Privacy-Preserving Multi-Point Traffic Volume Measurement Through Vehicle-to-Infrastructure Communications

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Abstract—Traffic volume measurement is critical in vehicular networks. Existing research on traffic volume measurement mainly focuses on single-point traffic statistics. In this paper, we switch our view from single-point to multi-point and study the important problem of privacy-preserving multi-point traffic volume measurement in vehicular cyber-physical systems (VCPSs), which complements the state of the art. While embracing automatic traffic data collection, which the VCPS provides through vehicle-to-infrastructure communications, we also need to accept the accompanying challenges: First, the privacy of all participating vehicles should be preserved as an inherent requirement of a VCPS; second, the measurement scheme should be efficient enough to fit today's large-scale vehicular networks. In this paper, we start from a novel scheme that measures traffic volume between two arbitrary points (locations) through variable-length bit array masking. Then, we extend the idea of variable-length bit array masking to address the more challenging problem of three-point traffic measurement and present a general framework to measure traffic among three or more locations. We also perform extensive simulations to demonstrate the superior performance, applicability, and scalability of our schemes.

Index Terms—Privacy, trajectory measurement, vehicular networks.

I. INTRODUCTION

TRAFFIC volume measurement is critical in vehicular networks and transportation engineering. In general, traffic volume statistics can be summarized into two categories: “single-point” statistics and “multi-point” statistics. Existing research on traffic volume measurement mainly focuses on single-point traffic statistics such as annual average daily traffic, which estimate the number of vehicles passing a specific point (geographical location) during some measurement period, and various prediction models [1]–[4] have been proposed to measure them using data recorded by roadside units (RSUs). Multi-point traffic statistics, by contrast, describe the number of vehicles traveling through multiple points (geographical locations) during a measurement period. Multi-point traffic statistics provide essential input to a variety of studies, such as estimating traffic link flow distribution for investment plan, calculating road exposure rates for safety analysis, and characterizing turning movements at intersections for signal timing determination [5]. In this paper, we switch our view from single-point to multi-point and study the important problem of privacy-preserving multi-point traffic measurement in vehicular cyber-physical systems (VCPSs), which complements the state of the art. Our goal is to utilize the VCPS for automatic traffic data collection through vehicle-to-infrastructure communications and measure multi-point traffic while preserving vehicles’ privacy.

Greatly advanced by new technologies in vehicular communications and networking [6]–[10], the VCPS has emerged as one of the most promising research areas in road networks. It integrates wireless communications and on-board computers into transportation systems to enhance road safety and improve driving experience [11], [12]. In particular, the IEEE has standardized dedicated short-range communications (DSRC) under IEEE 802.11p [13], which supports transmitting/receiving messages between vehicles and RSUs. A great advantage that the VCPS provides is automatic traffic data collection: Each vehicle simply transmits its ID as it passes each RSU. From the IDs collected by all RSUs, we can easily figure out the multi-point traffic data. However, this straightforward approach leads to serious privacy breaching as it also tracks the entire moving history of vehicles. As more and more people are concerned about their privacy, any traffic measurement scheme to be deployed in the VCPS should consider travelers’ privacy as its top priority. The transportation authorities from different countries have put forward a number of principles to protect travelers’ privacy. An example is the “anonymity by design” principle required by IntelliDrive from the United States Department of Transportation [14]. Keeping the privacy requirement in mind, it is clearly not acceptable to have the vehicles report their unique identifiers. Other permanently or temporarily fixed numbers also bare the potential of giving away the vehicles’ trajectory. Therefore, the challenge is to design a measurement scheme in which a...
vehicle never transmits any unique identifier or any fixed number for privacy protection, with the random and de-identified information that the vehicle submits still able to support the traffic measurement among multiple different locations.

However, limited research work exists in the literature to address this problem. The most related studies are that of Lou and Yin [15] and our previous work [16]–[19]. The work of Lou and Yin tries to infer “two-point” statistics from “single-point” data, but the high computation overhead limits its practicability. More importantly, it is not designed for multiple points. Google announced providing real-time traffic data service in Google maps [20], but their approach cannot assure the vehicle’s privacy since it uses the Global Positioning System and Wi-Fi in phones to track locations [21]. Our previous work [16] utilizes an encryption method to preserve vehicles’ privacy and measures two-point traffic based on the encrypted vehicle IDs. The computation efficiency is improved to $O(n_x n_y)$ for each pair of RSUs, where $n_x$ and $n_y$ denote the number of vehicles passing them, respectively. This is better than the work in [15], but the overhead is still too high for today’s large-scale road networks. Motivated by the works in [22] and [23], we propose a new approach in [17], which further improves the computation efficiency to $O(n_x + n_y)$ through the design of fixed-length bit arrays. However, the paper makes an unrealistic assumption about traffic similarity and uses bit arrays of equal length at different RSUs to encode the passing vehicles, such that the bit arrays from two RSUs can be bitwise compared to extract a statistical result for two-point traffic volume. The scheme works great when all RSUs observe comparable numbers of vehicles. However, in reality, the traffic volume at different RSUs greatly varies. For example, according to the 2012 yearly traffic volume report from the New York State Department of Transportation [24], major intersections in New York have hundreds of thousands of cars passing by every day, whereas light-traffic intersections only have a few hundred cars passing by during the same period. Considering this more realistic situation where different RSUs observe varied traffic, the performance in [17] dramatically decreases in terms of both vehicle privacy and measurement accuracy, which, therefore, limits its practicability. As a continuous effort in improving efficiency, privacy, and accuracy, we design variable-length bit array masking in [19] to remove the similar traffic assumption. However, it only handles two-point traffic.

This journal paper makes a significant new advance to handle multi-point traffic measurement, which extends and generalizes over our previous two-point scheme [19]. As far as we know, it is the first study of the privacy-preserving multi-point traffic measurement problem that measures the traffic passing through multiple locations. We propose novel solutions based on variable-length bit arrays for privacy-preserving multi-point traffic measurement, which tackle the efficiency, privacy, accuracy, and generalization problems encountered by all previous solutions. We begin by introducing our two-point traffic measurement scheme, then extend our idea of variable-length bit array masking to address the more challenging three-point traffic measurement problem, and eventually present a general multi-point traffic measurement framework to measure traffic volume among more than two points (locations). We demonstrate the superior performance, applicability, and scalability of our solutions through mathematical proof and extensive simulations.

II. PROBLEM STATEMENT

A. Problem Definition

We consider a VCPS involving three groups of entities: vehicles, RSUs, and a central server, with the latter two forming the infrastructure. Vehicles and RSUs each has a unique ID and is equipped with computing and communication capabilities. Vehicles can communicate with RSUs in real time via DSRC [13]. RSUs are connected to the central server through wired or wireless means, and they report information collected from vehicles to the central server periodically.

Given any $d$ locations where RSUs are installed, we define the set of vehicles that pass all the $d$ locations during some measurement period $T$ as the $d$-point traffic flow. We want to measure the number of vehicles in the flow, which is called the $d$-point traffic volume. For example, the two-point traffic volume among a set of two RSUs $\{R_x, R_y\}$ measures the number of vehicles passing by both $R_x$ and $R_y$, whereas the three-point traffic volume among a set of three RSUs $\{R_x, R_y, R_z\}$ describes the number of vehicles passing by all three RSUs, i.e., $R_x, R_y$, and $R_z$. The problem is to measure the $d$-point traffic volume ($d > 1$) while protecting vehicles’ privacy. To achieve privacy protection, we need a solution in which a vehicle never transmits any unique identifier or any permanently or temporarly fixed data. Ideally, the information transmitted by the vehicles to the RSUs looks totally random, out of which neither the identity nor the trajectory of any vehicle can be pried with high probability. One typical application scenario is to measure multi-point traffic in a city with a typical measurement period of a day, where RSUs may be deployed at any interested locations in the city.

