is capable of performing RSA public key encryption in 0.0003 and 0.00097 seconds for 1024 and 2048-bit keys, respectively. To offer similar semantic expressiveness and prevent the need for \(2^n - 1\) keys (there are \(2^n -1\) nonempty subsets in a set of size \(N\)), we assume that each attribute in an ABE system has a corresponding RSA key pair. For simple policies, encryption under a single attribute/key is 300 and 93 times faster under RSA (0.0003 and 0.00097 vs 0.09 seconds). ABE’s thresholding primitive, however, allows much more efficient execution. For example, a policy requiring a threshold of 2 of 32 attributes has nearly identical execution times (0.1488 vs 0.2043 seconds) for both RSA-1024 and ABE with MNT curves and random oracles. RSA-2048 requires approximately 0.5 seconds to achieve the same ends. A system requiring 16 of 32 attributes would also require 0.2043 seconds for an ABE system; however, the equivalent RSA systems would require approximately 33.4 and 107.97 \text{days} over 46.6 million times slower, to achieve the same. ABE’s inherent expressibility makes it a practical means of constructing real attribute systems.

**6.1.1 HIPAA System Policy Analysis**

We now determine the cost incurred for implementing expressive policies. Encrypting with the policy \(P_O,C1\) from Section 5.1.1 requires an initial encryption of the principal Oppenheimer’s contact information using \(T_1(\text{ACME, Medicare D})\), a process requiring \(E_2\) time to complete. This ciphertext object is then independently re-encrypted with each of the following attributes: Dr, Rn, Billing, Rx. Each of these encryptions requires 1 attribute, and thus takes \(E_1\) time to complete. This policy could alternatively be implemented using two encryptions if the or construction is replaced with \(T_1(\text{Dr, Rn, Bill, Rx})\). Table 4 shows the timing values for this optimized policy, noted as \(P_O,C1 = E(E(C1, T_1(\text{ACME, Medicare D})), T_1(\text{Dr, Rn, Bill, Rx}))\).

Decryption data encrypted under \(P_O,C1\) requires two operations. The first decryption occurs with any of the following attributes: Dr, Rn, Billing, Rx. The second decryption, which enables recovery of the plaintext, requires decryption of \(T_1(\text{ACME, Medicare D})\). Table 4 shows average execution time.

### 6.1.2 Social Network Analysis

We now examine the cost of expressing policy as described in Section 5.2.1 for a social network application. Consider the time required to encrypt a message under Grant’s policy, \(P_G\). Grant’s information \(I_G\) is first independently encrypted under \(T_1(\text{Red Hair})\) and \(T_1(\text{Blonde})\). Both values, noted as \(I_G' = E(I_G, T_1(\text{Red Hair}))\) and \(I_G'' = E(I_G, T_1(\text{Blonde}))\) respectively, are then encrypted under \(T_1(\text{Female})\), yielding \(E(I_G', T_1(\text{Female}))\) and \(E(I_G'', T_1(\text{Female}))\). Note that the total number of encryptions can be halved if the or semantic is equivalently implemented as \(T_1(\text{Red Hair, Blonde})\). The total time to encrypt \(P_G\) is given by \(E(P_G)\).

Table 5 details the time required to perform the unoptimized operations required to formulate \(P_G\). These values represent the encryption and decryption operations for SS and MNT elliptic curves with and without random oracles.

In the case of decryption for \(P_G\), two decryptions are required. The decrypting party initially performs two decryptions with \(a_1\). From this, only \(a_1\) or \(a_2\) must be decrypted in order to recover the original plaintext. The total time to decrypt \(P_G\) is given by \(D(P_G)\).

