# Guided Multiple Hashing: Achieving Near Perfect Balance for Fast Routing Lookup 

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#### Abstract

The routing and packet forwarding function is at the core of the IP network-layer protocols. The throughput of a router is constrained by the speed at which the routing table lookup can be performed. Hash-based lookup has been a research focus in this area due to its $O(1)$ average lookup time, as compared to other approachs such as trie-based lookup which tends to make more memory accesses. With a series of prior multihashing developments, including $d$-random, 2 -left, and $d$-left, we discover that a new guided multi-hashing approach holds the promise of further pushing the envelope of this line of research to make significant performance improvement beyond what today's best technology can achieve. Our guided multi-hashing approach achieves near perfect load balance among hash buckets, while limiting the number of buckets to be probed for each key (address) lookup, where each bucket holds one or a few routing entries. Unlike the localized optimization by the prior approaches, we utilize the full information of multi-hash mapping from keys to hash buckets for global key-to-bucket assignment. We have dual objectives of lowering the bucket size while increasing empty buckets, which helps to reduce the number of buckets brought from off-chip memory to the network processor for each lookup. We introduce mechanisms to make sure that most lookups only require one bucket to be fetched. Our simulation results show that with the same number of hash functions, the guided multiplehashing schemes are more balanced than $d$-left and others, while the average number of buckets to be accessed for each lookup is reduced by $\mathbf{2 0 - 5 0 \%}$.


## I. Introduction

Hashing provides an efficient mechanism to organize, look up, and update a large information table based on a key value, which has been used in a wide variety of network applications [1], [2], [3]. In particular, hash-based lookup has been an important research direction on routing and packet forwarding, among the core functions of the IP network-layer protocols. While there are other alternative approaches for routing table lookup such as trie-based solutions, this paper's focus is on hash-based solutions, which have the advantages of simplicity and $O(1)$ average lookup time, whereas trie-based lookup tends to make much more memory accesses.

Single-hashing however suffers from the collision problem, where multiple keys are hashed to the same bucket and cause uneven distribution of keys among the buckets. It takes variable delays in looking up keys located in different buckets. For hash-based network routing tables [1], [2], [3], [6], it is critical
to perform fast lookup for the next hop routing information. In today's backbone routers, routing tables are often too big to fit into on-chip memory of a network processor. As a result, off-chip routing table access becomes the bottleneck for meeting the increasing throughput requirement on highspeed Internet [7], [8]. The unbalanced hash buckets further worsen the off-chip access. Today's memory technology is more efficient to fetch a contiguous block (such as a cache block) at once than individual data elements separately from off-chip memory. A heavy-loaded hash bucket may require two or more memory accesses to fetch all its keys. However, in order to accommodate the most-loaded bucket for a constant lookup delay, fetching a large memory block which can hold the highest number of keys in a bucket increases the critical memory bandwidth requirement, wastes the memory space, and lowers the network throughput [2], [6], [9], [10].

Methods were proposed to handle the hash collision problem for balancing the bucket load by reducing the maximum number of keys in a bucket among all buckets. One approach is to use multiple hashing such as $d$-random [11] which hashes each key to $d$ buckets using $d$ independent hash functions and stores the key into the least-loaded bucket. The 2-left scheme [1], [9] is a special case of $d$-random where the buckets are partitioned into left and right regions. When inserting a key, a random hash function is applied in each region and the key is allocated to the least-loaded bucket (to the left in case of a tie). The multiple-hashing approach balances the buckets and reduces the fetched bucket size for each key look up. However, without the knowledge of which bucket that a key is located, $d$-random (d-left) requires probing all $d$ buckets. As the bottleneck leans on the off-chip memory access, accessing multiple buckets slows down the hash table access and degrades the network performance [8], [10].

To remedy probing $d$ buckets, extended Bloom Filter[3] uses counters and extra pointers to link keys in multiple hashed buckets to avoid lookups of multiple buckets. However, it requires key replications and must handle complex key updates. The recently proposed Deterministic Hashing [6] applies multiple hash functions to an on-chip intermediate index table where the hashed bucket addresses are saved. By properly setting up the bucket addresses in the index table, the hashed buckets can be balanced. This approach incurs space
overhead and delays due to indirect access through the index table. In [10], an improved approach uses an intermediate table to record the hash function IDs, instead of the bucket addresses to alleviate the space overhead. In addition, it uses a single hash function to the index table to ease the update complexity. However, with limited index table and hashing functions, the achievable balance is also limited. In another effort to avoid collision, the perfect hash function sets more rigid goal to achieve one to one mapping between keys and buckets. It accomplishes the goal using complex hash functions encoded on-chip with significant space and additional delays [12], [13]. It also requires changes in the encoded hash function upon a hash table update.

In this paper, we propose a new multiple-hashing method, named guided multiple hashing, denoted as d-ghash, for balancing the hash buckets while limiting the number of bucket access for key lookup. Unlike d-random [11], which places each key into the least-loaded bucket progressively for localized balance among the hashed buckets, $d$-ghash achieves global balance among all buckets based on the complete placement information after all keys are hashed into buckets $d$ times using $d$ independent hash functions. In other words, the decision of individual key placement is deferred until the knowledge of the complete placement is available.

