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Single Bend Wiring*

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The problem of wiring pairs of points with wires having at most one bend is considered. We develop an $O(n^2)$ algorithm to determine whether or not a set of n point pairs can be wired in this manner on a single layer. When this is possible, our algorithm generates the layout. We show that determining the maximum number of point pairs that can be wired in with at most one bend is NP-hard. The problem of determining the minimum number of layers needed to wire a set of n point pairs is also NP-hard. © 1986 Academic Press, Inc.

1. SINGLE BEND WIRING

Wire routing (equivalently, *wiring*, *wire layout*, etc.) is the problem of defining precisely the conductor paths in a given *wiring medium* for a set of *wires*. A wire is a specification of a pair of points to be connected.[1]

In the most general wire routing case, the conductor paths can be curves. In fact, it is not unusual to find curved conductor paths in manual layouts. However, in automated systems, conductor paths are usually constrained to contain horizontal and vertical segments only. The resulting loss in generality is offset by the following advantages:

(1) The conductor paths can now be represented far more easily. General curves, on the other hand, are not amenable to compact representation on a computer. Note, however, that in the case of river routing, Tompa [TOMP80] has shown that general wiring need consider only straight line segments and arcs of circles of different radii. In this case, the restriction to horizontal and vertical segments does not significantly enhance the representation of wires in a computer.

(2) The length calculations are considerably simplified.

(3) Fabrication equipment such as plotters is usually capable of horizontal and vertical movements only. Curves and diagonal lines are simulated by a series of small, alternating horizontal and vertical movements, and take a long time to draw.

Wiring where each path is made up of horizontal and vertical segments only is called *rectilinear wiring*. It is often thought of as wiring on a grid. All the wire endpoints are grid points; each conductor path is constrained to lie along grid lines only; and conductor paths are not allowed to cross. The separation between grid lines reflects the minimum conductor separation necessary to avoid inductive cross-talk. Thus, the concept of wiring on a grid neatly captures a number of practical constraints inherent in wire routing on a single layer. Figure 1.1 shows an example of wiring on a grid.

Any point on a grid may be specified by its x- and y-coordinates. Assuming that consecutive grid lines are one unit apart, the coordinates of grid points are always natural numbers. Let $S = \{(u_i, v_i, x_i, y_i) | 1 \le i \le n\}$ be a set of n wires to be connected. Wire i is specified by requiring that the point (u_i, v_i) is to be connected to the point (x_i, y_i), $1 \le i \le n$. S is called a *wire set*. For the example of Fig. 1.1, $S = \{(2,3,6,3), (3,2,5,5), (2,4,3,4), (3,4,1,5)\}$.

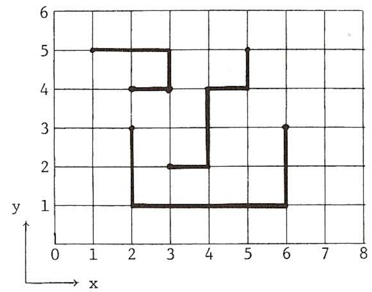

FIG. 1.1. Wiring on a grid.

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[1]With this terminology, the context of usage ensures that, in our discussion, there is no confusion between the problem specification (i.e., the wire) and the problem solution (i.e., the wiring or the wire layout). Another point is that in general, a *signal net* (a set of points to be interconnected) may be specified, rather than a wire. However, in fast circuit technologies (e.g., ECL), transmission line considerations impose rigid restrictions on the topology of the interconnection tree. Specifically, Steiner trees may not be permitted. In such cases, the nets are effectively decomposed into wires (before wire routing even begins) in ways that guarantee that all restrictions will be satisfied. We shall assume such an environment.

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