B. Threat Model

We assume that RSUs are semi-trusted: On the one hand, all RSUs are from trustworthy authorities, which can be enforced by authentication based on public key infrastructure, and RSUs will not be compromised. Vehicles can use the public key certificate broadcasted by RSUs, which they obtained from the trusted third parties, to verify the RSUs. On the other hand, the authorities may exploit the information collected by RSUs to track individual vehicles when they need to do so. For instance, if a vehicle transmits any unique identifier upon each query, that identifier can be used for tracking purposes.

Note that there are also other ways to track a vehicle, for example, tailgating the vehicle or setting cameras near RSUs to take photos and using image processing to recognize it. These methods are beyond the scope of this paper. In this paper, we focus on preventing automatic tracking caused by the traffic measurement scheme itself.

We also assume that a special medium access control (MAC) protocol such as SpooMAC [25] is applied to support privacy preservation such that the MAC address of a vehicle is not fixed. Vehicles may pick a MAC address randomly from a large
space for one-time use when needed. Through this, vehicles can report information to RSUs for traffic flow measurement without revealing their true identities.

C. Performance Metrics

We consider three performance metrics to evaluate a traffic measurement scheme: computation overhead, measurement accuracy, and preserved privacy.

1) Computation Overhead: It includes the computation overhead for each vehicle per RSU en route, and for each RSU per passing vehicle, and for the central server to measure the multi-point traffic volume among an arbitrary set of RSUs.

2) Measurement Accuracy: Let \( n_c \) be the actual multi-point traffic volume among a set of RSUs and \( \hat{n}_c \) be the estimator for \( n_c \). We measure the accuracy of a multi-point traffic measurement scheme by evaluating the bias and standard deviation of \( \hat{n}_c/n_c \). Clearly, a good measurement scheme should have close-to-zero bias and relatively small standard deviation.

3) Preserved Privacy: The essence of privacy preservation in multi-point transportation traffic measurement is to give the adversary only a limited chance of identifying partially or fully any trajectory of any vehicle. Accordingly, we quantify the privacy of a scheme through a parameter \( p \) that satisfies the following requirement: The probability for any “trace” of any vehicle to not be identified must be at least \( p \), where a trace of a vehicle is a pair of RSUs it has passed by. A larger value of \( p \) means better privacy. Intuitively, a scheme with \( p = 0.9 \) is better than a scheme with \( p = 0.5 \) in terms of privacy because the latter gives the adversary a better chance to link traces of a vehicle to obtain its trajectory since it allows the traces to be identified with a higher probability, i.e., \( 1 - p \).

Note that our privacy definition agrees with the privacy requirements as proposed in [26] and [27]. In [26], different privacy metrics [27], [28] are surveyed to characterize the vehicles’ privacy level. In contrast to the anonymity set analytical models [27], which vary as the traffic conditions change, it is easier to judge the privacy level of a traffic measurement scheme through a single quantitative metric of probability that fits the global system and applies to various traffic conditions and scenarios. In [28], the overall probability for an adversary to follow a vehicle from the origin to the destination (OD data) with an entropy perspective is considered. However, we believe that stronger privacy, which considers the probability for the trajectory of a vehicle (as opposed to the narrower OD data) to not be identified by any adversary, is desirable for VCPs. For example, the identity of a vehicle may be revealed at some location (not necessarily at the origin or the destination of its trip), e.g., through a photograph triggered by the vehicle rushing a red light or by a police car stopping the vehicle. These circumstances are not covered by the privacy definition in [28] but are captured by ours.

III. TWO-POINT TRAFFIC MEASUREMENT

Here, we introduce our privacy-preserving two-point traffic measurement scheme in [19], which is designed based on variable-length bit arrays, a novel “unfolding” technique, and a formally derived MLE estimator. We first describe the two measurement phases in the proposed scheme and then analyze its performance.

A. Online Coding Phase

Our two-point scheme consists of two phases: online coding phase for storing de-identified vehicle information in bit arrays of RSUs and offline decoding phase for measuring two-point traffic volume between two arbitrary RSUs based on the reported bit arrays. In the following, we explain the first phase.

Each RSU \( R_x \) maintains a counter \( n_x \), which keeps track of the total number of passing vehicles during the current measurement period. \( R_x \) also maintains a bit array \( B_x \) with length \( m_x \) to mask vehicle identities. We require the lengths of all bit arrays to be a power of 2, i.e., \( m_x \) must be in the form of \( 2^k \), to facilitate the comparisons of varied-length bit arrays (more explanation later). We set the value of \( m_x \) to be \( m_x = 2^\lceil \log_2(n_x \times f) \rceil \), where \( \hat{n}_x \) is the expected traffic at \( R_x \) during the measurement period based on history average traffic at the same location and the same time, and \( f \) is a system-wide parameter whose value affects the tradeoff between measurement accuracy and level of privacy. Clearly, \( m_x \) is the smallest integer that is a power of 2 and no less than \( \hat{n}_x \times f \). At the beginning of each measurement period, \( n_x \) and all bits in \( B_x \) are set to zeros.

Each vehicle \( v \) has a logical bit array \( LB_v \), which consists of \( s \) bits randomly selected from an imaginary array \( B \), whose size \( m_x \) is equal to that of the largest bit array among all RSUs, where \( s \ll m_x \). The indices of these bits in \( B \) are \( H(v \oplus K_v \oplus X[0]) \ldots, H(v \oplus K_v \oplus X[s]) \), where \( \oplus \) is the bitwise XOR, \( H(\ldots) \) is a hash function whose range is \( [0, m_x) \), \( X \) is an integer array of randomly chosen constants to arbitrarily alter the hash result, and \( K_v \) is the private key of \( v \) to protect its privacy.

Table I lists some frequently used notations in this paper. Given the notations and data structures, online coding works as follows. RSUs broadcast queries in preset intervals (e.g., once a second), ensuring that each passing vehicle receives at least one query and, at the same time, giving enough time for the vehicle to reply. Collisions can be resolved through a well-established carrier-sense multiple access or time-division multiple access protocol, which are not the focus of this paper. Every query that an RSU sends out includes the RSU’s RID, its public key certificate, and the size of its bit array. Suppose a vehicle, whose ID is \( v \), receives a query from an RSU, whose ID is \( R_x \) and the bit array size is \( m_x \). It first verifies the certificate to authenticate the RSU. After verifying that \( R_x \) is from trustworthy authority, \( v \) will randomly select a bit from its logical bit array \( LB_v \) by computing an index \( b = H(v \oplus K_v \oplus X[H(R_x \oplus t) \mod s]) \), where \( t \) is the current time stamp. Then, \( v \) generates an index \( b_v \) in the range of \( [0, m_x) \) corresponding to \( b \), where \( b_v = b \mod m_x \), and sends \( b_v \) to \( R_x \). Upon receiving the index \( b_v \), \( R_x \) will first increase its counter \( n_x \) by 1 and then set the \( b_v \)th bit in \( B_x \) to 1. Therefore, the overall effect that \( v \) produces on \( R_x \) is

\[
n_x = n_x + 1
\]

\[
B_x[H(v \oplus K_v \oplus X[H(R_x \oplus t) \mod s]) \mod m_x] = 1.
\]
TABLE I
FREQUENTLY USED NOTATIONS

