Optimal Leaf Sequencing with Elimination of Tongue-and-Groove Underdosage

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Abstract. The individual leaves of a multileaf collimator (MLC) have a tongue-and-groove or stepped-edge design to minimize leakage radiation between adjacent leaves. This design element has a drawback that it creates areas of underdosages in intensity modulated photon beams unless a leaf trajectory is specifically designed such that for any two adjacent leaf pairs, the direct exposure under the tongue-and-groove is equal to the lower of the direct exposures of the leaf pairs. In this work, we present a systematic study of the optimization of leaf sequencing algorithm for segmental multileaf collimator beam delivery that completely eliminates areas of underdosages due to tongue-and-groove or stepped-edge design of the MLC. Simultaneous elimination of tongue-and-groove effect and leaf interdigitation is also studied. This is an extension of our previous work (Kamath *et al* 2003) in which we described a leaf sequencing algorithm that is optimal for monitor unit (MU) efficiency under most common leaf movement constraints that includes minimum leaf separation. Compared to our previously published algorithm (without constraints), the new algorithms increase the number of sub-fields by appriximately 21% and 25%, respectively, but are optimal in MU efficiency for unidirectional schedules.

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1. Introduction

Intensity modulated radiation therapy (IMRT) delivered with multi-leaf collimator (MLC) in the step-and-shoot mode uses multiple static MLC segments to achieve intensity modulation. The sides of each leaf of a MLC have a protruding tongue or a step on one side that fits into a similar groove of the adjacent leaf. This results in different radiological path lengths across different parts of the leaves. Galvin *et al* (1993a) first described that the different radiological path lengths manifest themselves as varying doses in a plane perpendicular to the leaf motion. The low dose region between two adjacent leaves was classified as the tongue-and-groove effect. In an IMRT treatment using an MLC, the tongue-and-groove effect occurs when the tongue, or the groove or both for the most time during treatment delivery cover the overlapping region between two adjacent pairs of leaves. As pointed out by many investigators, the tongue-and-groove arrangement always results in underdosages of as much as 10-25% in the treatment fields in both static and dynamic multileaf collimation (Galvin *et al* 1993a, b, Chui *et al* 1994, Mohan 1995, Wang *et al* 1996, Sykes and Williams 1998).

Several recent publications (van Santvoort and Heijmen 1996, Webb *et al* 1997, Convery and Webb 1998, Dirkx *et al* 1998, Xia and Verhey 1998) have shown that the tongue-and-groove effect can be significantly reduced by synchronization of the leaves. However, the cost of leaf synchronization is usually an increase in the total number of sub fields and monitor units. van Santvoort and Heijmen (1996) propose an algorithm to eliminate tongue-and-groove effects for DMLC treatment plans. Although they note that their algorithm increases the number of monitor units, they do not examine the optimality or suboptimality of the plans they obtain. We recently published a paper (Kamath *et al* 2003) that gave mathematical formalisms and rigorous proofs of leaf sequencing algorithms for segmental multileaf collimation, which maximize MU efficiency. We proved that our leaf sequencing algorithms that explicitly account for minimum leaf separation obtain feasible unidirectional solutions that are optimal. We now extend that work to develop algorithms that explicitly account for leaf interdigitation and the tongue-and-groove effect and are optimal in MU efficiency for unidirectional schedules. We show also that the algorithm of van Santvoort and Heijmen (1996) obtains optimal dynamic multileaf collimation treatment schedules.

2. Methods

2.1. Discrete Profile

We consider delivery of optimal radiation intensity map produced by the inverse treatment planning system. It is important to note that the intensity profile output from the optimizer is piecewise continuous. The intensity matrix from the optimizer generally has a spatial resolution that is similar to the smallest beamlet size. The beamlet size ranges from 5-10 mm. As described in our earlier work (Kamath *et al* 2003) $I(x)$ is the desired intensity profile. The discretized profile from the optimizer gives the intensity values at sample points $x_0, x_1, x_2, \ldots, x_m$. We assume that the sample points are uniformly spaced and that $\Delta x = x_{i+1} - x_i, 0 \le i < m$. $I(x)$ is assigned the value $I(x_i)$ for $x_i \le x < x_{i+1}$, for each i. Now, $I(x_i)$ is our desired intensity profile, i.e., $I(x_i)$ is a measure of the number of MUs for which x_i, $0 \le i < m$, needs to be exposed. Figure 1 shows a discretized profile, which is the output from the optimizer. The discretized profile is delivered either with the Segmental Multileaf Collimation (SMLC) method or with Dynamic Multileaf Collimation (DMLC). An SMLC sequence can be transformed to a dynamic leaf sequence by allowing both leaves to start at the same point and close together at the same point, so that they sweep across the same spatial interval. We develop our theory for the SMLC delivery.

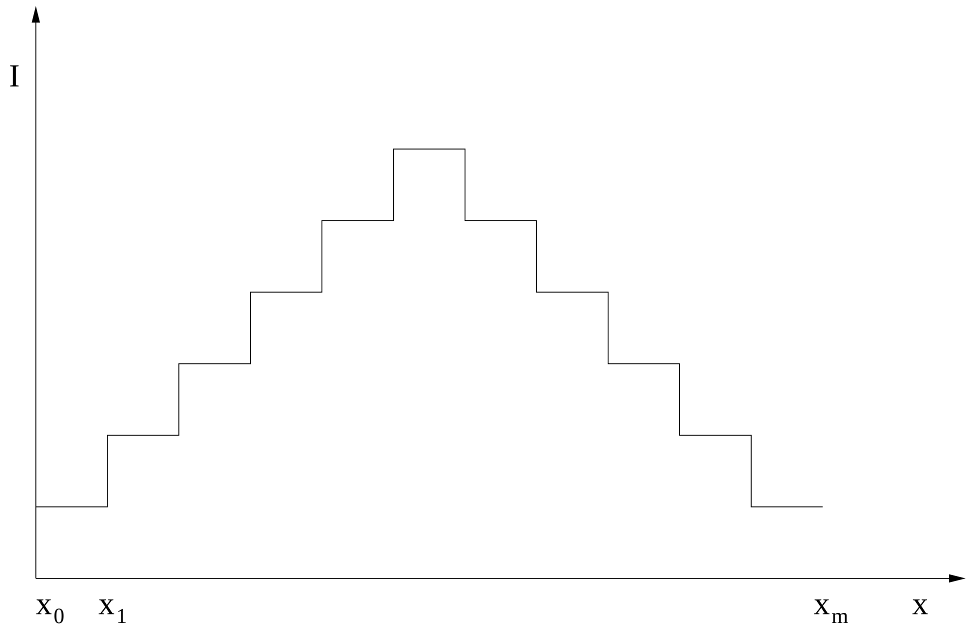

Figure 1. Discretized profile

2.2. Movement of Leaves

In our analysis we will assume that the beam delivery begins when the pair of leaves is at the left most position. The initial position of the leaves is x_0. In this paper, we assume that leaves may move only from left to right. That is unidirectional leaf movement is assumed. Figure 2 illustrates the leaf trajectory during SMLC delivery. Let $I_l(x_i)$ and $I_r(x_i)$ respectively denote the amount of Monitor Units (MUs) delivered when the left and right leaves leave position x_i. Consider the motion of the left leaf. The left leaf begins at x_0 and remains here until $I_l(x_0)$ MUs have been delivered. At this time the left leaf is moved to x_1, where it remains until $I_l(x_1)$ MUs have been delivered. The left leaf then moves to x_3 where it remains until $I_l(x_3)$ MUs have been delivered. At this time, the left leaf is moved to x_6, where it remains until $I_l(x_6)$ MUs have been delivered. The final movement of the left leaf is to x_7, where it remains until $I_l(x_7) = I_{max}$ MUs have been delivered. At this time the machine is turned off. The total therapy time, $TT(I_l, I_r)$, is the time needed to deliver I_{max} MUs. Note that we use the term therapy time to refer to the beam-on time. The right leaf starts at x_2; moves to x_4 when $I_r(x_2)$ MUs have been delivered; moves to x_5 when $I_r(x_4)$ MUs have been delivered and so on. Note that the machine is off when a leaf is in motion. We make the following observations:

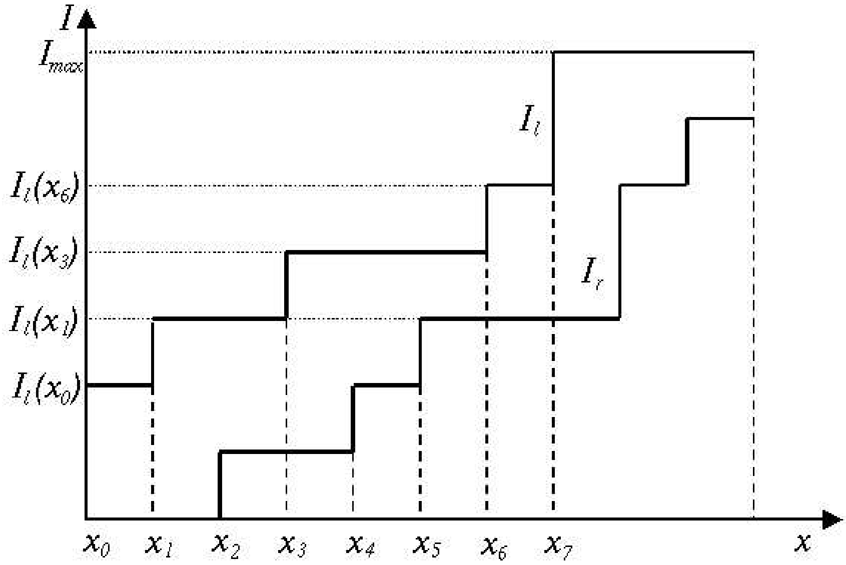

Figure 2. Leaf trajectory during SMLC delivery

(i) All MUs that are delivered along a radiation beam along x_i before the left leaf passes x_i fall on it. Greater the x value, later the leaf passes that position. Therefore $I_l(x_i)$ is a non-decreasing function.

(ii) All MUs that are delivered along a radiation beam along x_i before the right leaf passes x_i, are blocked by the leaf. Greater the x value, later the leaf passes that position. Therefore $I_r(x_i)$ is also a non-decreasing function.

From these observations we notice that the net amount of MUs delivered at a point is given by $I_l(x_i) - I_r(x_i)$, which must be the same as the desired profile $I(x_i)$.

2.3. Optimal Algorithm without Constraints

2.3.1. Optimal Algorithm for Single Pair of Leaves. Once the desired intensity profile, $I(x_i)$ is known, the single leaf pair problem becomes that of determining the individual *intensity profiles* to be delivered by the left and right leaves, I_l and I_r such that:

$$I(x_i) = I_l(x_i) - I_r(x_i), 0 \leq i \leq m \tag{1}$$

We refer to (I_l, I_r) as the *treatment plan* (or simply *plan*) for I. Once we obtain the plan, we will be able to determine the movement of both left and right leaves during the therapy. For each i, the left leaf can be allowed to pass x_i when the source has delivered $I_l(x_i)$ MUs. Also, we can allow the right leaf to pass x_i when the source has delivered $I_r(x_i)$ MUs. A plan is *unidirectional* if $I_l(x)$ and $I_r(x)$ are unique, $x_0 \leq x \leq x_m$, i.e., each leaf passes over each point only once. Ma *et al* (1998) present an algorithm (Figure 3) that obtains unidirectional plans. They prove that the plans obtained are optimal in therapy time (Theorem 1). Their proof relies on the results of Boyer and Strait (1997), Spirou and Chui (1994) and Stein *et al* (1994). Kamath *et al* (2003) provide a simpler proof that also yields Corollary 1. Kamath *et al* (2003) also prove Lemma 1.

Algorithm SINGLEPAIR

$I_l(x_0) = I(x_0)$
$I_r(x_0) = 0$
  For $j = 1$ to m do
    If $(I(x_j) \geq I(x_{j-1})$
      $I_l(x_j) = I_l(x_{j-1}) + I(x_j) - I(x_{j-1})$
      $I_r(x_j) = I_r(x_{j-1})$
    Else
      $I_r(x_j) = I_r(x_{j-1}) + I(x_{j-1}) - I(x_j)$
      $I_l(x_j) = I_l(x_{j-1})$
  End for

Figure 3. Obtaining a unidirectional plan

Theorem 1 *Algorithm SINGLEPAIR obtains plans that are optimal in therapy time even when bidirectional leaf movement is permitted.*

Corollary 1 *Let $I(x_i)$, $0 \leq i \leq m$ be a desired profile. Let $I_l(x_i)$ and $I_r(x_i)$, $0 \leq i \leq m$ be the left and right leaf profiles generated by Algorithm SINGLEPAIR. $I_l(x_i)$ and $I_r(x_i)$, $0 \leq i \leq m$ define optimal therapy time unidirectional left and right leaf profiles for $I(x_i)$, $0 \leq i \leq j$.*

Lemma 1 *Let (I_L, I_R) be any treatment plan for I.*
(a) $\Delta(x_i) = I_L(x_i) - I_l(x_i) = I_R(x_i) - I_r(x_i) \geq 0, 0 \leq i \leq m$.
(b) $\Delta(x_i)$ is a non-decreasing function.

2.3.2. Optimal Algorithm for Multiple Leaf Pairs. We use a single pair of leaves to deliver intensity profiles defined along the axis of the pair of leaves. However, in a real application, we need to deliver intensity profiles defined over a 2-D region. We use Multi-Leaf Collimators (MLCs) to deliver such profiles. An MLC is composed of multiple pairs of leaves with parallel axes. Figure 4 shows an MLC that has three pairs of leaves - $(L1, R1), (L2, R2)$ and $(L3, R3)$. $L1, L2, L3$ are left leaves and $R1, R2, R3$ are right leaves. Each pair of leaves is controlled independently. If there are no constraints on the leaf movements, we divide the desired profile into a set of parallel profiles defined along the axes of the leaf pairs. Each leaf pair i then delivers the plan for the corresponding intensity profile $I_i(x)$. The set of plans of all leaf pairs forms the solution set. We refer to this set as the *treatment schedule* (or simply *schedule*). A schedule in which all plans are unidirectional is a *unidirectional schedule*. Only unidirectional schedules are considered in this paper.

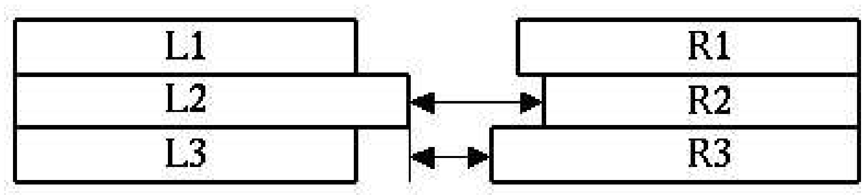

Figure 4. Inter-pair minimum separation constraint

Assume we have n pairs of leaves. For each pair, we have m sample points. The input is represented as a matrix with n rows and m columns, where the ith row represents the desired intensity profile to be delivered by the ith pair of leaves. We apply Algorithm SINGLEPAIR to determine the optimal plan for each of the n leaf pairs. This method of generating schedules is described in Algorithm MULTIPAIR (Figure 5). Since the individual plans of the leaf pairs are optimal in therapy time, so is the resulting schedule (Kamath *et al* (2003)).