There are two criteria for placing the keys into buckets. First, the maximum number of keys in buckets should be as small as possible. We define the optimal bucket load as the lower bound on the maximum number of keys in a bucket, denoted as $\Omega_{p}$. Clearly, $\Omega_{p}=\lceil n / m\rceil$, where $n$ is number of keys and $m$ is the number of buckets. $d$-ghash ensures that the maximum number of keys in a bucket is close to $\Omega_{p}$. In order to achieve this, it assigns each key to one of the hashed buckets and removes duplicates from other buckets after the complete placement information is known. Reassignment may be needed for buckets exceeding the optimal load. Second, in order to reduce bucket access, $d$-ghash creates as many empty buckets as possible, called companion empty buckets, or $c$ empty buckets. A small array called the empty array is built to indicate if a bucket is a c-empty bucket. As it is unnecessary to access an empty bucket to find a key, the number of the bucket accesses for a lookup is reduced.

To further reduce lookup time, a target array is established to guide the lookup of a key. Basically it records the hash function ID that each key uses to locate the assigned bucket. We call the $d$-ghash scheme with a target array the enhanced $d$-ghash, to separate from the base d-ghash that does not have the target array. Upon looking up a key, the hash function ID is retrieved from the target array based on the key value, and then the corresponding hash bucket is fetched from the offchip memory. Only when the key is absent from this bucket, the remaining buckets are examined. Collision may happen in the target array. Using a sufficient target array, however, it has limited impact as we will discuss in Section IV.

In summary, the proposed $d$-ghash makes two important contributions. First, $d$-ghash achieves near perfect balance
using multiple hash functions. By deferring individual key assignment, it is able to achieve global balance. Second, $d$ ghash further utilizes and arranges the available bucket space to reduce the number of buckets required to look up a key. It creates as many empty buckets as possible when removing duplicate keys during the assignment to avoid probing such empty buckets. It also records the hash function to the bucket where the key is located to guide the lookup. These two techniques together can reduce the bucket access significantly in comparison with other multiple-hashing approaches.

Performance evaluation shows significant advantages of using the guided multiple-hashing approach. Given different ratios of the number of keys, buckets, and hash functions, $d$-ghash out-balances existing single-hash and d-left hashing schemes by producing smaller bucket load among all buckets. For example, given 200,000 keys, $d$-ghash can achieve perfect balance with a maximum of one key per bucket using 4 hash functions and 275,000 buckets while other scheme cannot obtain the perfect balance even with 500,000 buckets. Meanwhile, enhanced d-ghash has the ability to reduce $20-50 \%$ of the bucket access for a key lookup in comparison with the $d$ left approaches. We also show that a similar conclusion can be made when applying $d$-ghash to several real network routing tables [7]. Results show that enhanced 4-ghash reduces 37$50 \%$ bucket access of 4-left for a key lookup and enhanced 2ghash reduces $20-23 \%$ bucket access of 2 -left. Simulations and experiments based on real routing table trace also demonstrate the robustness of our algorithm.

The remaining paper is organized as follows. We will first provide the background and motivation of using multiple hash functions to accomplish specific objectives in the hash table in Section II. Section III describes the detailed algorithm of the proposed guided multiple-hashing method. This is followed by the performance evaluation and comparison of the state-of-the-art hashing methods in Section IV. Section V provides a routing table experiment based on real Internet traces. Related works are give in Section VI and Section VII concludes the paper.

## II. Background and Motivation

In this section, we describe the challenges of a hashingbased information table using a single hash function. We also bring up the motivation and applications of using a multiplehashing approach for organizing and accessing a hash table. It is well-known that hash collision is an inherent problem when a single random hash function is used, which causes uneven distribution of keys among the hash buckets in a nondeterministic fashion. The multiple-hashing technique, on the other hand, uses $d$ independent hash functions to place a key into one of $d$ possible buckets. The criteria of selecting the target bucket for placement is flexible and can be controlled to accomplish a specific objective. One well-known objective of using multiple hash functions is load balancing, i.e. to balance the keys in the buckets [1], [5], [9], [11], [14], [15]. To achieve this objective with two hash functions, for example, each key
is placed in the bucket with smaller number of keys. The power of balancing the buckets with two choices was reported in [9]. In comparison with single hashing, the maximum number of keys in a bucket can be reduced from $((1+o(1)) \ln n / \ln \ln n)$ to $(\ln \ln n / \ln 2+O(1))$ when placing $n$ keys into $n$ buckets [11]. However, due to the two choices, the exact location of a key is unknown and both buckets need to be probed to look up a key.