<table>
<thead>
<tr>
<th>Notation</th>
<th>Meaning</th>
</tr>
</thead>
<tbody>
<tr>
<td>$R_x, R_y$</td>
<td>single-point traffic volume of RSUs $R_x, R_y$, respectively</td>
</tr>
<tr>
<td>$n_{xy}, n_{xz}, n_{yz}$</td>
<td>two-point traffic volume between two RSUs, $R_x$ and $R_y$, $R_x$ and $R_z$, $R_y$ and $R_z$, respectively</td>
</tr>
<tr>
<td>$\hat{n}<em>{xy}, \hat{n}</em>{xz}, \hat{n}_{yz}$</td>
<td>MLE estimator of the two-point traffic volume $n_{xy}, n_{xz}, n_{yz}$, respectively</td>
</tr>
<tr>
<td>$B_{xy}, B_{xz}, B_{yz}$</td>
<td>bit arrays of RSUs $R_x, R_y, R_z$, respectively</td>
</tr>
<tr>
<td>$m_x, m_y, m_z$</td>
<td>sizes of bit arrays $B_x, B_y, B_z$, respectively</td>
</tr>
<tr>
<td>$m_{xy}, m_{xz}, m_{yz}$</td>
<td>size of the bit array $B_{xy}, B_{xz}, B_{yz}$, respectively</td>
</tr>
<tr>
<td>$U_x, U_y, U_s$</td>
<td>number of zeros in bit arrays $B_x, B_y, B_s$, respectively</td>
</tr>
<tr>
<td>$U_{xy}, U_{xz}, U_{yz}$</td>
<td>number of zeros in bit arrays $B_{xy}, B_{xz}, B_{yz}$, respectively</td>
</tr>
<tr>
<td>$V_x, V_y, V_s$</td>
<td>ratio of zeros in bit arrays $B_x, B_y, B_s$, respectively</td>
</tr>
<tr>
<td>$V_{xy}, V_{xz}, V_{yz}$</td>
<td>ratio of zeros in bit arrays $B_{xy}, B_{xz}, B_{yz}$, respectively</td>
</tr>
<tr>
<td>$LB_u$</td>
<td>size of the logical bit array of vehicle $u$</td>
</tr>
<tr>
<td>$\ell, f_x, f_y, f_z$</td>
<td>single-point load factor, ratio of an RSU’s bit array size over its traffic volume, e.g., $f_x = \frac{m_x}{n_x}$</td>
</tr>
<tr>
<td>$\tilde{f}$</td>
<td>fixed load factor for all RSUs in our design</td>
</tr>
<tr>
<td>$m$</td>
<td>fixed bit array size for all RSUs in [17], i.e., $m_i = m, \forall i$</td>
</tr>
</tbody>
</table>

Note that the same vehicle may transmit different bit indexes at two RSUs. The probability for this to happen is $1 - (1/s)$, which is larger when the size $s$ of $LB_u$ is larger. Different vehicles may send the same index because their logical bit arrays share bits from $B_z$. As any vehicle does not have to transmit any fixed number in support of traffic measurement, we improve privacy protection. This is true even when there is a single vehicle passing through two RSUs.

B. Offline Decoding Phase

At the end of each measurement period, all RSUs will send their counters and bit arrays to the central server, which first updates the history average single-point traffic volume for the RSUs to take into account the traffic data in the current measurement period and then measures the two-point traffic volume between two arbitrary RSUs based on the reported counters and bit arrays.

Suppose the set of vehicles that pass RSU $R_x$ ($R_y$) is denoted as $S_x$ ($S_y$) with cardinality $|S_x| = n_x$ ($|S_y| = n_y$). Clearly, the set of vehicles that pass both RSUs $R_x$ and $R_y$ is $S_x \cap S_y$. Denote its cardinality as $n_{xy}$, i.e., $|S_x \cap S_y| = n_{xy}$, which is the value that we want to measure. Denote the size of the bit array $B_x (B_y)$ stored in RSU $R_x (R_y)$ as $m_x (m_y)$. Without loss of generality, we assume that $m_x \leq m_y$. Given the counters $n_x$ and $n_y$, and bit arrays $B_x$ and $B_y$, the server measures $n_{xy}$ as follows.

First, our previous work [17] shows that when two bit arrays have the same length, we are able to combine them through bitwise OR and produce a good estimate for the two-point traffic volume. Now, we have to deal with two bit arrays of different lengths. To combine the information of the two arrays through bitwise OR, the central server expands the smaller bit array $B_x$ to the same size of $B_y$ through a process called “unfolding,” which is simply duplicating $B_x$ multiple times until it reaches the size of $B_y$. Because $m_x$ and $m_y$ are both powers of 2 and $m_x \leq m_y$, it will always be true that $m_y$ is divisible by $m_x$, which means that we can unfold $B_x$ to the size of $B_y$ by duplicating $B_x$ for $m_y/m_x$ times. (When we derive the new formula for estimating the two-point traffic volume, we will mathematically account for the impact of duplication.) The “unfolded” bit array of $B_x$ is denoted as $B_x^u$. Specifically

$$B_x^u[i] = B_x[i \mod m_x] \quad \forall i \in [0, m_y).$$

Second, the server takes a bitwise OR operation on $B_x^u$ and $B_y$ to obtain a new bit array $B_{xy}$, i.e.,

$$B_{xy}[i] = B_x^u[i] \lor B_y[i] \quad \forall i \in [0, m_y).$$

The bitwise OR operation is granted since the two bit arrays, i.e., $B_x^u$ and $B_y$, are of the same size. Through requiring the size of all bit arrays to be a power of 2, we facilitate the comparison of varied-length bit arrays: The overall computation overhead to compare $B_x$ and $B_y$ is just $O(m_y)$, in contrast to $O(m_x \times m_y)$ without the “power of 2” requirement.

Finally, given $B_{xy}, B_x (B_x^u)$, and $B_y$, the central server uses the following formula to estimate the two-point traffic volume between $R_x$ and $R_y$:

$$\hat{n}_{xy} = \frac{\ln(V_{xy}) - \ln(V_x) - \ln(V_y)}{\ln\left(1 - \frac{m_x}{m_y}\right) - \ln\left(1 - \frac{m_y}{m_y}\right)}$$

where $V_{xy}, V_x,$ and $V_y$ are random variables that represent the fraction of zero bits in $B_{xy}, B_x, B_y$, respectively. Their values can be easily found by counting the number of zeros in $B_{xy}, B_x,$ and $B_y$, which are denoted by $U_{xy}, U_x,$ and $U_y$, respectively, and dividing them by the bit array size $m_x, m_y,$ and $m_y$. That is, $V_{xy} = U_{xy}/m_y, V_x = U_x/m_x$, and $V_y = U_y/m_y$. Note that the fraction of zero bits in $B_x^u$ is the same as $B_x$. 


C. Performance Analysis

1) Measurement Accuracy: In [19], we have demonstrated that \( \hat{n}_{xy} \) is an MLE estimator of \( n_{xy} \) and mathematically analyzed the measurement accuracy of our two-point scheme through the bias and standard deviation of \( \hat{n}_{xy} / n_{xy} \). We also perform extensive simulations, which show that our scheme can indeed achieve very accurate measurement results. See [19] for detailed derivation, proof, and analysis.

2) Computation Overhead: Clearly, the computation overhead for the vehicles and RSUs of this scheme are comparable to that in [17]. In this scheme, when a vehicle \( v \) passes an RSU \( R_x \), vehicle \( v \) only needs to compute two hashes to obtain an index of a random bit, and RSU \( R_x \) only needs to set one bit in its bit array \( B_x \), as described in Section III-A. Hence, the computation overhead for each vehicle per RSU as well as for each RSU per vehicle is both \( O(1) \).