Algorithm MULTIPAIR
For($i = 1; i \leq n; i++$)

Apply Algorithm SINGLEPAIR to the ith pair of leaves to obtain plan (I_{il}, I_{ir}) that delivers the intensity profile $I_i(x)$.

End For

Figure 5. Obtaining a schedule

3. Optimal Algorithm with Interdigitation and Tongue-and-Groove Constraints

3.1. Interdigitation Constraint

In practical situations, there are some constraints on the movement of the leaves. The minimum separation constraint requires that opposing pairs of leaves be separated by at least some distance (S_{min}) at all times during beam delivery. In some MLCs this constraint is applied not only to opposing pairs of leaves, but also to opposing leaves of neighboring pairs. For example, in Figure 4, $L1$ and $R1$, $L2$ and $R2$, $L3$ and $R3$, $L1$ and $R2$, $L2$ and $R1$, $L2$ and $R3$, $L3$ and $R2$ are pairwise subject to the constraint. We use the term *intra-pair minimum separation constraint* to refer to the constraint imposed on an opposing pair of leaves and *inter-pair minimum separation constraint* to refer to the constraint imposed on opposing leaves of neighboring pairs. The *inter-pair minimum separation constraint* with $S_{min} = 0$ is of special interest and is referred to as the *interdigitation constraint.*

3.2. Tongue-and-Groove Underdosage Effect

In most commercially available MLCs, there is a tongue-and-groove arrangement at the interface between adjacent leaves. A cross section of two adjacent leaves is depicted in Figure 6. The width of the tongue-and-groove region is l. The area under this region gets underdosed due to the mechanical arrangement. Figure 7 shows a beams-eye view of the region to be treated by two adjacent leaf pairs, t and $t+1$. Consider the shaded rectangular areas $A_t(x_i)$ and $A_{t+1}(x_i)$ that require exactly $I_t(x_i)$ and $I_{t+1}(x_i)$ MUs to be delivered, respectively. The tongue-and-groove overlap area between the two leaf pairs over the sample point x_i, $A_{t,t+1}(x_i)$, is colored black. Let the amount of MUs delivered in $A_{t,t+1}(x_i)$ be $I_{t,t+1}(x_i)$. Ignoring leaf transmission, the following lemma is a consequence of the fact that $A_{t,t+1}(x_i)$ is exposed only when both $A_t(x_i)$ and $A_{t+1}(x_i)$ are exposed.

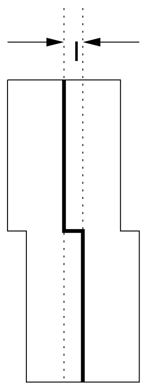

Figure 6. Cross section of leaves

Lemma 2 $I_{t,t+1}(x_i) \leq \min\{I_t(x_i), I_{t+1}(x_i)\}$, $0 \leq i \leq m$, $1 \leq t < n$.

Schedules in which $I_{t,t+1}(x_i) = \min\{I_t(x_i), I_{t+1}(x_i)\}$ are said to be free of tongue-and-groove underdosage effects.

Unless treatment schedules are carefully designed, it is possible that $I_{t,t+1}(x_i) << \min\{I_t(x_i), I_{t+1}(x_i)\}$ for some i and t. For example, in a schedule in which $I_{tr}(x_i) = 30$, $I_{tl}(x_i) = 50$, $I_{(t+1)r}(x_i) = 50$ and $I_{(t+1)l}(x_i) = 60$, we have $I_{t,t+1}(x_i) = I_{tl}(x_i) -$

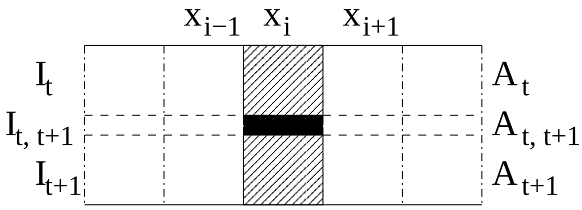

Figure 7. Tongue-and-groove effect

$I_{(t+1)r}(x_i) = 50 - 50 = 0$. Note that in this case, $\min\{I_t(x_i), I_{t+1}(x_i)\} = I_{(t+1)l}(x_i) - I_{tl}(x_i) = 60 - 50 = 10$. It is clear from this example that $I_{t,t+1}(x_i)$ could be 0 even when $\min\{I_t(x_i), I_{t+1}(x_i)\}$ is arbitrarily large.

3.3. Algorithms

Kamath *et al* (2003) present an algorithm that generates a schedule that satisfies inter-pair minimum separation constraint. The schedule is optimal in therapy time. However, it does not account for the tongue-and-groove effect. In this section, we present two algorithms. Algorithm TONGUEANDGROOVE generates minimum therapy time unidirectional schedules that are free of tongue-and-groove underdosage and maybe used for MLCs that do not have a interdigitation constraint. Algorithm TONGUEANDGROOVE-ID generates minimum therapy time unidirectional schedules that are free of tongue-and-groove underdosage while simultaneously satisfying the interdigitation constraint and is for MLCs that have an interdigitation constraint.

The following lemma provides a necessary and sufficient condition for a unidirectional schedule to be free of tongue-and-groove underdosage effects.

Lemma 3 *A unidirectional schedule is free of tongue-and-groove underdosage effects if and only if,*

(a) $I_t(x_i) = 0$ *or* $I_{t+1}(x_i) = 0$*, or*

(b) $I_{tr}(x_i) \leq I_{(t+1)r}(x_i) \leq I_{(t+1)l}(x_i) \leq I_{tl}(x_i)$*, or*

(c) $I_{(t+1)r}(x_i) \leq I_{tr}(x_i) \leq I_{tl}(x_i) \leq I_{(t+1)l}(x_i)$*,*

$0 \leq i \leq m$, $1 \leq t < n$.

Proof: It is easy to see that any schedule that satisfies the above conditions is free of tongue-and-groove underdosage effects. So what remains is for us to show that every schedule that is free of tongue-and-groove underdosage effects satisfies the above conditions. Consider any such schedule. If condition (a) is satisfied at every i and t, the proof is complete. So assume i and t such that $I_t(x_i) \neq 0$ and $I_{t+1}(x_i) \neq 0$ exist. We need to show that either (b) or (c) is true for this value of i and t. Since the schedule is free of tongue-and-groove effects,

$$I_{t,t+1}(x_i) = \min\{I_t(x_i), I_{t+1}(x_i)\} > 0 \quad (2)$$

From the unidirectional constraint, it follows that $A_{t,t+1}(x_i)$ first gets exposed when both right leaves pass x_i, and it remains exposed till the first of the left leaves passes x_i. Further, if a left leaf passes x_i before a neighboring right leaf passes x_i, $A_{t,t+1}(x_i)$ is not exposed at all. So,

$$I_{t,t+1}(x_i) = \max\{0, I_{(t,t+1)l}(x_i) - I_{(t,t+1)r}(x_i)\} \tag{3}$$

where $I_{(t,t+1)r}(x_i) = \max\{I_{tr}(x_i), I_{(t+1)r}(x_i)\}$ and $I_{(t,t+1)l}(x_i) = \min\{I_{tl}(x_i), I_{(t+1)l}(x_i)\}$. From 2 and 3, it follows that

$$I_{t,t+1}(x_i) = I_{(t,t+1)l}(x_i) - I_{(t,t+1)r}(x_i) \tag{4}$$

Consider the case $I_t(x_i) \geq I_{t+1}(x_i)$. Suppose that $I_{tr}(x_i) > I_{(t+1)r}(x_i)$. It follows that $I_{(t,t+1)r}(x_i) = I_{tr}(x_i)$ and $I_{(t,t+1)l}(x_i) = I_{(t+1)l}(x_i)$. Now from 4, we get

$$\begin{aligned} I_{t,t+1}(x_i) &= I_{(t+1)l}(x_i) - I_{tr}(x_i) \\ &< I_{(t+1)l}(x_i) - I_{(t+1)r}(x_i) \\ &= I_{t+1}(x_i) \\ &\leq I_t(x_i) \end{aligned} \tag{5}$$