Another known objective of multiple hashing sets an opposite criteria for reducing the fill factor of the hash buckets [9], [16]. The fill factor is measured by the ratio of nonempty buckets. Instead of placing a key in the bucket with smaller number of keys for load balancing, this approach places the key in the bucket with non-zero number of keys. The objective of this placement is to maximize the amount of empty buckets. One potential application is to apply the low fill-factor hashing method to a Bloom filter [9], [16]. A Bloom filter [17] is an efficient bit-map data structure $B[1], \ldots, B[m]$ with $m$ bits for performing membership queries of a set of $n$ keys, $S=\left\{x_{1}, x_{2}, \ldots, x_{n}\right\} . B[1], \ldots, B[m]$ are set to zero initially. Each key $x$ is hashed to $B k$ times using $k$ independent hash functions $H_{1}, H_{2}, \ldots, H_{k}$ and set $B\left[H_{i}(x)\right]=1$ where $1 \leq i \leq k$. When querying whether $y$ is a member in the key set, we can check if all $B\left[H_{i}(y)\right]=1,1 \leq i \leq k$. Any zero in $B\left[H_{i}(y)\right]$ guarantees $y$ is not a member. A false positive occurs when $y$ satisfies the membership query, but is actually not a member. With more zeros remained in the Bloom filter, the critical false positive rate can be reduced. To create more zeros in establishing the Bloom filter, however, multiple sets of hash functions are needed for different keys since all the hashed $k$ bits for each key must be set during the setup of the Bloom filter. Therefore, the multiple hashing concept is actually applied for choosing a set of hash functions out of multiple groups to maximize the number of zeros in the Bloom filter after recording $k$ ' 1 's for every key.

To demonstrate the power of multiple hashing in accomplishing different objectives for the hash table, we compare the simulation results of four hashing schemes: single hashing (single-hash), 2-hash with load balancing (2-left), 4-hash with load balancing (4-left), and 2-hash with maximum zero buckets (2-max-0). We simulate 200,000 randomly generated keys to be hashed to 100,000 buckets. The distribution of keys in buckets is plotted in Figure 1. We can observe substantial differences in the key distribution among the four hashing schemes. The maximum number of keys in a bucket reach to ten for single-hash and 2-max-0. Meanwhile, 2-max-0 produces 2.5 times empty buckets than single-hash does. 2left and 4-left are more balanced with four and three as the maximum numbers of keys in a bucket, respectively. It's easy to see that increasing the number of hash functions from two to four helps improving the balance.

In this paper, we investigate a new objective for allocating keys in hash buckets using the multiple-hashing approach. The goal is to balance the keys in the buckets to reduce the bucket load so that the amount of data in a bucket to be fetched

TABLE I
NOTATION AND DEFINITION

| $n$ | Total number of keys |
| :---: | :--- |
| $m$ | Total number of buckets |
| $B[i]$ | Set of keys in $i$-th bucket |
| $v(B[i])$ | Number of keys of the $i$-th bucket |
| $s$ | indices of the buckets in $B$ sorted in ascending order <br> of $v(B[i])$ |
| $H_{i}$ | $i$-th hash function |
| $\Omega_{p}$ | Optimal bucket load, $\lceil n / m\rceil$ |
| $\Omega_{a}$ | Achievable bucket load |
| $n_{u}$ | Total number of keys in under-loaded buckets <br> (bucket load less than $\left.\Omega_{a}\right)$ |
| $b_{u}$ | Number of under-loaded buckets |
| $\theta$ | Memory usage ratio |



Fig. 2. A simple d-ghash table with 5 keys, 8 buckets and 2 hash functions. (The shaded bucket is a c-empty bucket. The final key assignment is as illustrated.
from off-chip memory is minimized. In addition, we want the average number of buckets to be probed for looking up a key to be as small as possible to alleviate the disadvantage of multiple hashing, which needs to probe multiple buckets.

## III. d-ghash Hashing Scheme

In this section, we describe the detailed algorithms of the guided multiple-hashing scheme that consists of a setup algorithm, a lookup algorithm, and an update algorithm. Assume we have $m$ buckets $B_{1}, \ldots, B_{m}$ and $d$ independent hash functions $H_{1}, \ldots, H_{d}$. Each key $x$ is hashed and placed into all $d$ buckets, $B_{H_{i}(x)}, 1 \leq i \leq d$. The set of keys in bucket $B_{i}$ is denoted by $B[i]$, and the number of keys in bucket $B_{i}$ is $v(B[i]), 1 \leq i \leq m$. The bucket load $\Omega_{a}$ is defined as the maximum number of keys in any bucket. We define the memory usage ratio as: $\theta=\left(\Omega_{a} \times m\right) / n$ to indicate the memory requirement of the hash table. Other terminologies are self-explanatory and are listed in Table I. For better illustration of d-ghash, we use a simple hashing table with 5 keys and 8 buckets. All keys are hashed to the buckets using two hashing functions, where buckets $B_{0}$ to $B_{7}$ have $1,0,1,2,3,0,1,2$ keys as indicated by the arrows in Figure 2.

## A. The Setup Algorithm

Since the objective is to minimize the bucket load while approaching to a single bucket access per lookup, the setup algorithm needs to satisfy two criteria: (1) achieving near


Fig. 1. Distribution of keys in buckets of four hashing algorithms.
perfect balance, and (2) maximizing the number of $c$-empty buckets. Recall that a c-empty bucket serves as a multiplehashing target of one or more keys, but the key(s) is placed into other alternative buckets that make c-empty bucket access unnecessary.