### 6.2 Ciphertext Size and User Key Length

Ciphertext size and key length are important to some classes of applications, e.g., in high traffic volume or low bandwidth networks or on resource poor devices. Here, we briefly detail the size of ciphertexts and the size of a user’s private key. Specifically, we quantify ciphertext length and key size for different symmetric groupings. A bilinear map that is asymmetric has the property of \(E : \mathbb{G} \times \mathbb{G} \rightarrow \mathbb{G}_T\). This type of bilinear map is said have symmetric groups. A bilinear map that is asymmetric has the following form: \(e : \mathbb{G}_1 \times \mathbb{G}_2 \rightarrow \mathbb{G}_T, \mathbb{G}_1 \neq \mathbb{G}_2\). SS curves are characterized by having symmetric bilinear groups. Both \(\mathbb{G}\) and \(\mathbb{G}_T\) require 512 bits to be represented. MNT curves are characterized by having asymmetric bilinear groups. \(\mathbb{G}_1\)

---

**Table 4: Average performance (sec) for \(P_{O, C1}\)**

<table>
<thead>
<tr>
<th>No Rand Oracles</th>
<th>Rand Oracles</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>SS</td>
</tr>
<tr>
<td>(E)</td>
<td>0.13</td>
</tr>
<tr>
<td>(D)</td>
<td>0.31</td>
</tr>
<tr>
<td>(E(P_{O,C1}))</td>
<td>0.09</td>
</tr>
<tr>
<td>(D(P_{O,C1}))</td>
<td>0.18</td>
</tr>
</tbody>
</table>

**Table 5: Average performance (sec) for \(P_G\)**

<table>
<thead>
<tr>
<th>No Rand Oracles</th>
<th>Rand Oracles</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>SS</td>
</tr>
<tr>
<td>(E)</td>
<td>0.07</td>
</tr>
<tr>
<td>(D)</td>
<td>0.04</td>
</tr>
<tr>
<td>(E(P_G))</td>
<td>0.28</td>
</tr>
<tr>
<td>(D(P_G))</td>
<td>0.08</td>
</tr>
</tbody>
</table>
is represented with 170 bits while $G_2,G_T$ are represented with 510 bits.

Each attribute $i$ possessed by a principal corresponds to two private key components $d_i$ and $D_i$. For SS curves both of these components are members of $G$. For MNT curves $D_i \in G_1,d_i \in G_2$. This yields (for a private key with $n$ attributes):

$$\text{Supersingular KeySize}(n) = 2 \cdot n \cdot 512 \text{bits} \tag{10}$$

$$\text{MNT KeySize}(n) = (170 + 510) \cdot n \text{bits} \tag{11}$$

A ciphertext $C$ scales with the number of attributes it contains as follows. A ciphertext with $n$ attributes is composed of $C', C''$, and $n$ elements $C_i$. For SS curves $C'', C_i \in G$ and $C' \in G_T$. For MNT curves $C'' \in G_2, C' \in G_T$, and $C_i \in G_1$. This yields (for a $n$ attribute ciphertext):

$$\text{Supersingular CTLen}(n) = (n + 2) \cdot 512 \text{ bits} \tag{12}$$

$$\text{MNT CTLen}(n) = 2 \cdot 510 + 170m \text{ bits} \tag{13}$$

### 6.3 Attribute Revocation Issues

We now address some of the practical issues relevant to constructing ABE-based systems. An in-depth discussion of the implementation and the associated parameters is provided in Appendix A.

Revocation of users and keys in systems is a well studied but nontrivial problem [21]. Revocation is even more difficult in attribute systems, given that each attribute is conceivably possessed by multiple different users, whereas public/private key pairs are uniquely associated with a single principal. While an in-depth discussion of revocation is out of the scope of this paper, we give a brief overview of one method by which revocation could be implemented.

One revocation technique would require each attribute to contain a time frame within which it is valid. For instance, one method by which revocation could be implemented.

Before an attribute/user can be purged, the administrator must be notified, and the system administrator periodically releasing the latest version of attributes and periodically reissue user keying information. Removal of an attribute from this system would be accomplished by the administrator not releasing the latest version of the attribute. Similarly, revoking an attribute from an individual requires the administrator to withdraw the updated attribute in the user's private key. There are significant trade-offs between the load placed upon the administrator and the amount of time that can elapse before an attribute/user can be purged. We therefore leave more efficient solutions to future work.

### 7. RELATED WORK

Securing the sharing of information between groups is a fundamental problem that arises in numerous applications. Such applications include multilevel security, secure multicast, collaborative online communities, and distributed file systems. The fundamental importance of the secure exchange of information has resulted in a wide range of solutions.