As for the central server, the task it performs is a little bit more complicated than that in [17], but the computation overhead is comparable. First, the server unfolds the smaller bit array \( B_x \) to \( B^*_x \), which has the same size as \( B_x \). This operation costs \( O(m_y) \) time. Second, it performs a bitwise OR over two or more bit arrays, i.e., \( B^*_x \) and \( B_y \), to create a new bit array \( B_{xy} \) of size \( m_y \), which also costs \( O(m_y) \) time. Finally, the server counts the number of zeros in \( B_x \), \( B_y \), and \( B_{xy} \), which takes \( O(m_y) \) time as well. Therefore, the overall computation overhead for the server to measure the traffic volume between a pair of RSUs, i.e., \( R_x \) and \( R_y \), is \( O(m_y) \), where \( m_y \) is the size of the larger bit array of the two RSUs. Since [17] assumes that \( m_x = m_y = m \) and its computation overhead for the server is \( O(m) \), one can see that this scheme indeed achieves comparable computation overhead as in [17].

3) Preserved Privacy: In [19], we have analyzed the privacy of our two-point scheme through mathematical derivation and numerical analysis, which demonstrate that our scheme well preserves vehicles’ privacy. Here, we directly give the formula for the privacy \( p \) of this scheme and refer interested readers to [19] for detailed derivation and analysis. Thus

\[
p = \frac{1}{1 - P(\bar{A})} \times \left[ (1 - \frac{1}{m_x})^{n_{xy}} - (1 - \frac{1}{m_x})^{n_x} \right] \times \left[ (1 - \frac{1}{m_y})^{n_{xy}} - (1 - \frac{1}{m_y})^{n_y} \right]
\]

(6)

where \( P(\bar{A}) \) is given in

\[
P(\bar{A}) = \left( 1 - \frac{1}{m_x} \right)^{n_x} \times C_1^{n_{xy}} + \left( 1 - \frac{1}{m_y} \right)^{n_y}
- \left( 1 - \frac{1}{m_x} \right)^{n_x} \left( 1 - \frac{1}{m_y} \right)^{n_y} \times C_2^{n_{xy}}
\]

(7)

and \( C_1 \) and \( C_2 \) are both constants with values

\[
C_1 = \frac{1}{s} \times \frac{1 - m_y}{1 - m_x} + \left( 1 - \frac{1}{s} \right)
\]

(8)

\[
C_2 = \frac{1}{s} \times \frac{1 - m_x}{1 - m_y} + \left( 1 - \frac{1}{s} \right)
\]

(9)

Note that if we set \( m_x = m_y = m \) in (6), we get the same formula as in [17]. This is natural, since [17] is just a special case of this scheme.

IV. MULTI-POINT TRAFFIC MEASUREMENT

A. From Two-Point to Multi-Point

In the previous section, we have presented our privacy-preserving two-point traffic measurement scheme, which can easily fit today’s large-scale road networks. To serve for a broader spectrum of applications in transportation engineering, we are motivated to generalize our design to address the more challenging problem of multi-point traffic measurement.

Here, we will show how to extend our idea of variable-length bit array masking to address three-point traffic measurement, which observes the potential of further generalization to solve multi-point traffic measurement. Intuitively, if we can unfold two bit arrays to obtain statistical results related to the two-point traffic volume, we should also be able to unfold three or more bit arrays to get a statistical estimator for the multi-point traffic volume. The measurement process should be similar: Vehicles report random indexes from their logical bit arrays to mark RSUs’ varied-length bit arrays, and the central server performs unfolding and bitwise OR operations on three or more bit arrays to obtain statistical results related to the multi-point traffic volume. If an MLE estimator can also be mathematically derived from those statistical results, it will be easy for the central server to compute the multi-point traffic volume.

In the remaining part of this section, we follow the above thinking to develop our privacy-preserving three-point traffic measurement scheme. We first explain the two measurement phases, validate the MLE estimator used to measure three-point traffic volume, and then analyze its performance. Finally, we generalize our two-point and three-point schemes and present a general framework for multi-point traffic measurement.

B. Privacy-Preserving Three-Point Traffic Measurement

1) Online Coding Phase: The online coding phase of our three-point scheme is exactly the same as our two-point scheme. Each RSU \( R_x \) maintains a counter \( n_x \) to record the total number of passing vehicles and a bit array \( B_x \) with length \( m_x = 2^{[\log_2(n_x \times f)]} \) to collect vehicles “masked” data, where \( \bar{n}_x \) is the expected traffic volume in \( R_x \), and \( f \) is a system-wide load factor, whose value is the same for all RSUs. At the beginning, \( n_x \) and all bits in \( B_x \) are set to zeros. For privacy protection, each vehicle \( v \) also has a logical bit array \( L B_v \) consisting of \( s \) bits randomly selected from an imaginary array \( B \), whose size \( m_s \) is equal to that of the largest bit array among all RSUs, where \( s \ll m_s \). The bit indexes in \( B_v \) are \( H(v \oplus K_v \oplus X[0]) \ldots H(v \oplus K_v \oplus X[s-1]) \). Some frequently used notations can be found in Table I.

Vehicles and RSUs cooperate to automatically collect “masked” traffic data. When a vehicle \( v \) receives a query from an RSU \( R_x \), whose bit array is \( B_x \) with size \( m_x \), it first verifies \( R_x \). Once \( R_x \) is authenticated, \( v \) randomly selects a bit from \( L B_v \) by computing an index \( b = H(v \oplus K_v \oplus X[0])(H(R_x \oplus t) \mod s) \), where \( t \) is the current time stamp, then generates an
index \( b_x = b \mod m_x \) in the range of \([0, m_x)\), and finally sends \( b_x \) to \( R_x \). Upon receiving index \( b_x \), \( R_x \) increases its counter \( n_x \) by 1 and sets the \( b_x \)-th bit in \( B_x \) to \( 1 \).

2) Offline Decoding Phase: At the end of each measurement period, all RSUs will send their counters and bit arrays to the central server, which first updates the history single-point traffic data for the RSUs to take into account the current measurement period. Then, the server will measure the three-point traffic volume among three arbitrary RSUs based on the reported counters and bit arrays, which incurs a little bit more work than the two-point scheme (due to the third involving RSU). However, the measurement process is similar, and the computation overhead is also comparable to the two-point case.

We first define some notations (also summarized in Table I). We denote the set of vehicles passing RSUs \( R_x, R_y \), and \( R_z \) as \( S_x, S_y \), and \( S_z \) with cardinality \( |S_x| = n_x, |S_y| = n_y, \) and \( |S_z| = n_z \). Respectively. The set of vehicles that pass the set of three RSUS \( \{R_x, R_y, R_z\} \) is \( S_x \cap S_y \cap S_z \). Denote its cardinality as \( n_{xyz} \), i.e., \( n_{xyz} = |S_x \cap S_y \cap S_z| \), which is the value that we want to measure. The set of vehicles passing both \( R_x \) and \( R_y \) is \( S_x \cap S_y \), whose size is denoted as \( n_{xy} \), i.e.,\( n_{xy} = |S_x \cap S_y| \). Similarly, we have \( n_{xz} = |S_x \cap S_z|, \) \( n_{yz} = |S_y \cap S_z| \). In addition, we denote the size of the bit arrays \( B_x, B_y, \) and \( B_z \) stored in RSUs \( R_x, R_y, \) and \( R_z \) as \( m_x, m_y, \) and \( m_z \), respectively. Without loss of generality, we assume that \( m_x \leq m_y \leq m_z \).

Given above notations, the central server measures \( n_{xyz} \) by performing the four steps of unfolding and bitwise operation below and then computing the MLE estimator in (14).