Given $n$ keys and $m$ buckets, a perfect-balance hashing scheme achieves optimal bucket load $\Omega_{p}=\lceil n / m\rceil$. However, the perfect balance may not be achieved under our or other multi-hashing schemes because even with multiple hashing, some buckets may still probabilistically be under-loaded, i.e. zero or less than $\Omega_{p}$ keys are hashed to the bucket. And this translates to some other buckets being squeezed with more keys. Increasing the number of hash functions reduces under-loaded buckets and helps approaching to the perfect balancing. Our simulation shows that with 4 hash functions, the achievable balance is the same as or very close to $\Omega_{p}$.

The first step in the setup algorithm is to estimate $\Omega_{a}$, the achievable balance. The idea is to count the number of underloaded buckets and the number of keys inside. If the remaining buckets can not hold the rest of the keys with $\Omega_{a}$ keys in each bucket, we increase $\Omega_{a}$ by one. Details are shown in Algorithm 1. We then use $\Omega_{a}$ as the benchmark bucket load for key assignment. We sort all buckets in $B$ resulting a sorted index array, $s$, such that $v(B[s(i)]) \leq v(B[s(i+1)], 1 \leq i \leq m-1$. In the simple example of Figure $2, \Omega_{a}=\Omega_{p}=\lceil n / m\rceil=1$.

The next step is key assignment, which consists of two procedures, creating c-empty buckets, and balancing key assignment. For creating $c$-empty buckets, the procedure removes duplicate keys starting from the most-loaded buckets to maximize their services as companion buckets to reduce the bucket access. A key can be safely removed from a bucket if it exists in other bucket(s). As shown in Algorithm 2, the procedure goes through all buckets whose initial load are greater than $\Omega_{a}$ and tries to remove keys from them. In the illustrated example

```
Algorithm 1: Achievable \(\Omega_{a}\)
    initialize \(\Omega_{a}=\lceil n / m\rceil\);
    \(n_{u}=0 ; b_{u}=0 ; S=\emptyset ;\)
    for \(i=1\) to \(m\) do
        // count buckets and keys in under-loaded buckets
        if \(v(B[i])<\Omega_{a}\) then
        \(n_{u}=n_{u}+\) no. of distinct keys in \(B[i]\) not in \(S\);
            \(b_{u}=b_{u}+1\);
            add distinct keys in \(B[i]\) to \(S\);
    \(n_{k}=n-n_{u} ; b_{k}=\left\lceil n_{k} / \Omega_{a}\right\rceil ;\)
    if \(b_{u}+b_{k}>m\) then
        \(\Omega_{a}=\Omega_{a}+1\); go back to (2);
```

    Algorithm 2: Create \(c\)-empty buckets
    set count \((\) key \()=d\) for all keys;
    repeat based on the sorted list \(s\)
        find next most-loaded bucket with largest \(v(B[s(i)])\);
        if \(v(B[s(i)])>\Omega_{a}\) then
            for each key \(x\) in bucket do
                        if \(\operatorname{count}(x)>1\) then
                        delete key in the bucket;
                        \(\operatorname{count}(x)=\operatorname{count}(x)-1\);
    in Figure 2, all 3 keys in $B_{4}$ are successfully removed and $B_{4}$ becomes empty. Next, we check $B_{3}$ and $B_{7}$, each of which has 2 keys. Note that both $K_{2}$ and $K_{4}$ in $B_{3}$ can be removed if $B_{3}$ is emptied first. As a result, $K_{2}$ and $K_{3}$ cannot be removed from $B_{7}$ and exceed $\Omega_{a}$. All buckets with the bucket load exceeding $\Omega_{a}$ will be a target for reallocation as described next.
After emptying the buckets, the key assignment procedure assigns each key to a bucket starting from the least-loaded bucket as described in Algorithm 3. Once a key is assigned, its duplicates are removed from the remaining buckets. During

```
Algorithm 3: Balanced key assignment
    // Assign keys:
    repeat based on the sorted list \(s\)
        find next unassigned bucket with smallest \(v(B[s(i)])\);
        if \(v(B[s(i)]) \leq \Omega_{a}\) then
            assign all keys in \(B[s(i)]\);
            remove keys in \(B[s(i)]\) from other buckets;
            update \(s\);
    until all buckets \(B[i]\) are either assigned or \(v(B[i])>\Omega_{a}\);
    // Reassign overflow buckets:
    repeat based on the sorted list s
        find next maximum load bucket \(v(B[s(i)])>\Omega_{a}\);
        for each key \(x\) in \(B[s(i)]\) do
            if exists \(i\), such that \(0<v\left(B\left[H_{i}(x)\right]\right)<\Omega_{a}\)
                then reassign \(x\) to \(B\left[H_{i}(x)\right]\);
            else choose the smallest of all \(v\left(B\left[H_{i}(x)\right]\right)\)
                then reassign \(x\) to that bucket;
    if all buckets \(B[i]\) satisfies \(v(B[i]) \leq \Omega_{a}\), then stop;
    if any bucket with \(v(B[i])>\Omega_{a}\)
        go back to (11) and iterate \(r\) times;
    if exceed \(r\) times then
            \(\Omega_{a}=\Omega_{a}+1 ;\) redo key assignment;
```