So $I_{t,t+1}(x_i) < \min\{I_t(x_i), I_{t+1}(x_i)\}$, which contradicts 2. So

$$I_{tr}(x_i) \leq I_{(t+1)r}(x_i) \tag{6}$$

Now, suppose that $I_{tl}(x_i) < I_{(t+1)l}(x_i)$. From $I_t(x_i) \geq I_{t+1}(x_i)$, it follows that $I_{(t,t+1)l}(x_i) = I_{tl}(x_i)$ and $I_{(t,t+1)r}(x_i) = I_{(t+1)r}(x_i)$. Hence, from 4, we get

$$\begin{aligned} I_{t,t+1}(x_i) &= I_{tl}(x_i) - I_{(t+1)r}(x_i) \\ &< I_{(t+1)l}(x_i) - I_{(t+1)r}(x_i) \\ &= I_{t+1}(x_i) \\ &\leq I_t(x_i) \end{aligned} \tag{7}$$

So $I_{t,t+1}(x_i) < \min\{I_t(x_i), I_{t+1}(x_i)\}$, which contradicts 2. So

$$I_{tl}(x_i) \geq I_{(t+1)l}(x_i) \tag{8}$$

From 6 and 8, we can conclude that when $I_t(x_i) \geq I_{t+1}(x_i)$, (b) is true. Similarly one can show that when $I_{t+1}(x_i) \geq I_t(x_i)$, (c) is true. ■

Lemma 3 is equivalent to saying that the time period for which a pair of leaves (say pair t) exposes the region $A_{t,t+1}(x_i)$ is completely contained by the time period for which pair $t+1$ exposes region $A_{t,t+1}(x_i)$, or vice versa, whenever $I_t(x_i) \neq 0$ and $I_{t+1}(x_i) \neq 0$. Note that if either $I_t(x_i)$ or $I_{t+1}(x_i)$ is zero the containment is not necessary. We will refer to the necessary and sufficient condition of Lemma 3 as the *tongue-and-groove constraint condition*. Schedules that satisfy this condition will be said to satisfy the tongue-and-groove constraint. van Santvoort and Heijmen (1996) present an algorithm that generates schedules that satisfy the tongue-and-groove constraint for DMLC.

Xia and Verhey (1998) claim that every schedule that violates the interdigitation constraint also violates the tongue-and-groove constraint. We demonstrate with a counterexample that this is not necessarily the case. The intensity matrix shown in Figure 8(a) can be exposed in a single segment as shown in Figure 8(b). The segment is free of tongue-and-groove constraint violations, while it clearly violates the interdigitation constraint.

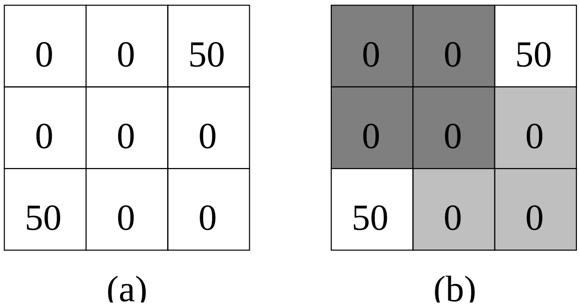

Figure 8. The intensity matrix shown in (a) can be treated using a single segment with 50 MUs as shown in (b). Areas shaded dark are covered by left leaves and those shaded light are covered by right leaves. Areas not shaded are exposed. Interdigitation constraint violation occurs though there is no tongue-and-groove violation.

3.3.1. Elimination of tongue-and-groove effect. Note that the schedule generated by Algorithm MULTIPAIR may violate the tongue-and-groove constraint. If the schedule has no tongue-and-groove constraint violations, it is the desired optimal schedule. If there are violations in the schedule, we eliminate all violations of the tongue-and-groove constraint starting from the left end, i.e., from x_0. To eliminate the violations, we modify those plans of the schedule that cause the violations. We scan the schedule from x_0 along the positive x direction looking for the least x_w at which there exist leaf pairs u, t, $t \in \{u-1, u+1\}$, that violate the constraint at x_w. After rectifying the violation at x_w we look for other violations. Since the process of eliminating a violation at x_w, may at times, lead to new violations at x_w, we need to search afresh from x_w every time a modification is made to the schedule. However, we will prove a bound of $O(n)$ on the number of violations that can occur at x_w. After eliminating all violations at a particular sample point, x_w, we move to the next point, i.e., we increment w and look for possible violations at the new point. We continue the scanning and modification process until no tongue-and-groove constraint violations exist. Algorithm TONGUEANDGROOVE (Figure 9) outlines the procedure.

Let $M = ((I_{1l}, I_{1r}), (I_{2l}, I_{2r}), \ldots, (I_{nl}, I_{nr}))$ be the schedule generated by Algorithm MULTIPAIR for the desired intensity profile.
Let $N(p) = ((I_{1lp}, I_{1rp}), (I_{2lp}, I_{2rp}), \ldots, (I_{nlp}, I_{nrp}))$ be the schedule obtained after Step iv of Algorithm TONGUEANDGROOVE is applied p times to the input schedule M. Note that $M = N(0)$.

To illustrate the modification process we use examples. To make things easier, we only show two neighboring pairs of leaves. Suppose that the $(p+1)$th violation occurs between the leaves of pair u and pair $t = u+1$ at x_w. Note that $I_{tlp}(x_w) \neq I_{ulp}(x_w)$, as otherwise, either (b) or (c) of Lemma 3 is true. In case $I_{tlp}(x_w) > I_{ulp}(x_w)$, swap

Algorithm TONGUEANDGROOVE

(i) $x = x_0$

(ii) While (there is a tongue-and-groove violation) do

(iii) Find the least x_w, $x_w \geq x$, such that there exist leaf pairs u, $u+1$, that violate the tongue-and-groove constraint at x_w.

(iv) Modify the schedule to eliminate the violation between leaf pairs u and $u+1$.

(v) $x = x_w$

(vi) End While

Figure 9. Obtaining a schedule under the tongue-and-groove constraint

u and t. Now, we have $I_{tlp}(x_w) < I_{ulp}(x_w)$. *In the sequel, we refer to these u and t values as the u and t of Algorithm TONGUEANDGROOVE.* From Lemma 3 and the fact that a violation has occurred, it follows that $I_{trp}(x_w) < I_{urp}(x_w)$. To remove this tongue-and-groove constraint violation, we modify (I_{tlp}, I_{trp}). The other profiles of $N(p)$ are not modified.

The new plan for pair t, $(I_{tl(p+1)}, I_{tr(p+1)})$ is as defined below.
If $I_{ulp}(x_w) - I_{tlp}(x_w) \leq I_{urp}(x_w) - I_{trp}(x_w)$, then

$$I_{tl(p+1)}(x) = \begin{cases} I_{tlp}(x) & x_0 \leq x < x_w \\ I_{tlp}(x) + \Delta I & x_w \leq x \leq x_m \end{cases} \tag{9}$$

where $\Delta I = I_{ulp}(x_w) - I_{tlp}(x_w)$. $I_{tr(p+1)}(x) = I_{tl(p+1)}(x) - I_t(x)$, where $I_t(x)$ is the target profile to be delivered by the leaf pair t.
Otherwise,

$$I_{tr(p+1)}(x) = \begin{cases} I_{trp}(x) & x_0 \leq x < x_w \\ I_{trp}(x) + \Delta I' & x_w \leq x \leq x_m \end{cases} \tag{10}$$

where $\Delta I' = I_{urp}(x_w) - I_{trp}(x_w)$. $I_{tl(p+1)}(x) = I_{tr(p+1)}(x) + I_t(x)$, where $I_t(x)$ is the target profile to be delivered by the leaf pair t.
The former case is illustrated in Figure 10 and the latter is illustrated in Figure 11. Note that our strategy for plan modification is similar to that used by van Santvoort and Heijmen (1996) to eliminate a tongue-and-groove violation for dynamic multileaf collimator plans.