Step 1: The server unfolds \( B_x \) to the same size of \( B_y \) and \( B_z \) by taking a bitwise operation on the unfolded bit array and \( B_y \) to obtain a new bit array \( B_{xy} \) of size \( m_y \). More specifically

\[
B_{xy}[i] = B_x[i \mod m_x] \lor B_y[i] \quad \forall i \in [0, m_y),
\]

(10)

Step 2: The server unfolds \( B_x \) to the same size of \( B_z \) and \( B_z \) by taking a bitwise operation on the unfolded bit array and \( B_z \) to obtain a new bit array \( B_{xz} \) of size \( m_z \). More specifically

\[
B_{xz}[i] = B_x[i \mod m_x] \lor B_z[i] \quad \forall i \in [0, m_z),
\]

(11)

Step 3: The server unfolds \( B_y \) to the same size of \( B_z \) and takes a bitwise operation on the unfolded bit array and \( B_z \) to obtain a new bit array \( B_{yz} \) of size \( m_z \). More specifically

\[
B_{yz}[i] = B_y[i \mod m_y] \lor B_z[i] \quad \forall i \in [0, m_z),
\]

(12)

Step 4: The server unfolds \( B_x \) and \( B_y \) to the same size of \( B_z \) and takes a bitwise operation on the two unfolded bit arrays \( B_x, B_y \) and \( B_z \) to obtain a new bit array \( B_{xyz} \) of size \( m_z \). More specifically

\[
B_{xyz}[i] = B_x[i \mod m_x] \lor B_y[i \mod m_y] \lor B_z[i] \quad \forall i \in [0, m_z).
\]

(13)

Finally, given \( B_x, B_y, B_z, B_{xy}, B_{xz}, B_{yz}, \) and \( B_{xyz} \), the MLE formula that the central server uses to estimate the three-point traffic volume of RSUs \( R_x, R_y, \) and \( R_z \) is

\[
\hat{n}_{xyz} = \frac{W}{\ln \left(1 - \frac{1}{m_x}\right) + \ln(C_3) - \ln(C_4) - 2 \ln(C_5)}
\]

(14)

where \( W \) is a function of zero ratios in the bit arrays, i.e.,

\[
W = \ln V_{xyz} + \ln V_{x} + \ln V_y + \ln V_z - \ln V_{xy} - \ln V_{xz} - \ln V_{yz}
\]

(15)

and \( C_3, C_4, \) and \( C_5 \) are constants whose values are

\[
C_3 = \frac{1}{s} \times \left(1 - \frac{s - 1}{s} \times \frac{1}{m_z}\right)
\]

(16)

\[
C_4 = 1 - \frac{s - 1}{s} \times \frac{1}{m_y}
\]

(17)

\[
C_5 = 1 - \frac{s - 1}{s} \times \frac{1}{m_z}
\]

(18)

In (15), \( V_{xyz}, V_x, V_y, V_z, V_{xy}, V_{xz}, \) and \( V_{yz} \) are random variables that represent the fraction of zero bits in \( B_{xyz}, B_x, B_y, B_z, B_{xy}, B_{xz}, \) and \( B_{yz} \), respectively. Their values can be easily found by counting the number of zeros in the bit arrays, which are denoted by \( U_{xyz}, U_x, U_y, U_{xy}, U_{xz}, \) and \( U_{yz} \), respectively, and dividing them by the corresponding bit array size. For example, \( V_{xyz} = U_{xyz} / m_z, V_x = U_x / m_x, \) and \( V_{xy} = U_{xy} / m_y \).

3) Derivation of the MLE Estimator \( \hat{n}_{xyz} \): Now, we follow the MLE method to derive \( \hat{n}_{xyz} \) given by (14). The derivation process is similar to the two-point scheme [19]; we first derive the probability \( q(n_{xyz}) \) for an arbitrary bit in \( B_{xyz} \) to be “0” and use \( q(n_{xyz}) \) to establish the likelihood function \( L \) to observe \( U_{xyz} \)’s “0” bits in \( B_{xyz} \). Finally, maximizing \( L \) with respect to \( n_{xyz} \) will give the MLE estimator, i.e., \( \hat{n}_{xyz} \).

Consider an arbitrary bit \( b \) in \( B_{xyz} \). Let \( A_b \) be the event that the \( b \)-th bit in \( B_{xyz} \) remains “0,” then \( q(n_{xyz}) \) is the probability for \( A_b \) to occur. Note that the set of all vehicles passing \( R_x \) and/or \( R_y \) and/or \( R_z \) (i.e., \( S_x \cup S_y \cup S_z \)) can be partitioned into seven sets: \( S_x \cap S_y \cap S_z, S_x \cap S_y \cap S_z, S_x \cap S_y \cap S_z, S_x \cap S_y \cap S_z, S_x \cap S_y \cap S_z, S_x \cap S_y \cap S_z, \) and \( S_x \cap S_y \cap S_z \). Consider the vehicles in each partition. Clearly, event \( A_b \) is equivalent to the combination of the following seven events.

1) Event \( H_1 \): For vehicles passing \( R_x, R_y, \) and \( R_z \) (i.e., in set \( S_x \cap S_y \cap S_z \)), none of them have chosen bit \( b \) mod \( m_x \) in \( B_x \) or bit \( b \) mod \( m_y \) in \( B_y \) or bit \( b \) in \( B_z \). Otherwise, bit \( b \) in \( B_{xyz} \) will be “1” according to (13). There are \( n_{xyz} \) vehicles in the set \( S_x \cap S_y \cap S_z \), and Fig. 1 shows the decision tree for each individual car \( v \in S_x \cap S_y \cap S_z \) to not set those bits. For \( R_x \), \( v \) should choose \( b \) mod \( m_x \neq b \) mod \( m_x \), and the probability is clearly \( 1 - (1/m_x) \) (root node in Fig. 1).

Given its selection of \( b_1 \) in \( R_y \), it has two choices in \( R_y \): First, as shown in the left node of the second level in Fig. 1, with a probability of \( 1/s, v \) selects the same bit \( b_1 \) in \( R_y \) (hence will not set bit \( b \) mod \( m_y \) in \( B_y \)); second, as shown in the right node
of the same level, with a probability of $1 - (1/s)$, $v$ chooses a separate bit $b_2$ randomly from its logical bit array $LB_v$, and the conditional probability for $b_2 \mod m_y \neq b \mod m_y$ is $1 - (1/m_y).

Now, we examine the choices for $v$ to not set bit $b$ in $B_v$ for RSU $R_v$ given its previous selections at $R_x$ and $R_y$ (the five nodes at the bottom level of Fig. 1). Under its first choice at $R_y$, to not set bit $b$ in $B_v$, $v$ can either choose the same bit $b_1$ with a probability of $1/s$ (node #1) or select a separate bit $b_2$ randomly from $LB_v$ with a probability of $1 - (1/s)$, and the conditional probability for $b_3 \neq b$ is $1 - (1/m_z)$ (node #2). Under its second choice at $R_y$, $v$ can have three choices to not set bit $b$ in $B_v$: 1) With a probability of $1/s$, $v$ chooses $b_1$ in $R_x$ (node #3); 2) with a probability of $1/s$, $v$ chooses $b_2$ in $R_x$ (node #4); 3) with a probability of $1 - (2/s)$, $v$ chooses a separate bit $b_3$ randomly from $LB_v$, and the conditional probability for $b_1 \neq b$ is $1 - (1/m_z)$ (node #5).