the assignment, buckets with more than $\Omega_{a}$ keys are skipped in order to maintain the achievable balance. A re-assignment of the buckets with load greater than $\Omega_{a}$ is necessary after all the buckets are assigned. During re-assignment, the keys in the overflow buckets are attempted to be relocated to other buckets. In our experiment, we use Cuckoo Hashing [5] to relocate keys from an overflow bucket to an alternative bucket using multiple hashing functions. If all alternative buckets are full, an attempt is made to make room in the alternative buckets. For simplicity, however, such attempts stop after $r$ tries, where $r$ can be any heuristic number. A larger r brings better balance at the expense of longer setup time. In the illustrated example, $K_{2}$ in $B_{7}$ is relocated to $B_{3}$ to reduce the bucket load of $B_{7}$, hence, the optimal load is achieved.

In case that the perfect balance is not achievable, $\Omega_{a}$ is incremented by one and the key assignment procedure repeats. It is important to note that the priority of the key assignment is to achieve perfect balance. Therefore, the keys that are previously removed from an empty bucket can be reassigned back in order to accomplish the perfect balance such that the number of keys are less than or equal to $\Omega_{a}$ in all buckets. It is also important to know that in order to reduce the bucket load, we can decrease the ratio of $n / m$, i.e. to increase the number of buckets for a fixed number of keys. However, increasing the number of buckets inflates the memory space requirement as the memory usage ratio can be calculated by $\theta=\left(\Omega_{a} \times m\right) / n$ for a constant bucket size for efficient fetch of a bucket from off-chip memory.

```
Algorithm 4: Key lookup
    the lookup key \(x\) is hashed using \(\left\{H_{i}\right\}\);
    use \(\left\{H_{i}(x)\right\}\) as indices to get \(d\) bits from empty array;
    let \(e=\) number of 1 s in \(d\) bits from the empty array;
    retrieve a hash function ID \(t\) from the target array;
    if \(e=1\) then fetch the only non-empty bucket;
    else if \(e>1\) then
        lookup the bucket \(B\left[H_{t}(x)\right]\);
        if \(x\) is not found then
            lookup remaining nonempty buckets till found
```


## B. The Lookup Algorithm

In order to speed up the lookup of keys, we introduce a data structure called the empty array, which is a bit array of size $m$ indicating whether a bucket is empty or not. If a bit in the empty array is ' 0 ', it means that the corresponding bucket is empty; otherwise it is not empty. Upon looking up a key $x$, the bits of indices $H_{1}(x), \ldots, H_{d}(x)$ in the empty array are checked. If there is only one of the hashed buckets is non-empty, we simply fetch that bucket and thus complete a lookup. If there are two or more non-empty buckets, we access them one by one until we find the key. Algorithm 4 gives the details. In the worst case, all $d$ bits are ones and $d$ buckets are examined before we find the key. As discussed above, creating c-empty buckets helps reduce bucket accesses per lookup, thus alleviates the lookup cost.
To further enhance our algorithm, we introduce another data structure, the target array, to record the hash function ID once a key is hashed to two or more non-empty buckets. To separate from the algorithm described above, we call it enhanced $d$ ghash algorithm. The algorithm only using the empty array is called base d-ghash algorithm. The recorded ID indicates the bucket that the key is most-likely located. The empty array has $m$ bits while the size of the target array varies depending on the number of keys. Suppose $m=200 \mathrm{~K}$, and we use $200 \mathrm{~K}-$ entry target array, then the empty array takes 25 KB and target array 25 KB for enhanced 2 -ghash and 50 KB for enhanced 4ghash. These two small arrays can be placed on chip for fast access. Multiple keys may collide in the target array. When a collision occurs, the priority of recording the target hashing function is given to the key which hashes to more non-empty buckets. Given a fixed number of keys, we can adjust the number of buckets ( $m$ ) and hash functions ( $d$ ) to achieve a specific goal of the bucket size and the number of buckets to be fetched for looking up a key. More discussions will be in Section IV.

## C. The Update Algorithm

There are three common types of hash table updates: insertion, deletion, and modification. It is straightforward to delete or to modify a key in the hash table. For deletion, the key is probed first by fetching the bucket from off-chip memory. The key is then removed from the bucket before the bucket is written back to memory. If the key is the last one in the
bucket, the corresponding bit in the empty array is set to zero. For modification of the associated record of a key, the key and its associated record are fetched. The new record replaces the old one before the bucket is written back to memory. Those two types of updates do not involve the modification of the target array.