Since $(I_{tl(p+1)}, I_{tr(p+1)})$ differs from (I_{tlp}, I_{trp}) for $x \geq x_w$ there is a possibility that $N(p+1)$ is involved in tongue-and-groove violations for $x \geq x_w$. Since none of the other leaf profiles are changed from those of $N(p)$ no tongue-and-groove constraint violations are possible in $N(p+1)$ for $x < x_w$. One may also verify that since I_{tl0} and I_{tr0} are non-decreasing functions of x, so also are I_{tlp} and I_{trp}, $p > 0$.

Lemma 4 *Let $F = ((I'_{1l}, I'_{1r}), (I'_{2l}, I'_{2r}), \ldots, (I'_{nl}, I'_{nr}))$ be any unidirectional schedule for the desired profile that satisfies the tongue-and-groove constraint. Let $S(p)$, be the following assertions.*

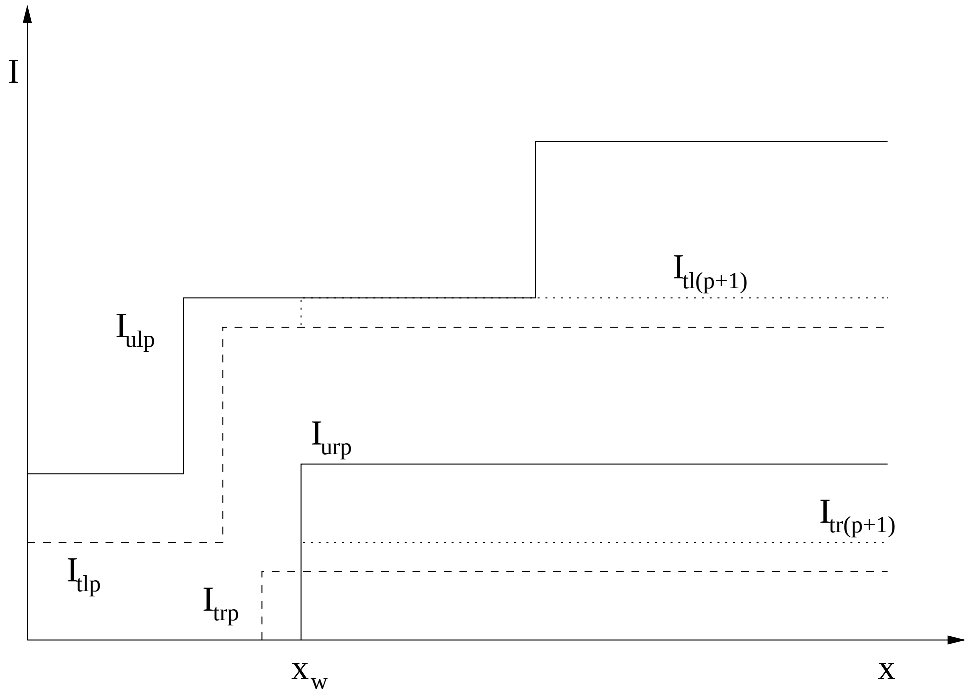

Figure 10. Tongue-and-groove constraint violation: case1

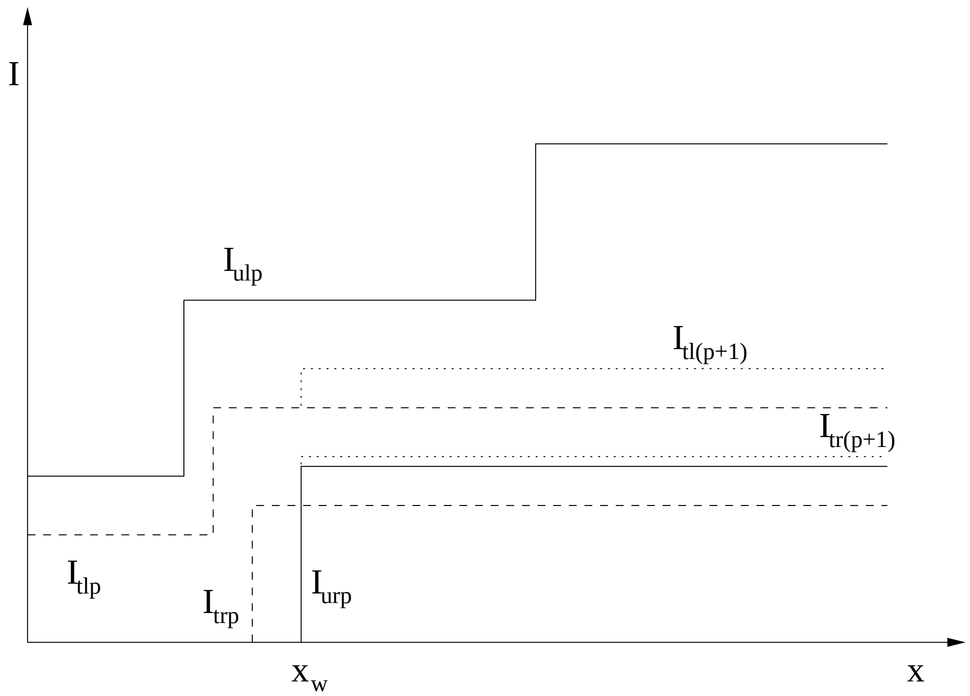

Figure 11. Tongue-and-groove constraint violation: case2 (close parallel dotted and solid line segments overlap, they have been drawn with a small separation to enhance readability)

(a) $I'_{il}(x) \geq I_{ilp}(x)$, $0 \leq i \leq n, x_0 \leq x \leq x_m$

(b) $I'_{ir}(x) \geq I_{irp}(x)$, $0 \leq i \leq n, x_0 \leq x \leq x_m$

$S(p)$ *is true for* $p \geq 0$.

Proof: The proof is by induction on p.

(i) Consider the base case, $p = 0$. From Corollary 1 and the fact that the plans $(I_{il0}, I_{ir0}), 0 \leq i \leq n$, are generated using Algorithm SINGLEPAIR, it follows that $S(0)$ is true.

(ii) Assume $S(p)$ is true. Suppose Algorithm TONGUEANDGROOVE finds a next violation and modifies the schedule $N(p)$ to $N(p+1)$. Suppose that the next violation occurs between leaf pairs u and t, $t \in \{u-1, u+1\}$. Hence, $I_{tlp}(x_w) < I_{ulp}(x_w)$. We modify pair t's plan for $x \geq x_w$, to eliminate the violation. All other plans in the schedule remain unaltered. Therefore, to establish $S(p+1)$ it suffices to prove that

$$I'_{tl}(x) \geq I_{tl(p+1)}(x), x \geq x_w \tag{11}$$

$$I'_{tr}(x) \geq I_{tr(p+1)}(x), x \geq x_w \tag{12}$$

We need prove only one of these two relationships since $I'_{tl}(x) - I'_{tr}(x) = I_{tl(p+1)}(x) - I_{tr(p+1)}(x), x_0 \leq x \leq x_m$ (i.e., $I'_{tl}(x) - I_{tl(p+1)}(x) = I'_{tr}(x) - I_{tr(p+1)}(x)$). We now consider pair t's plan for $x \geq x_w$ and show that Equation 11 is always true. This, in turn, implies that $S(p+1)$ is true whenever $S(p)$ is true and hence completes the proof.