Traditional access control mechanisms can be categorized into three groups: mandatory access control (MAC) [11], discretionary access control (DAC) [18, 28], and role-based access control (RBAC) [13, 27]. In MAC, an administrative mechanism enforces centralized access control on every object. Systems implementing DAC require the owner of an object to dictate policy. Under RBAC, a user’s role in an organization inherently dictates their ability to access and manipulate data. Each role in an RBAC system is associated with a set of permissions required to carry out that role. While these mechanisms are highly effective at controlling access for systems under a single administrative authority, they have been largely unsuccessful at providing the same for unconnected and distributed environments.

ABE can enforce access control policy in such environments because it cryptographically binds objects to their policies. Only users possessing the requisite set of attributes are able to view and/or manipulate data. The ability to make policy portable through cryptography is not new. Several works have attempted to use a public key infrastructure (PKI) [15] or secure group communications mechanisms [7, 20] to provide similar access control mechanisms. The difficulty with applying standard cryptographic techniques is that they are designed to control access to single groups. In real systems, however, users are often members of multiple groups. Unique keys must therefore be assigned or negotiated for each of the subgroups for which a user is a member.

Such solutions do not scale for complex organizations with significant communication across groups. In contrast, users in ABE-based systems automatically belong to every possible attribute subset group without the need for additional keying.

By using cryptographic mechanisms that are in and of themselves able to express complex policies, ABE-based systems become a highly practical means of ensuring the efficient and secure exchange of information between groups.

### 8. CONCLUSION

This paper has presented a novel secure information management architecture and implementation. We extended existing constructions for attribute-based encryption (ABE) and promoted them as a practical systems building block. The needs of complex attribute applications were met via the introduction of a policy system and an associated implementation for its enforcement. We illustrated the infrastructure through the creation and performance evaluation of two applications: a HIPAA compliant distributed file system and a social network. A further empirical study shows that a careful selection of parameters and use of construction optimizations can lead to significant cost savings. These analyses demonstrate that our approach is an attractive solution for securely managing information in large, loosely-coupled, distributed systems.

### 9. REFERENCES


Figure 12: Components of attribute-based cryptosystem API.

APPENDIX

A. ABE SYSTEMS DESIGN & ISSUES

Figure 12 illustrates the architecture of ABE systems built using our ABE API. Specifically there are two main components: the ABE API and the application code. ABE API – We have created the ABE API to enable rapid development of systems and applications which use attribute-based cryptography. Our API uses the PBC library [19] to implement our attribute-based cryptography. This C language API has been specifically designed to enable a programmer with no knowledge of ABE to quickly write applications; the complex cryptography inherent to ABE is entirely handled by the API.

For didactic purposes we present the API as four distinct modules: attribute-based cryptography, standard cryptography, serialization, and utility functions. Attribute-based Cryptography – The majority of application level code interacts with the API through the attribute-based cryptography component. This module was specifically designed for ease of use, consisting of seven simple functions: SetupSystem, CreateUser, NewAttribute, GiveAttribute, Key_Generation, Encryption, and Decryption.

The SetupSystem function creates and initializes a new attribute-based cryptosystem. Specifically, this instantiates two key structures: global_params and authority_priv. global_params contains global parameters required to perform encryption and decryption operations. authority_priv contains the master secret, from which all attribute keys are defined. authority_priv must be kept secret in order to ensure the security of the system.

SetupSystem must be given pbc_param_file_name, the name of an XML file defining an elliptic curve from which all of the API’s ABE cryptography is formulated. Included with the API are two such parameter files, a_param.xml (Supersingular curve) and c159_param.xml (MNT curve). Supersingular curves are optimized for fast cryptographic pairings, and MNT curves are optimized to result in small cryptographic group elements.

The nature of ABE cryptography is such that every ci-
phertext in a given cryptosystem is of a fixed length $n$. The user can specify what this length is by providing the API with $\text{ct}$.\_\text{len}$.

To increase the flexibility of the API, $\text{Setup.\_System}$ creates several “default” attributes. The default attributes can be included in a ciphertext to take the place of non-default attributes, enabling the user to create ciphertexts with less than $\text{ct}$.\_\text{len}$ attributes.