The key insertion is slightly complicated. All hashed buckets are probed and the key is inserted into the least-loaded, nonempty bucket with the number of keys $<\Omega_{a}$. If all non-empty buckets are full, the key is inserted into an empty bucket if it is also hashed. The empty array is updated accordingly. In case that all hashed buckets are full, the Cuckoo Hashing is applied to make a room for the new key, i.e., "rehashing" a key in one of the hashed buckets to another alternative bucket. During key relocations, both the empty and the target arrays are updated accordingly.

There are two options in case a key cannot be inserted without breaking the property of $v(B[i]) \leq \Omega_{a}$, i.e., all its hashed/rehashed bucket loads are greater than or equal to $\Omega_{a}$. First, set $\Omega_{a}=\Omega_{a}+1$ and insert the key normally; Second, initiate an off-line process to re-setup the table. Normally, the possibility that a key cannot be inserted is small, and we should use the second option to prevent the bucket size from growing fast. However, if this operation happens very frequently, it implies that most of the buckets are "full", i.e. the average number of keys in buckets are approaching $\Omega_{a}$. In this case we should use the first option. By increasing the maximum load by one, all buckets gain one extra space to store another key.

## IV. Performance Evaluation

The performance evaluation is based on simulations for seven hashing schemes: single-hash, 2-left, 4-left, base 2ghash, enhanced 2-ghash, base 4-ghash, and enhanced 4ghash. Note that we do not include $d$-random in the evaluation, because it is outperformed by d-left both in terms of the bucket load and the number of bucket accesses per lookup. We simulate 200,000 randomly generated keys to be hashed into 100,000 to 500,000 buckets. To test the new multiple hashing idea, we adopt the random hash function in [18] which uses a few shift, or, and addition operations on the key to produce multiple hashing results. For relocation, we try to relocate keys in no more than ten buckets to other alternative buckets in the Setup Algorithm and no more than two in the Update Algorithm. We first compare the bucket load and the average number of bucket accesses per lookup by varying $n / m$. Then we normalize the number of keys per lookup based on the memory usage ratios to understand the memory overhead for different hashing schemes. In addition, we demonstrate the effectiveness of creating c-empty buckets to reduce the bucket access. We also give a sensitivity study on the number of bucket accesses per lookup with respect to the size of the target array. Lastly, we evaluate the robustness of $d$-ghash scheme by using two simple probabilistic models.

Figure 3 displays the bucket loads of the hashing schemes.


Fig. 3. Bucket loads for the five hashing schemes. The enhanced d-ghash scheme and base d-ghash scheme has the same bucket load.

Note that enhanced d-ghash and base d-ghash have the same bucket load. The only difference between the two is that enhanced d-ghash uses a target array to reduce the number of bucket accesses per lookup. The results show that $d$-ghash has the least bucket load, and hence achieves best balance among the buckets. This is followed by $d$-left. More hash functions improve the balance for both $d$-ghash and d-left. With 275,000 buckets, 4-ghash accomplishes perfect balance with the bucket load of a single key. No other simulated scheme can achieve such balance with up to 500,000 buckets. 2-ghash performs slightly better than 4 -left as the former needs 150,000 buckets to reduce the bucket load to two keys while the latter requires 175,000 buckets. This result demonstrates the power of $d$ ghash in balancing the keys over that of d-left. The single-hash scheme is the worst. The bucket load is six even with 500,000 buckets. Note that bucket load is an integer, but we slightly adjust the integer values to separate the curves of different schemes for easy read.

In Figure 4, we evaluate the lookup efficiency of the seven hashing schemes. Single-hash only accesses one bucket per lookup. The $d$-left scheme looks up a key from the left-most bucket. In case that the key is not found, the next bucket to the right is accessed until the key is located. Since the key is always placed in the left-most bucket to break a tie, the number of bucket accesses per lookup is quite low, $1.68 \sim$ 2.36 for 4 -left and $1.27 \sim 1.44$ for 2 -left. The base 4 -ghash and base 2-ghash reduce the number of bucket accesses per lookup to $1.25 \sim 2.18$ and $1.11 \sim 1.44$ respectively with a 5$34 \%$ and $0-14 \%$ reduction. With a target array of $1.5 n$ entries, the enhanced 4-ghash and the enhanced 2-ghash can further reduce the number of bucket accesses per lookup to as low as $1.03 \sim 1.23$ and $1.01 \sim 1.11$ respectively with a $38-51 \%$ and $21-24 \%$ reduction.

It is interesting to see that the number of bucket accesses per lookup for $d$-ghash does not decrease continuously when the number of buckets increases. We can observe a sudden jump at $m=125,000$ and $m=275,000$ for 4 -ghash. This is due to the fact that the optimal bucket load drops from three to two when $m=125,000$ and from two to one when $m=275,000$. As the average number of keys per bucket is very close to the optimal bucket load, it is hard to create $c$ -


Fig. 4. Number of bucket accesses per lookup for d-ghash.


Fig. 5. Average number of keys per lookup based on memory usage ratio. empty buckets. Therefore, there are sudden decreases in the amount of cempty buckets at those two points. As a result, 4-ghash experiences more bucket access for key lookup. The same reason goes for 2-ghash.