Suppose that $I_{ulp}(x_w) - I_{tlp}(x_w) \leq I_{urp}(x_w) - I_{trp}(x_w)$. Then, $I_{ulp}(x_w) - I_{urp}(x_w) \leq I_{tlp}(x_w) - I_{trp}(x_w)$, i.e., $I_u(x_w) \leq I_t(x_w)$. Clearly, in a schedule F, which is free of tongue-and-groove violation between pairs u and t at x_w, only the ordering $I'_{tr}(x_w) \leq I'_{ur}(x_w) \leq I'_{ul}(x_w) \leq I'_{tl}(x_w)$ is possible (refer Lemma 3) in this scenario (the exception being when $I_u(x_w) = I_t(x_w)$, in which case all the quantities in the ordering are equal). From this ordering, $I'_{tl}(x_w) \geq I'_{ul}(x_w)$. From the induction hypothesis, $I'_{ul}(x_w) \geq I_{ulp}(x_w) = I_{ul(p+1)}(x_w)$. From Equation 9, $I_{tl(p+1)}(x_w) = I_{ulp}(x_w) = I_{ul(p+1)}(x_w)$. Hence, $I'_{tl}(x_w) \geq I_{tl(p+1)}(x_w)$ when $I_{ulp}(x_w) - I_{tlp}(x_w) \leq I_{urp}(x_w) - I_{trp}(x_w)$. A symmetric argument can be presented to show that $I'_{tl}(x_w) \geq I_{tl(p+1)}(x_w)$ when $I_{ulp}(x_w) - I_{tlp}(x_w) > I_{urp}(x_w) - I_{trp}(x_w)$. So $I'_{tl}(x_w) \geq I_{tl(p+1)}(x_w)$.

It remains to be proved that $I'_{tl}(x_i) \geq I_{tl(p+1)}(x_i)$, $w < i \leq m$. Suppose for a contradiction that $\exists v > w$, $I'_{tl}(x_v) < I_{tl(p+1)}(x_v)$. Let $\Delta I'' = I'_{tl}(x_w) - I_{tl0}(x_w)$. Note that $I'_{tl}(x_w) \geq I_{tl(p+1)}(x_w)$ and so $\Delta I'' \geq I_{tl(p+1)}(x_w) - I_{tl0}(x_w) = I_{tl(p+1)}(x_v) - I_{tl0}(x_v)$ (from the working of Algorithm TONGUEANDGROOVE). Define a new plan (I''_{tl}, I''_{tr}) as follows:

$$I''_{tl}(x_i) = \begin{cases} I_{tl0}(x_i) & i < w \\ I'_{tl}(x_i) - \Delta I'' & w \leq i \leq m \end{cases}$$

$$I''_{tr}(x_i) = \begin{cases} I_{tr0}(x_i) & i < w \\ I'_{tr}(x_i) - \Delta I'' & w \le i \le m \end{cases}$$

Note that $I''_{tl}(x_w) = I'_{tl}(x_w) - \Delta I'' = I_{tl0}(x_w) \ge I_{tl0}(x_{w-1}) = I''_{tl}(x_{w-1})$. Similarly, $I''_{tr}(x_w) \ge I''_{tr}(x_{w-1})$. So (I''_{tl}, I''_{tr}) is a plan for t. Also, $I''_{tl}(x_v) = I'_{tl}(x_v) - \Delta I'' \le I'_{tl}(x_v) - I_{tl(p+1)}(x_v) + I_{tl0}(x_v)$ (since $\Delta I'' \ge I_{tl(p+1)}(x_v) - I_{tl0}(x_v)$ as explained above). From this and our assumption that $I'_{tl}(x_v) < I_{tl(p+1)}(x_v)$, it follows that $I''_{tl}(x_v) < I_{tl0}(x_v)$. Since plan (I_{tl0}, I_{tr0}) was generated using Algorithm SINGLEPAIR, $I''_{tl}(x_v) < I_{tl0}(x_v)$ violates Corollary 1. So our assumption was wrong and hence Equation 11 is always true.

■

3.3.2. Elimination of tongue-and-groove effect and interdigitation. As we have pointed out, the elimination of tongue-and-groove constraint violations does not guarantee elimination of interdigitation constraint violations. Therefore the schedule generated by Algorithm TONGUEANDGROOVE may not be free of interdigitation violations. The algorithm we propose for obtaining schedules that simultaneously satisfy both constraints, Algorithm TONGUEANDGROOVE-ID, is similar to Algorithm TONGUEANDGROOVE. The only difference between the two algorithms lies in the definition of the constraint condition. To be precise we make the following definition.

Definition 1 *A unidirectional schedule is said to satisfy the tongue-and-groove-id constraint if*

(a) $I_{tr}(x_i) \le I_{(t+1)r}(x_i) \le I_{(t+1)l}(x_i) \le I_{tl}(x_i)$, *or*

(b) $I_{(t+1)r}(x_i) \le I_{tr}(x_i) \le I_{tl}(x_i) \le I_{(t+1)l}(x_i)$,

for $0 \le i \le m$, $1 \le t < n$.

The only difference between this constraint and the tongue-and-groove constraint is that this constraint enforces condition *(a)* or *(b)* above to be true at all sample points x_i including those at which $I_t(x_i) = 0$ and/or $I_{t+1}(x_i) = 0$.

Lemma 5 *A schedule satisfies the tongue-and-groove-id constraint iff it satisfies the tongue-and-groove constraint and the interdigitation constraint.*

Proof: It is obvious that the tongue-and-groove-id constraint subsumes the tongue-and-groove constraint. If a schedule has a violation of the interdigitation constraint, $\exists i, t$, $I_{(t+1)l}(x_i) < I_{tr}(x_i)$ or $I_{tl}(x_i) < I_{(t+1)r}(x_i)$. From Definition 1, it follows that schedules that satisfy the tongue-and-groove-id constraint do not violate the interdigitation constraint. Therefore a schedule that satisfies the tongue-and-groove-id constraint satisfies the tongue-and-groove constraint and the interdigitation constraint.
For the other direction of the proof, consider a schedule O that satisfies the tongue-and-groove constraint and the interdigitation constraint. From the fact that O satisfies the tongue-and-groove constraint and from Lemma 3 and Definition 1, it only remains to be proved that for schedule O,

(a) $I_{tr}(x_i) \leq I_{(t+1)r}(x_i) \leq I_{(t+1)l}(x_i) \leq I_{tl}(x_i)$, or

(b) $I_{(t+1)r}(x_i) \leq I_{tr}(x_i) \leq I_{tl}(x_i) \leq I_{(t+1)l}(x_i)$,

whenever $I_t(x_i) = 0$ or $I_{t+1}(x_i) = 0$, $0 \leq i \leq m$, $1 \leq t < n$.
When $I_t(x_i) = 0$,

$$I_{tl}(x_i) = I_{tr}(x_i) \tag{13}$$

Since O satisfies the interdigitation constraint,

$$I_{tr}(x_i) \leq I_{(t+1)l}(x_i) \tag{14}$$

and

$$I_{(t+1)r}(x_i) \leq I_{tl}(x_i) \tag{15}$$

From Equations 13, 14 and 15, we get $I_{(t+1)r}(x_i) \leq I_{tr}(x_i) = I_{tl}(x_i) \leq I_{(t+1)l}(x_i)$. So (b) is true whenever $I_t(x_i) = 0$. Similarly, (a) is true whenever $I_{t+1}(x_i) = 0$. Therefore, O satisfies the tongue-and-groove-id constraint. ■

Algorithm TONGUEANDGROOVE-ID finds violations of the tongue-and-groove-id constraint from left to right in exactly the same manner in which Algorithm TONGUEANDGROOVE detects tongue-and-groove violations. Also, the violations are eliminated as before, i.e., as prescribed by Equations 9 and 10 and illustrated in Figures 10 and 11, respectively. Algorithm TONGUEANDGROOVE-ID is shown in Figure 12. All notation used in the algorithm and the related discussion in the remainder of Section 3.3.2 is also the same as that used in Section 3.3.1 and corresponds directly to the usage in Algorithm TONGUEANDGROOVE.