Note that the probabilities in the given analysis are all conditional probabilities given that the ancestor nodes have been chosen. To sum up, the probability of $H_1$ is

$$Q_1 = \left\{ \left(1 - \frac{1}{m_x}\right) \left[ \frac{1}{s} \times \left(1 - \frac{s - 1}{m_z} \times \frac{1}{m_y}\right) \right] + \left(1 - \frac{1}{s}\right) \left(1 - \frac{1}{m_y}\right) \left(1 - \frac{s - 2}{m_z} \times \frac{1}{m_y}\right) \right\} n_{xyz} = \left(1 - \frac{1}{m_x}\right) n_{xyz} C_4^{n_{xyz}}. \tag{19}$$

II) Event $H_2$: For vehicles passing only $R_x$ and $R_y$ (i.e., in set $S_x \cap S_y - S_z$), none of them have chosen bit $(b \mod m_x)$ in $B_x$, or bit $(b \mod m_y)$ in $B_y$. We analyze the probability of each individual vehicle to not set those two bits at $R_x$ and $R_y$, which is exactly the same as that for Event $E_1$ of our two-point analysis in [19]. Since there are $n_{xy} - n_{xyz}$ cars in the set $S_x \cap S_y - S_z$, the probability of $H_2$ is

$$Q_2 = \left\{ \left(1 - \frac{1}{m_x}\right) \left[ \frac{1}{s} + \left(1 - \frac{1}{s}\right) \left(1 - \frac{1}{m_y}\right) \right] \right\} n_{xyz} = \left(1 - \frac{1}{m_x}\right) n_{xyz} C_4^{n_{xyz}}. \tag{20}$$

III) Event $H_3$: For vehicles passing only $R_y$ and $R_z$ (i.e., in set $S_x \cap S_y - S_z$), none of them have chosen bit $(b \mod m_x)$ in $B_x$ or bit $b$ in $B_z$. There are $n_{xz} - n_{xyz}$ cars in the set $S_x \cap S_y - S_z$. Similar to $H_2$, we get the probability of $H_3$ as

$$Q_3 = \left\{ \left(1 - \frac{1}{m_x}\right) \left[ \frac{1}{s} + \left(1 - \frac{1}{s}\right) \left(1 - \frac{1}{m_y}\right) \right] \right\} n_{xz} - n_{xyz} = \left(1 - \frac{1}{m_x}\right) n_{xz} - n_{xyz} C_5^{n_{xz} - n_{xyz}}. \tag{21}$$

IV) Event $H_4$: For vehicles passing only $R_x$ and $R_z$ (i.e., in set $S_y \cap S_x - S_z$), none of them have chosen bit $(b \mod m_y)$ in $B_y$ or bit $b$ in $B_z$. There are $n_{yz} - n_{xyz}$ cars in the set $S_y \cap S_x - S_z$. Similar to $H_2$, we get the probability of $H_4$ as

$$Q_4 = \left\{ \left(1 - \frac{1}{m_y}\right) \left[ \frac{1}{s} + \left(1 - \frac{1}{s}\right) \left(1 - \frac{1}{m_z}\right) \right] \right\} n_{yz} - n_{xyz} = \left(1 - \frac{1}{m_y}\right) n_{yz} - n_{xyz} C_5^{n_{yz} - n_{xyz}}. \tag{22}$$

V) Event $H_5$: For vehicles passing only $R_x$ (i.e., in set $S_x - S_y - S_z$), none of them have chosen bit $(b \mod m_x)$ in $B_x$. There are $n_x - n_{xy} - n_{xz} + n_{xyz}$ cars in the set $S_x - S_y - S_z$, and each of them has a probability of $1 - (1/m_z)$ to not set bit $(b \mod m_z)$ in $B_x$. Therefore, the probability of $H_5$ is

$$Q_5 = \left(1 - \frac{1}{m_z}\right) n_x - n_{xy} - n_{xz} + n_{xyz}. \tag{23}$$

VI) Event $H_6$: For vehicles passing only $R_y$ (i.e., in set $S_y - S_x - S_z$), none of them have chosen bit $(b \mod m_y)$ in $B_y$. There are $n_y - n_{xy} - n_{yz} + n_{xyz}$ cars in the set $S_y - S_x - S_z$, and each of them has a probability of $1 - (1/m_y)$ to not set bit $(b \mod m_y)$ in $B_y$. Hence, the probability of $H_6$ is

$$Q_6 = \left(1 - \frac{1}{m_y}\right) n_y - n_{xy} - n_{yz} + n_{xyz}. \tag{24}$$

VII) Event $H_7$: For vehicles passing only $R_z$ (i.e., in set $S_z - S_x - S_y$), none of them have chosen bit in $B_z$. There are $n_z - n_{xz} - n_{yz} + n_{xyz}$ cars in the set $S_z - S_x - S_y$, and each of them has a probability of $1 - (1/m_z)$ to not set bit $b$ in $B_z$. Therefore, the probability of $H_7$ is

$$Q_7 = \left(1 - \frac{1}{m_z}\right) n_z - n_{xz} - n_{yz} + n_{xyz}. \tag{25}$$

Combining the given analysis, we obtain the probability $q(n_{xyz})$ for bit $b$ in $B_{xyz}$ to remain “0” as

$$q(n_{xyz}) = Q_1 \times Q_2 \times Q_3 \times Q_4 \times Q_5 \times Q_6 \times Q_7 = C_4^{n_{xyz}} \times C_3^{n_{xy} - n_{xyz}} \times C_5^{n_{xz} + n_{yz} - 2n_{xyz}} \times \left(1 - \frac{1}{m_x}\right) \times \left(1 - \frac{1}{m_y}\right) n_{xy} - n_{xyz} \times \left(1 - \frac{1}{m_z}\right) \times \left(1 - \frac{1}{m_y}\right) n_x - n_{xz} - n_{yz} + n_{xyz}. \tag{26}$$
Similar to our two-point analysis in [19], we know that for any bit in $B_z$, the probability for it to remain “0” after $n_z$ vehicles, each choosing a random bit from $B_z$, is
\[
q(n_z) = \left(1 - \frac{1}{m_z}\right)^{n_z}
\]
and the expected values for $V_z$ and $V_{xyz}$ are
\[
E(V_z) = E\left(\frac{U_z}{m_z}\right) = m_z \times q(n_z) = q(n_z)
\]
\[
E(V_{xyz}) = E\left(\frac{U_{xyz}}{m_z}\right) = m_z \times q(n_{xyz}) = q(n_{xyz}).
\]
From [19], we also get
\[
\left(1 - \frac{s-1}{1 - \frac{1}{m_y}}\right)^{n_{xy}} = \frac{V_{xy}}{V_x \times V_y}.
\]
Similarly, we have
\[
\left(1 - \frac{s-1}{1 - \frac{1}{m_y}}\right)^{n_{xz}} = \frac{V_{xz}}{V_x \times V_z}
\]
\[
\left(1 - \frac{s-1}{1 - \frac{1}{m_y}}\right)^{n_{yz}} = \frac{V_{yz}}{V_y \times V_z}.
\]
Substituting [19, eqs. (10)–(13)] as well as (27)–(32) to (26) and replacing $E(V_z)$, $E(V_y)$, $E(V_x)$, and $E(V_{xyz})$ with their instance values $V_z$, $V_y$, $V_x$, and $V_{xyz}$, respectively, we have
\[
V_{xyz} = \frac{V_{xy} \times V_{xz} \times V_{yz}}{V_x \times V_y \times V_z} \times \left[\frac{(1 - \frac{1}{m_z}) \times C_3}{C_4 \times C_5}\right]^{n_{xyz}}.
\]
Finally, solving (33) gives the MLE estimator $\hat{n}_{xyz}$, as described in (14).