In order to reduce the bucket load for a fixed number of keys, we can increase the number of buckets. However, increasing buckets inflates the memory space requirement. In Figure 5, we plot the average number of keys per lookup based on the memory usage ratio, where the average number of keys is the product of the bucket load and the average number of buckets per look up. The results clearly show the advantage of the $d$ ghash scheme. Enhanced 4-ghash accomplishes a single key per bucket with 275,000 buckets which are only $37 \%$ more than the number of keys. With slightly larger than one key per lookup, enhanced 4-ghash requires the least amount of memory to achieve close to one key access per lookup.

Besides the perfect balance, $d$-ghash creates $c$-empty buckets to maximize the number of keys hashing to empty buckets. Figure 6 shows the effectiveness of the c-empty buckets for reducing the bucket access. In this figure, $y$-axis indicates the average number of non-empty buckets that each key is hashed into. In comparison with $d$-left, $d$-ghash reduces nonempty buckets more significantly, resulting in smaller number of bucket access. For $d$-left, the number of non-empty buckets


Fig. 6. The average number of non-empty buckets for looking up a key. This parameter is the same for enhanced d-ghash scheme and base d-ghash scheme.


Fig. 7. Sensitivity of the number of bucket accesses per lookup for enhanced 4-ghash with respect to the target array size.
decreases as the number of buckets increases. This is due to the fact that $d$-left assigns each key to the least-loaded bucket. $d$ ghash, on the other hand, creates c-empty buckets by removing keys away from those buckets with more keys hashed into. As a result, there are fewer non-empty buckets for looking up each key. It is interesting to observe that the ability to create c-empty buckets depends heavily on the optimal bucket load and the ratio of keys and buckets. For example, when the number of buckets is 250,000 , the optimal bucket load is 2 for 200,000 keys that leaves plenty of room to create many c-empty buckets. However, when the number of buckets increases to 275,000 , the optimal bucket load drops to 1 that leaves little room for the c-empty buckets. Hence, the average number of non-empty buckets increases for each key to be hashed into.

Moreover, we show a sensitivity study of bucket access per lookup with respect to the size of the target array. We vary the size of the target array from $n$ to $2 n$ entries using enhanced 4-ghash and the result is shown in Figure 7. As expected, larger target array reduces the collision, resulting in a smaller number of bucket accesses. We pick $1.5 n$ entries as the target array size in earlier simulations which has the best tradeoff in terms of space overhead and bucket access per lookup.

Finally, we evaluate the robustness of our scheme. We first set up a table using 200,001 keys, 200,000 buckets, and 300,000 target array entries. The achievable bucket load $\Omega_{a}$ is 2 in this setting. We simulate two update models:


Fig. 8. Changes in the number of bucket accesses per lookup and rehash percentage for two update models using enhanced 4-ghash. Bucket accesses per lookup lines correspond to the left Y axis. Rehash percentage lines correspond to the right Y axis.
(1) Balanced Update: $33 \%$ insertion, $33 \%$ deletion, and $33 \%$ modification; and (2) Heavy Insertion: $40 \%$ insertion, 30\% deletion, and $30 \%$ modification. We run for 600 K updates and record the rehash percentage of all the update operations and the number of bucket accesses per lookup. The results are presented in Figure 8. The top two lines reflect the number of bucket accesses per lookup under Heavy Insertion model and Balanced Update model respectively. We notice increases for both lines. The number of bucket accesses per lookup increases continuously to 1.37 for Heavy Insertion, an increase of $25 \%$ than the original number. While for Balanced Update, the number first increases up to 1.25 and then drops to 1.21 , with an increase of $10 \%$ in the end. The bottom two lines are rehash percentages of the whole update operations. These two lines give a clear view that if the insertion is heavy, we will come across more rehashes. For Balanced Update, the rehash percentage stays almost the same at $0.5 \%$. There is a slight increase in Heavy Insertion rehash percentage. Since the rehash percentages for both models are less than $2 \%$ and the rehash operation involves keys in no more than two buckets, we believe $d$-ghash is able to handle these rehashes without incurring too much delay.

## V. Routing Table Experiment

Finally, we apply our algorithm to a real routing table application. We use five routing tables downloaded from the Internet backbone routers: as 286 (KPN Internet Backbone), as513 (CERN, European Organization for Nuclear Research), as 1103 (SURFnet, the Netherlands), as 4608 (Asia Pacific Network Information Center, Pty. Ltd.), and as 4777 (Asia Pacific Network Information Center) [7], with 276K, 291K, $279 \mathrm{~K}, 283 \mathrm{~K}, 281 \mathrm{~K}$ prefixes respectively after removing the redundant prefixes.