The $\text{New.\_Attribute}$ function is used to add a new attribute, whose name is specified by $\text{att}$.\_\text{name}$, to the universe of attributes in the system. Upon completion of this function the new attribute’s name and the hash of its name can be made publicly available. At this point the new attribute can be used for encryption operations.

The $\text{Create.\_User}$ function adds a user named $\text{user}$.\_\text{id}$ to the system. This function instantiates $\text{user}$.\_\text{id}$, a structure which stores the user’s name, the user’s default and non-default attribute information, and a polynomial. Each user is given a unique polynomial. Tying each user’s per-attribute keying information to their polynomial prevents users from colluding in order to attain more attributes.

The $\text{Give.\_Attribute}$ function is used to give a user a new attribute. Specifically this function is used to update the user’s attribute data structures and does not generate any keying information. The $\text{Key.\_Generation}$ function is used to create a user’s keying information based on the attributes that they possess. Keeping key generation separate enables the $\text{Give.\_Attribute}$ function to be executed with fewer trust assumptions than is needed to perform the $\text{Key.\_Generation}$ function.

The $\text{Encryption}$ function is used by a user to create a new ciphertext, $\text{ciphertext}$. The user specifies, $\text{message}$, a string they would like to encrypt and, $\text{uid}$, a list of attributes that they would like to encrypt to. The user can encrypt with at most $\text{ct}$.\_\text{len}$ attributes. The API will pad the ciphertext with as many default attributes as is necessary to make the ciphertext contain a total of $\text{ct}$.\_\text{len}$ attributes. A list of the attributes used to perform encryption are included in each ciphertext in order for the party performing decryption to know which attributes are required to decrypt the message.

Encryption is significantly more complicated than the API’s function calls would seem to indicate. Specifically, the ABE constructions mandate that a ciphertext’s payload must be a group element. To enable ABE to carry non-group element payloads we use the Key Encapsulation Mechanism (KEM). In our API, KEM takes a group element payload and uses SHA-1 to convert it into a HMAC key and an AES key. The AES key is then used to encrypt the user’s message.

Decryption decrypts a ciphertext encrypted by the $\text{Encryption}$. This process begins with the decrypting party verifying that they have the required attributes. The party performing decryption will then use their attributes to decrypt the ciphertext in order to obtain the AES and HMAC key. The party will then use the HMAC key to verify the ciphertext. If the ciphertext can be verified, then the AES key will be used to decrypt the actual payload.

Standard Cryptography – In addition to the attribute-based cryptography we have also used standard cryptographic tools. The implementation of these tools are contained in the $\text{crypto}\_\text{utility}$ and KEM code. The $\text{crypto}\_\text{utility}$ code implements some of the low level cryptographic operations required by ABE. KEM implements all of the operations required to enable ABE to encrypt non-group member payloads.

Serialization – The serialization routines enable the API data structures to be written out to disk for long term storage. There are two different implementations of this functionality. Serialization stores API data structures into byte-encoded files. XMLSerialization stores API data structures into XML files. XMLSerialization is human readable, has better platform independence, and is more fault tolerant. Serialization results in slightly less disk space.

Utility Functions – The utility functions are a group of functions that increase the ease of programming with the attribute-based cryptography API. Included in the utility routines are functions that print API data structures and conversion routines.

B. ATTRIBUTE-BASED ENCRYPTION

For our system we use a variant of the Sahai-Waters Large Universe system [26](Section 6) which we now describe.

In this construction we will make use of a bilinear group $G$ of prime order $p$. The group will have an efficiently computable bilinear map $e: G \times G \rightarrow G_T$ that maps two elements from the bilinear group into an element of the “target group”. The salient feature of these groups is that if $g$ is a generator of $G$ then for all $a,b \in Z_p$ we have that $e(g^a, g^b) = e(g, g)^{ab}$. We refer the reader to the IBE paper of Boneh-Franklin [5] for more details on bilinear groups.

The Sahai-Waters constructions works by computing a bilinear map between $k$ components of the ciphertext with corresponding pieces of the private key. The result of these are interpolated using the secret sharing method of Shamir (in the exponent). We first define the following Lagrangian coefficients, which we will use in our construction, as the following function over $Z_p$:

$$\Delta_i, S(X) = \prod_{j \in S, j \neq i} \frac{x - j}{i - j}.$$ 

Additionally, we will assume all systems will work in some predetermined bilinear group $G$ of appropriate size.