4) **Computation Overhead:** Note that the online coding phase works exactly the same as our two-point scheme; hence, the computation overhead for the vehicles and RSUs of our three-point scheme is exactly the same as the two-point scheme. For both schemes, when a vehicle $v$ passes an RSU $R_x$, $v$ only needs to compute two hashes to obtain an index of a random bit, and $R_x$ only needs to set one bit in its bit array $B_z$. Hence, the computation overhead for each vehicle per RSU as well as for each RSU per passing vehicle are both $O(1)$.

Our three-point and two-point schemes diverge from the offline decoding phase, where the central server performs a little bit more task for three-point traffic measurement: It takes four “unfolding” and bitwise OR operations instead of one. Similar to our two-point analysis, in our three-point scheme, the “unfolding” and bitwise OR operation in step 1 costs $O(m_y)$ time, and steps 2, 3, and 4 each costs $O(m_z)$ time, leading to an overall computation overhead of $O(m_y + m_z)$, where $m_z$ is the size of the largest bit array among the three RSUs. One can see that our three-point scheme is also very efficient.

5) **Preserved Privacy:** Since the way RSUs collect data from passing vehicles in our three-point scheme is no different from our two-point scheme, the preserved privacy is also the same. For both schemes, the privacy $p$, satisfying the requirement that the probability for any “trace” of any vehicle not to be identified must be at least $p$, is actually the conditional probability that states to what degree observing a same bit to be set in both bit arrays of two RSUs does not represent a common vehicle passing by both RSUs (a piece of a vehicle’s trace). The reason is that the only information a vehicle $v$ ever reports to an RSU is a bit index drawn from the same common pool uniformly at random, and the adversary can only attempt to identify the trace of a vehicle through the observation of the bits that are chosen by the vehicles to be set as “1” in both RSUs. Therefore, the preserved privacy of our three-point scheme is also given by (6), with the same outstanding conclusions as our two-point scheme [19].

C. **Generalization to Multi-Point Traffic Measurement**

We have proposed two schemes for privacy-preserving traffic measurement, which can efficiently measure the traffic volume among an arbitrary set of two or three RSUs. Below, we generalize our design to a multi-point traffic measurement framework for measuring traffic covering $d > 2$ locations and discuss its performance as $d$ increases.

1) **Framework:** Similar to the two-point and three-point schemes, our general scheme to measure $d$-point traffic includes two phases: online coding phase for RSUs to collect de-identified vehicle information through varied-length bit arrays and offline decoding phase for the central server to compute $d$-point traffic among an arbitrary set of $d$ RSUs based on the bit arrays. The online coding phase works exactly the same as our two-point and three-point schemes.

The offline decoding phase is also similar. At the end of each measurement period, all RSUs will send their counters and bit arrays to the central server. To compute the $d$-point traffic volume among an arbitrary set of $d$ RSUs, $\{R_1, \ldots, R_d\}$, the central server will perform a series of “unfolding” and bitwise OR operations in between the bit arrays of the $d$ RSUs to generate a series of statistical results (more specifically, the zero ratios of the resulting bit arrays) that are related to the $d$-point traffic volume. Again, if an MLE estimator can be derived based on these statistical results, the central server can easily compute the $d$-point traffic volume. Therefore, the key is to establish the relationship between the zero ratios of the bitwise ORed bit arrays and the $d$-point traffic volume.

Before deriving this relationship, we first define some notations. We denote the set of $d$ RSUs as $S_d$, i.e., $S_d = \{R_i, \ldots, R_d\}$. Without loss of generality, we assume $m_1 \leq m_2 \leq \cdots \leq m_d$, where $m_i$ is the size of the bit array $B_i$ in $R_i$, $1 \leq i \leq d$. For an arbitrary set $S \subset S_d$ of RSUs, we unfold their bit arrays to the same size of the largest bit array among $S$ and perform a bitwise OR over the unfolded bit arrays to obtain a new bit array $B_S$, whose zero ratio is $V_S$. Denote the set of vehicles passing all RSUs in $S$ as $V_S$ with cardinality $N_S = |V_S|$. Clearly, we want to measure $N_S$.

Given an arbitrary bit $b$ in $B_S$, the probability for $b$ to be “0” after an arbitrary vehicle $v \in V_S$ marks bits for all RSUs in $S$ is denoted as $P_S$. Similar to our two-point and three-point
schemes, we can derive the overall probability \( q(N_{S_d}) \) for an arbitrary bit \( b \) in \( B_{S_d} \) to be “0” after online coding as

\[
q(N_{S_d}) = P_{S_d}^{N_{S_d} - (R_d)} \times \prod_{1 \leq i \leq d} P_{S_d}^{N_{S_d} - (R_i)} \times \prod_{1 \leq i < j \leq d} P_{S_d}^{N_{S_d} - (R_i)} - N_{S_d} - (R_j) + N_{S_d} \\
\cdots \times \prod_{1 \leq i < j < k \leq d} P_{S_d}^{N_{S_d} - (R_i)} - N_{S_d} - (R_j) - N_{S_d} - (R_k) + N_{S_d}
\]

(34)

where each term captures the probability for bit \( b \) in \( B_{S_d} \) to be “0” after the set of vehicles passing only \( l \) \((d \geq l \geq 1)\) RSUs in \( S_l \) mark bits in the bit arrays, and the superscript in each term denotes the corresponding vehicle set cardinality derived from the inclusion–exclusion principle.

Given the above analysis, we present Algorithm 1 to iteratively derive the MLE estimator \( \hat{N}_{S_d} \), whose correctness can be easily proved through mathematical induction, which we omit.

**Algorithm 1 Iterative Algorithm to Derive the MLE Estimator \( \hat{N}_{S_d} \)**

1: **Inputs:** \( d, P_1, P_2, P_3, \{m_i\}_{1 \leq i \leq d}, \{N_{(R_i)}\}_{1 \leq i \leq d}, \hat{N}_{S_d} \)
2: **Initialize:** \( P_{S_1} \leftarrow P_1, P_{(R_1)} \leftarrow P_2, P_{(R_2)} \leftarrow P_3, \hat{P}_{S_2} \leftarrow \{P_{S_1}, P_{(R_1)}, P_{(R_2)}\} \)
3: \( \hat{N}_{S_d} \leftarrow \hat{N}_{S_d}, \hat{N}_{S_d} \leftarrow \{N_{S_2}, N_{(R_1)}, N_{(R_2)}\} \)
4: for \( j \leftarrow 2 \) to \( d - 1 \) do
5: **Step 1:** Use decision tree as Fig. 1 to obtain \( P_{S_{j+1}} \)
6: **Step 2:** Use \( P_{S_{j+1}} \) and \( \hat{P}_{S_j} \) to update \( \hat{P}_{S_{j+1}} = \{P_{S_{j+1}}\} \cup \{\hat{P}_{S_{j+1}}\} \)
7: **Step 3:** Use \( \hat{P}_{S_{j+1}} \) to update \( \hat{N}_{S_{j+1}} = \{N_{S_{j+1}}\} \cup \{\hat{N}_{S_{j+1}}\} \)
8: **Step 4:** Use \( \hat{N}_{S_{j+1}} \) and formula (34), and replace \( q(N_{S_{j+1}}) = E(V_{S_{j+1}}) \) by its instance value \( V_{S_{j+1}} \), to get the MLE estimator \( \hat{N}_{S_{j+1}} = F_{j+1}(\{V_{S_{j+1}}\}) \)
9: end for