Algorithm TONGUEANDGROOVE-ID

(i) $x = x_0$

(ii) While (there is a tongue-and-groove-id violation) do

(iii) Find the least x_w, $x_w \geq x$, such that there exist leaf pairs u, $u+1$, that violate the tongue-and-groove-id constraint at x_w.

(iv) Modify the schedule to eliminate the violation between leaf pairs u and $u+1$.

(v) $x = x_w$

(vi) End While

Figure 12. Obtaining a schedule under both the constraints

Lemma 6 *Let* $F = ((I'_{1l}, I'_{1r}), (I'_{2l}, I'_{2r}), \ldots, (I'_{nl}, I'_{nr}))$ *be any unidirectional schedule for the desired profile that satisfies the tongue-and-groove-id constraint. Let* $S(p)$*, be the following assertions.*

(a) $I'_{il}(x) \geq I_{ilp}(x)$*,* $0 \leq i \leq n, x_0 \leq x \leq x_m$

(b) $I'_{ir}(x) \geq I_{irp}(x)$*,* $0 \leq i \leq n, x_0 \leq x \leq x_m$

$S(p)$ is true for $p \geq 0$.

Proof: The proof is by induction on p.

(i) Consider the base case, $p = 0$. From Corollary 1 and the fact that the plans $(I_{il0}, I_{ir0}), 0 \leq i \leq n$, are generated using Algorithm SINGLEPAIR, it follows that $S(0)$ is true.

(ii) Assume $S(p)$ is true. Suppose Algorithm TONGUEANDGROOVE-ID finds a next violation and modifies the schedule $N(p)$ to $N(p+1)$. Suppose that the next violation occurs between leaf pairs u and t, $t \in \{u-1, u+1\}$. As in the proof of Lemma 4, we only need prove either Equation 11 or Equation 12 to complete this proof. We complete the proof for the following three cases that are exhaustive.

case 1: $I_t(x_w) \neq 0$ and $I_u(x_w) \neq 0$.
The remainder of the proof for this case is the same as that of Lemma 4.

case 2: $I_t(x_w) = 0$.
In this case, $I_{tlp}(x_w) = I_{trp}(x_w)$. Since $I_{ulp}(x_w) \geq I_{urp}(x_w)$, we have $I_{ulp}(x_w) - I_{tlp}(x_w) \geq I_{urp}(x_w) - I_{trp}(x_w)$. The modification prescribed by Equation 10 is applicable. Note that if $I_{urp}(x_w) - I_{trp}(x_w) = I_{ulp}(x_w) - I_{tlp}(x_w)$, Equation 9 is the same as Equation 10. In particular,

$$I_{tr(p+1)}(x_w) = I_{trp}(x_w) + I_{urp}(x_w) - I_{trp}(x_w) = I_{urp}(x_w) \tag{16}$$

Since $I_t(x_w) = 0$,

$$I_{tr(p+1)}(x_w) = I_{tl(p+1)}(x_w) \tag{17}$$

From Equations 16 and 17,

$$I_{urp}(x_w) = I_{tl(p+1)}(x_w) \tag{18}$$

Since F satisfies the interdigitation constraint, the left leaf of pair t does not pass x_w before the right leaf of pair u passes x_w. So,

$$I'_{tl}(x_w) \geq I'_{ur}(x_w) \tag{19}$$

From $S(p)$ and Equation 18, we get,

$$I'_{ur}(x_w) \geq I_{urp}(x_w) = I_{tl(p+1)}(x_w) \tag{20}$$

Equations 19 and 20 yield

$$I'_{tl}(x_w) - I_{tl(p+1)}(x_w) \geq 0 \tag{21}$$

Lemma 1b implies,

$$I'_{tl}(x) - I_{tl}(x) \geq I'_{tl}(x_w) - I_{tl}(x_w), x \geq x_w \tag{22}$$

Subtracting $I_{tl(p+1)}(x)$ from Equation 22, and rearranging terms we get

$$I'_{tl}(x) - I_{tl(p+1)}(x) \geq I'_{tl}(x_w) - I_{tl}(x_w) + I_{tl}(x) - I_{tl(p+1)}(x), x \geq x_w \tag{23}$$

From Equations 9 and 10 and the working of Algorithm TONGUEANDGROOVE-ID, it follows that

$$I_{tl(p+1)}(x) - I_{tl}(x) = I_{tl(p+1)}(x_w) - I_{tl}(x_w), x \geq x_w \tag{24}$$

From Equations 23, 24 and 21, we get

$$I'_{tl}(x) - I_{tl(p+1)}(x) \geq I'_{tl}(x_w) - I_{tl(p+1)}(x_w) \geq 0, x \geq x_w \tag{25}$$

Therefore,

$$I'_{tl}(x) \geq I_{tl(p+1)}(x), x \geq x_w \tag{26}$$

case 3: $I_u(x_w) = 0$.

The proof is similar to that of case 2.

■

3.4. Efficient Implementation of the Algorithms

In the remainder of this section we will use 'algorithm' to mean Algorithm TONGUEANDGROOVE or Algorithm TONGUEANDGROOVE-ID and 'violation' to mean tongue-and-groove constraint violation or tongue-and-groove-id constraint violation (depending on which algorithm is considered) unless explicitly mentioned.

The execution of the algorithm starts with schedule M at $x = x_0$ and sweeps to the right, eliminating violations from the schedule along the way. The modifications applied to eliminate a violation at x_w, prescribed by Equations 9 and 10, modify one of the violating profiles for $x \geq x_w$. From the unidirectional nature of the sweep of the algorithm, it is clear that the modification of the profile for $x > x_w$ can have no consequence on violations that may occur at the point x_w. Therefore it suffices to modify the profile only at x_w at the time the violation at x_w is detected. The modification can be propagated to the right as the algorithm sweeps. This can be done by using an $(n \times m)$ matrix A that keeps track of the amount by which the profiles have been raised. $A(j, k)$ denotes the cumulative amount by which the jth leaf pair profiles have been raised at sample point x_k from the schedule M generated using Algorithm MULTIPAIR. When the algorithm has eliminated all violations at each x_w, it moves to x_{w+1} to look for possible violations. It first sets the $(w + 1)$th column of the modification matrix equal to the wth column to reflect rightward propagation of the modifications. It then looks for and eliminates violations at x_{w+1} and so on.

The process of detecting the violations at x_w merits further investigation. We show that if one carefully selects the order in which violations are detected and eliminated, the number of violations at each x_w, $0 \leq w \leq m$ will be $O(n)$.