To handle the longest prefix matching problem, hash-based lookup adopts the controlled prefix expansion[19] along with other techniques [20], [21], [22]; it is observed that there are small numbers of prefixes for most lengths, and they can be


Fig. 9. Number of bucket accesses per lookup for experiments with five routing tables.
dealt with separately, for example, using TCAM, while other prefixes are expanded to a limited number of fixed lengths. Lookup will then be performed for those lengths. In this experiment, we expand the majority of prefixes (with lengths in the range of $[19,24]$ ) to two lengths: 22 bits and 24 bits. Assuming the small number of prefixes outside [19, 24] are handled by TCAM, we perform lookups against lengths 22 and 24 . Because there are more prefixes of 24 -bits long after expansion, we present the results for 24 -bit prefix lookup.

There are $159,444,159,813,159,395,159,173$ and 159,376 prefixes of 24 -bits long from the five routing tables, respectively. We use these prefixes to setup five tables separately and vary the number of buckets from 100 K to 250 K with a target array of 150 K entries.

We find the number of bucket accesses per lookup for $d$ ghash and d-left scheme. The results are obtained based on the average of the five tables. As shown in Figure 9, both base 4-ghash and base 2-ghash performs better than the respective $d$-left scheme. The maximum reduction rate for base 4 -ghash than 4 -left is about $36 \%$ when $m=210,000$ and base 2 ghash $12 \%$ when $m=250,000$. The average number of bucket accesses per lookup for enhanced 4 -ghash scheme is almost one bucket less than 4 -left, with up to $50 \%$ reduction. For enhanced 2-ghash, there is an average of $20 \%$ reduction over 2 -left. We also notice that there is a jump for 4 -ghash at $m=$ 220,000 and another one for 2-ghash at $m=130,000$. This is due to the change in $\Omega_{a}$, as mentioned before.
In the second experiment, we setup our hash tables with the routing table as 286 downloaded at Juanuary 1st, 2010 from [7] and use the collected update trace of the whole month of January, 2010 to simulate the update process. To make experiments simple, we also choose prefixes with the length of 24. There are 159,44424 -bit prefixes in the table. The update trace contains $1,460,540$ insertions and 1,458,675 deletions for those 24 -bit prefixes. We vary the number of buckets from 110 K to 150 K . For all these settings, the achievable bucket load $\Omega_{a}$ is 2 for enhanced 4 -ghash. We also use a fixed 150 K entry target array.

TABLE II
Routing table updates For enhanced 4-ghash


Fig. 10. Experiment with the update trace using enhanced 4-ghash. Bucket access per lookup line correspond to the left Y axis. Rehash percentage line correspond to the right Y axis.

As shown in Table II, if we use 110K buckets, we need a re-setup for the whole table. If we use 120 K buckets, we don't need a re-setup, but have to rehash $0.23 \%$ of the whole update operations, which is about $0.5 \%$ of the 1.4 million insertions. And if we increase the number of buckets, we will rehash less. When using 150 K buckets, we have close to $0.05 \%$ chance of rehash. We also show the lookup efficiency change in Figure 10 with $\mathrm{m}=150 \mathrm{~K}$. The update trace used has nearly the same number of insertions and deletions, which is similar to a Balanced Update model used in Section IV. We can view that the rehash percentage grows continually to $0.05 \%$. The number of bucket accesses per lookup increases and decreases through the update process, with a $7 \%$ increase in the end.

## VI. Related Work

Using multiple hash functions to place keys in buckets is known to better balance hash table buckets than using a single hash function. There are many studies using multiple hash functions to improve load balancing [1], [5], [9], [11], [14], [15]. The recent proposed Deterministic Hashing [6] uses multiple hash functions of each key to an on-chip intermediate index table where the hashed bucket addresses are saved. This approach incurs space overhead in building the index table and delays due to indirect access through the index table. In addition, it also faces complexity in handling key updates. In [10], an improved approach uses the same intermediate index table to balance the buckets. In contrast to these approaches which go through an intermediate table for balancing the buckets, d-ghash allocates keys directly into multiple hashed buckets and then remove duplicate keys to achieve a perfect balance. A minimal perfect hash function [13], [23] accomplishes a one-to-one collision-free mapping when the number of buckets matches the number of keys. The delay of searching for the perfect hash function as well as the difficulty in handling the complicated hash table update involved in the perfect hash
function itself make it difficult for practical use. The goal of the proposed $d$-ghash is not to create or use a minimal perfect hash function; instead, it accomplishes near-perfect balance among the hash buckets using a small number of simple hash functions.

## VII. Conclusion

A new guided multiple-hashing method, $d$-ghash is introduced in this paper. Unlike previous approaches which select the least-loaded bucket to place a key progressively, $d$-ghash achieves global balance by allocating keys into buckets after all keys are placed into buckets $d$ times using $d$ independent hash functions. $d$-ghash calculates the achievable perfect balance and removes duplicate keys to achieve this goal. Meanwhile, $d$-ghash reduces the number of bucket accesses for looking up a key by creating as many empty buckets as possible without disturbing the balance. Furthermore, d-ghash uses a table to encode the hash function ID for the bucket where a key is located to guide the lookup and to avoid extra bucket access. Simulation results show that $d$-ghash achieves better balance than existing approaches and reduces the number of bucket accesses significantly.

## VIII. Acknowledgment

This work is supported in part by the National Science Foundation under grant CNS 1115548.

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