In Algorithm 1, the inputs \( P_1, P_2, \) and \( P_3 \) are probability formulas given in [19, eqs. (6)–(8)], with the notations \( n_x, n_y, \) and \( n_c \) changed to \( N_{(R_1)}, N_{(R_2)}, \) and \( N_{(R_1, R_2)}, \) respectively. We first initialize the probability set \( \hat{P}_{S_2} \) and the vehicle cardinality set \( \hat{N}_{S_d} \) from the two-point derivation, which serves as the base case of our iterative algorithm. Then, the for-loop works iteratively, where the iteration \( j \) derives \( \hat{P}_{S_{j+1}} \) and \( \hat{N}_{S_{j+1}} \) based on \( \hat{P}_{S_j} \) and \( \hat{N}_{S_j} \) obtained from the previous iteration. Note that \( \hat{N}_{S_{j+1}} \) includes the MLE estimator \( \hat{N}_{S_{j+1}} \) as a function \( F_{j+1}(\{V_S\}_{j+1}) \) of the zero ratios, where the set \( \{V_S\}_{j+1} \) contains the zero ratio \( V_S \) of all \( S \subset S_{j+1}, S \neq \emptyset \). Therefore, when the for-loop completes, we will obtain the MLE estimator \( \hat{N}_{S_d} \) as a function \( F_d(\{V_S\}) \) of the zero ratios in the corresponding bitwise or red bit arrays.

2) **Discussion:** We conclude with a quick discussion about the performance of our general \( d \)-point \((d > 1)\) traffic measurement scheme. Clearly, since RSUs collect de-identified information from passing vehicles in the same way as our two-point and three-point schemes, the preserved privacy is also the same. Moreover, the computation overhead for vehicles and RSUs remains \( O(1) \). However, as \( d \) increases, the computation overhead for the central server to measure \( d \)-point traffic exponentially grows. Given \( d \) bit arrays of \( d \) RSUs, the central server needs to perform unfolding and bitwise or on every \( l \) \((2 \leq l \leq d)\) bit arrays to generate \( 2^d - d - 1 \) new bit arrays and compute the zero ratios in them and \( d \) original bit arrays, which costs an overall \( O(d^2 \times m_d) \) time.

In addition, as \( d \) increases, the measurement accuracy of our general scheme is expected to decrease. The reason is that, for each iteration \( j \) of the MLE derivation, an instance value of zero ratio \( V_{S_{j+1}} \) replaces its expected value \( q(N_{S_{j+1}}) = E(V_{S_{j+1}}) \) to get the MLE estimator \( \hat{N}_{S_{j+1}} \), which introduces a certain level of inaccuracy. This inaccuracy will accumulate as \( d \) increases. When \( d \) exceeds some value, e.g., 10, our \( d \)-point scheme may not work well as our current two-point and three-point schemes. However, in reality, the \( d \)-point traffic of interest usually has small values of \( d \), such as 2 or 3. Therefore, our general scheme is still sufficient to serve for most applications.

**V. SIMULATION RESULTS**

We perform simulations to evaluate the measurement accuracy of our solutions. In [19], we have compared our two-point scheme with two different settings, i.e., fixed bit array size \( m \) as in [17] versus fixed load factor \( f \) as in [19]. Hence, here, we focus on evaluating the measurement accuracy of our three-point scheme, also under the two different settings, i.e., fixed \( m \) versus fixed \( f \). Note that if we set \( m_x = m_y = m_z = m \) in (14), we can easily get the MLE formula for \( \hat{n}_{xyz} \) under the setting of fixed bit array size \( m \) for all RSUs.

We conduct two sets of simulations. The first set is to observe the accuracy of our three-point scheme when the single-point traffic volume of three RSUs are comparable, which means that the two settings, i.e., fixed \( m \) and fixed \( f \), are now equivalent. The simulations are controlled by the following parameters: \( n_x, n_y, n_z, n_{xy}, n_{xz}, n_{yz}, s, \) and \( f (f) \). Their values are chosen as follows: \( n_x = n_y = n_z = n \), where \( n = 50,000, 100,000, \) or 500,000, and \( n_{xy} \) varies from 0.01 to 0.5 \( n \), with a step size of 0.001 \( n \); \( s = 2, 5, 10 \), and \( m_x = m_y = m_z = m \) \((f_x = f_y = f_z = f)\) is chosen to achieve the optimal privacy \( p \) according to (6).

Figs. 2–4 show our simulation results when \( n = 50,000, 100,000, \) and 500,000, respectively. One can see that our three-point scheme is quite accurate under \( s = 2 \) (the measured three-point traffic volume \( n_{xyz} \) closely follows its real value \( n_{xyz} \) in the first plot of the three figures). With the increment of \( s \), the measurement results slightly diverge from their real values (see the last plot of the three figures), which means larger
values of \( s \) will bring in less-accurate measurement results. This conclusion is similar to what we get from the two-point traffic measurement scheme in [17]. Intuitively, if a vehicle \( v \) has a larger logical bit array, the chance for it to report the same bit index to different RSUs decreases, which means the common information collected by different RSUs is reduced. Therefore, the accuracy will also be affected for both the two-point and the three-point measurement. One can also observe that the measurement accuracy of our three-point scheme improves along with the increment of \( n \) (compare each plot in Fig. 2 with Fig. 4), which is a natural phenomenon since our estimator is derived from the statistical MLE method.

The second set of simulations is to observe the measurement accuracy of our three-point scheme when the single-point traffic volume of three RSUs may differ. When RSUs’ traffic volume is not the same, will the two settings, i.e., fixed \( m \) and fixed \( f \), begin to show differences as we expected? If so, how will the gap between RSUs’ single-point traffic volume influence the performance of our scheme under the two different settings? These are the questions to investigate.

Bearing these questions in mind, the second set of simulations is controlled by the following parameters: \( n_x, n_y, n_z, n_{xyz}, s, f, \) and \( m \). Their values are chosen as follows: \( n_x = 10,000, n_z = n_y = n_x \) or \( n_z = 4n_y = 16n_x \) or \( n_z = 8n_y = 64n_x, n_{xyz} \) varies from 0.01\( n_x \) to 0.5\( n_x \), with step size of 0.001\( n_x \), \( s \) is set to 2, 5, and 10. \( m \) is the fixed bit array size for all RSUs under the first setting, and \( f \) is the fixed load factor for all RSUs under the second setting. The values of \( m \) and \( f \) are chosen to guarantee minimum privacy of at least 0.5 under the two settings, respectively.

Figs. 5 and 6 show the simulation results for our three-point scheme with fixed bit array size \( m \) and fixed load factor.
Second Plot

Figs. 5 and 6); 2) when the traffic volume varies for different RSUs, our three-point scheme achieves far better accuracy under the fixed $f$ than under the fixed $m$, and the performance difference enlarges with the widening of the gap among the three RSUs’ single-point traffic volume (the second and third plots in Figs. 5 and 6). The two trends observed from the measurement results of our three-point scheme also coincide with those shown in our two-point scheme.

VI. CONCLUSION

In this paper, we focused on privacy-preserving multi-point traffic measurement, which serves for a broad spectrum of applications in transportation engineering. As far as we know, this work is the first to study the measurement of traffic covering more than two locations. Through variable-length bit arrays, we combine the automatic traffic collection by VCPSs with a rigorous statistical MLE methodology, to propose two novel efficient schemes for two-point and three-point traffic measurement. Our schemes can protect vehicles’ privacy and achieve sound measurement results. We also presented a general framework to measure traffic covering more than two locations. The proposed schemes have potential applications beyond vehicular networks, such as privacy-preserving traffic estimation in a subway system with tagged toll cards. It is also possible to use it for estimating the movement patterns of mobile users in a corporate wireless network.

REFERENCES


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