Lemma 7 *The algorithm can be implemented such that $O(n)$ violations occur at each x_w, $0 \leq w \leq m$.*

Proof: The bound is achieved using a two pass scheme at x_w. In pass one we check adjacent leaf pairs $(1, 2), (2, 3), \ldots, (n - 1, n)$, in that order, for possible violations at x_w. In pass two, we check for violations in the reverse order, i.e., $(n - 1, n), (n - 2, n - 1), \ldots, (1, 2)$. So each set of adjacent pairs $(i, i + 1)$, $1 \leq i < n$ is checked exactly twice for possible violations. It is easy to see that if a violation is detected in pass one, either the profile of leaf pair i or that of leaf pair $i + 1$ may be modified (raised) to eliminate the violation. However, in pass two only the profile of pair i may be modified. This is because the profile of pair i is not modified between the two times it is checked for violations with pair $i + 1$. The profile of pair $i + 1$, on the other hand, could have been modified between these times as a result of violations with pair $i + 2$. Therefore in pass two, only i can be a candidate for t (where t is as explained in the algorithm) when pairs $(i, i + 1)$ are examined. From this it also follows that when pairs $(i - 1, i)$ are subsequently examined in pass two, the profile of pair i will not be modified. Since there is no violation between adjacent pairs $(1, 2), (2, 3), \ldots, (i, i + 1)$ at that time and none of these pairs is ever examined again, it follows that at the end of pass two there can be no violations between pairs $(i, i + 1)$, $1 \leq i < n$. ■

Lemma 8 *For the execution of the algorithm, the time complexity is $O(nm)$.*

Proof: Follows from Lemma 7 and the fact that there are m sample points. ■

Theorem 2 *(a) Algorithms TONGUEANDGROOOVE and TONGUEANDGROOVE-ID terminate.*

(b) The schedule generated by Algorithm TONGUEANDGROOVE is free of tongue-and-groove constraint violations and is optimal in therapy time for unidirectional schedules.

(c) The schedule generated by Algorithm TONGUEANDGROOVE-ID is free of interdigitation and tongue-and-groove constraint violations and is optimal in therapy time for unidirectional schedules.

Proof: (a) Lemma 8 provides a polynomial upper bound ($O(n{*}m)$) on the complexity of Algorithms TONGUEANDGROOVE and TONGUEANDGROOVE-ID. The result follows from this.

(b) When Algorithm TONGUEANDGROOVE terminates, no tongue-and-groove violations remain. From this and Lemma 4, it follows that the schedule generated by Algorithm TONGUEANDGROOVE is optimal in therapy time for unidirectional schedules free of tongue-and-groove violations.

(c) When Algorithm TONGUEANDGROOVE-ID terminates, no tongue-and-groove-id violations remain and from Lemma 5 the final schedule satisfies the tongue-and-groove and interdigitation constraints. From this and Lemma 6, it follows that the schedule generated by Algorithm TONGUEANDGROOVE-ID is optimal in therapy time for unidirectional schedules free of both types of violations.

■

Theorem 3 *The schedule generated by the algorithm of van Santvoort and Heijmen (1996) is free of interdigitation and tongue-and-groove constraint violations and is optimal in therapy time for unidirectional DMLC schedules with this property.*

Proof: Similar to that of Theorem 2(c). ■

4. Experimental Validation

The algorithms were validated on a Varian 2100 C/D with 120-leaf MLC (Varian Medical Systems, Palo Alto, CA). The intensity maps of a 7-field head and neck plan from a commercial inverse treatment planning system (CORVUS 5.0, NOMOS Corporation, Cranberry, PA) were sequenced using Algorithm MULTIPAIR, which optimizes the MU efficiency, and Algorithm TONGUEANDGROOVE-ID, which eliminates the tongue-and-groove effect and interdigitation. The intensity maps have a bixel size of 1 cm x 1 cm and a 20% intensity step. Figure 13 shows the film measurement of the fluence maps of the AP field. The tongue-and-groove effect is readily seen in Figure 13(a), while it is completely eliminated in Figure 13(b) using Algorithm TONGUEANDGROOVE-ID. Table 1 compares the number of segments and the MU efficiencies of all three algorithms. The MU efficiency is defined as the ratio of the maximum fluence of intensity modulated field per MU to the fluence of an open field per MU. Compared to the leaf sequences with no constraints, the consideration of tongue-and-groove correction increased both the number of segments and MUs, with an average increase of 21% and 19%, respectively, for the 7 intensity maps considered here. With the additional elimination of interdigitation, the increases were 25% and 24%, respectively. Examination of all the sub fields of the leaf sequences generated with Algorithm TONGUEANDGROOVE-ID verified that no interdigitation constraint has been violated.

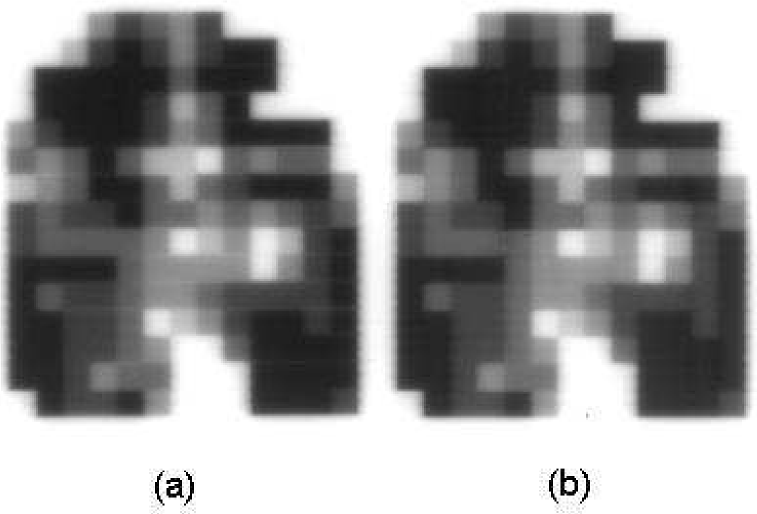

Figure 13. Film measurement of the AP field (field ID 1 in Table 1) of a seven-field head and neck plan. The optimized leaf sequences were generated without (Algorithm MULTIPAIR, (a)) and with tongue-and-groove-id correction (Algorithm TONGUEANDGROOVE-ID, (b)).

Field ID	1	2	3	4	5	6	7
MULTIPAIR							
# of Segments	11	8	13	15	14	10	10
MU Efficiency	0.47	0.63	0.40	0.35	0.37	0.40	0.51
TONGUEANDGROOVE							
# of Segments	14	10	15	21	14	13	11
MU Efficiency	0.37	0.51	0.35	0.25	0.37	0.40	0.40
% Segment # increase	27	25	15	40	0	30	10
% MU increase	26	24	15	37	0	0	28
TONGUEANDGROOVE-ID							
# of Segments	14	11	16	21	14	14	11
MU Efficiency	0.37	0.47	0.33	0.25	0.37	0.37	0.37
% Segment # increase	27	38	23	40	0	40	10
% MU increase	26	36	22	37	0	7	38

Table 1. Comparison of the number of segments and MU efficiency of the three leaf sequencing algorithms (MULTIPAIR, TONGUEANDGROOVE and TONGUEANDGROOVE-ID) for 7 intensity maps of a head and neck treatment plan generated from a commercial treatment planning system. The percent increases in the number of segments and MUs for Algorithms TONGUEANDGROOVE and TONGUEANDGROOVE-ID with respect to Algorithm MULTIPAIR are also shown. The average percent increases in the number of segments are 21% and 25%, respectively. The average percent increases in the number of MUs are 19% and 24%, respectively.

5. Conclusions

We have described mathematical formalism and rigorous proofs of leaf sequencing algorithms for segmental multileaf collimation, which maximize MU efficiency while completely eliminating the tongue-and-groove underdosage. Even though it has been shown that for a multiple field IMRT plan (≥ 5), the tongue-and-groove effect on the IMRT dose distribution is clinically insignificant (Deng *et al* 2001) due to the smearing effect of individual fields, yet it still can be problematic for a small number of fields and for the patient setup with minimal uncertainty. Compared to the unconstrained leaf sequencing algorithms, the presented methods yield leaf sequences, which decreases the MU efficiency a little. But they completely overcome tongue-and-groove underdosages. One of the methods also eliminates leaf interdigitation. Most importantly, mathematical proofs show that these algorithms are optimal in MU efficiency for unidirectional schedules.

Acknowledgments

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two pass implementation of the algorithm resolved all tongue-and-groove violations for test sets. This observation motivated us to formulate Lemma 7.

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