The cryptosystem follows:

$\text{Setup}(k)$: The setup algorithms first chooses a random exponent $y \in Z_p$ and lets the public parameter be $Y = e(g, g)^y$ and the threshold value $k$. It keeps the public key and the secret exponent $y$ as the master key.

$\text{KeyGen}(\text{S, MK})$: Let $H : \{0, 1\}^* \rightarrow Z_p$ be a collision-resistant hash function and let $T : Z_p \rightarrow G$ be a function that we will model as a random oracle [4].

First let $\Gamma$ be the set defined as $\Gamma = \bigcup_{g \in S} H(s)$. The set $\Gamma$ is essentially the set of the hash of all attributes. (Note that since $H$ is collision-resistant $\Gamma$ should contain $|S|$ unique elements of $Z_p$.) Then the authority will choose a new random degree $k - 1$ polynomial $q(x)$ over $Z_p$ such that $q(0) = y$ and for all $i \in \Gamma$ the authority chooses a random $r_i$. Then for all $i \in \Gamma$ the private keys components are:

$$D_i = g^{i (i - 1)/2}, d_i = g^{r_i}.$$ 

$\text{Encrypt}(\text{M, S, PK})$: The encryption algorithm first computes the set $\Gamma' = \bigcup_{g \in S'} H(s)$. Next, it chooses a random exponent $t \in Z_p$. The ciphertext is output as:

$$C = (C' = MY^t, C'' = g^t, \{C_i = T(i)^t : i \in \Gamma'\}).$$
Notice that both the size of the ciphertext and the encryption time grows linearly with the size of the set $S$.

**Decrypt($C, S', S, SK$):** The decryption algorithm first computes the sets $\Gamma$ and $\Gamma'$ as before. If the size of the intersection $|\Gamma \cap \Gamma'| < k$ the algorithm aborts, this will occur if the overlap between the private key attribute set $S$ and the ciphertext set $S'$ is below the threshold $k$. Otherwise it chooses an arbitrary set $U$ such that $|U| = k$ and $U \subseteq \Gamma \cap \Gamma'$. For each $i \in U$ the decryptor computes a temporary value

$$A_i = \frac{e(D_i, C'')}{e(d_i, C_i)} = \frac{e(g^{\delta(i)T(i)}r_i, g)}{e(g^{\delta(i)T(i)}r_i^\Gamma, g)} = e(g, g)^{tq(i)}.$$

This computation gives $k$ shares of the polynomial $tq(i)$ in the exponent. Using polynomial interpolation the algorithm recovers the blinding value $e(g, g)^{yt}$ and divides it out by computing:

$$M = C' / (A_i^{\Delta_{i,U}(0)}) = C' / e(g, g)^{tq(0)} = C' / e(g, g)^{y} = M.$$

The decryption algorithm interpolates a polynomial in the exponent using Shamir’s [29] secret sharing method. However, since a new random polynomial is chosen for each private key, the system is secure against collusion attacks such that different users are unable to combine their separate attributes.

The difference between the construction given here and that of Sahai and Waters is in the computation of the function $T(i)$. In their construction there is a upper bound, $n$, on the number of attributes that can label a ciphertext which is set at setup. The setup function publishes values $t_1, \ldots, t_n$. The function $T(i)$ is computed as:

$$T(i) = g^{x_i \prod_{j=1}^{n+1} t_{j,N}^{\Delta_{i,N}(i)}}$$

where $N$ is the set $\{1, \ldots, n+1\}$.

It is easily seen that the number of exponentiations required to compute $T(i)$ is equal to $n + 1$ in the original Sahai and Waters construction. We drastically reduce the computation overhead replacing the computation of $T$ with a hash function as a random oracle. A simple argument shows that the random oracle can be “programmed” such that the simulation in the security proof of Sahai and Waters goes through. We refer the reader to the literature [4,8] for further discussion on the random oracle model. In Section 6 we experimentally compare implementations of the original Sahai and Waters construction with our variant.