Approximation Algorithms For Wireless Sensor Deployment

Xiaochun Xu and Sartaj Sahni
Department of Computer and Information Science and Engineering,
University of Florida, Gainesville, FL 32611
{sahni,xxu}@cise.ufl.edu

April 21, 2006

Abstract

We develop an integer linear programming formulation to find a minimum cost deployment of sensors so at attain desired coverage of a target point set. Additionally ϵ-approximation algorithms and a polynomial time approximation scheme are proposed for the case of grid coverage. Experiments demonstrate the superiority of our proposed algorithms over earlier algorithms for point coverage of grids.

Keywords: Wireless sensor networks, point coverage, approximation algorithm.

1 Introduction

In a typical sensor network application, sensors are to be placed (or deployed) so as to monitor a region or a set of points. In some applications it is possible to select the sites where sensors are placed while in others (e.g., in hostile environments) we may simply scatter (e.g., air drop) a sufficiently large number of sensors over the monitoring region with the expectation that the sensors that survive the air drop will be able to adequately monitor the target region. When site selection is possible, we use *deterministic* sensor deployment and when site selection isn't possible, the deployment is *nondeterministic*. In some applications, it is desirable that the deployed collection of sensors be able to communicate with one another, either directly or indirectly via multihop communication. So, in addition to covering the region or set of points to be sensed, we often require the deployed collection of sensors to form a connected network. Instead, each sensor communicates directly with a base station that is situated within the communication range of all sensors.

In this paper, we focus on the problem of placing sensors at a subset of preselected sites so as to minimize sensor cost while providing a specified degree of coverage of the target sites. The sensors do not communicate with one another; rather each sensor communicates with a base station. We assume that power and sensor communication range are not design issues. That is, each of the feasible sensor sites has an abundant energy supply and each sensor has a sufficiently large transmission range to reach the base station from each of the preselected sites. A possible application of the problem considered here is in the deployment of chemical and radioactive sensors so as to monitor high risk targets that may be approximated as points; the cost of a sensor may be from hundreds of dollars to tens of thousands of dollars.

In Sectionsec:related, we review work related to the problem studied here.

2 Related Work

Iyengar and Brooks [26, 27] and Culler and Hong [12] provide good overviews of the breadth of sensor network research topics as well as of applications for sensor networks. Sensor network algorithms are reviewed in [50]. For a given placement of sensors, it is easy to check whether the collection covers the target region or point set and also whether the collection is connected. For the coverage property, we need to know the sensing range of individual sensors (we assume that a sensor can sense events that occur within a distance r, where r is the sensor's sensing range, from it) and for the connected property, we need to know the communication range, c, of a sensor. Zhang and Lou [73] have established the following necessary and sufficient condition for coverage to imply connectivity.

Theorem 1 *[Zhang and Lou [73]] When the sensor density (i.e., number of sensors per unit area) is finite, $c \geq 2r$ is a necessary and sufficient condition for coverage to imply connectivity.*

Wang et al. [62] prove a similar result for the case of q-coverage (each point is covered by at least q sensors) and q-connectivity (the communication graph for the deployed sensors is q connected).

Theorem 2 *[Wang et al. [62]] When $c \geq 2r$, q-coverage of a convex region implies q-connectivity.*

Notice that q-coverage with $q > 1$ affords some degree of fault tolerance, we are able to monitor all points so long as no more than $q - 1$ sensors fail. Huang and Tseng [25] develop algorithms to verify whether a sensor deployment provides q-coverage.

[23, 24] consider the case when the sensors are mobile and self deploy. A collection of mobile sensors may be placed into an unknown and potentially hazardous environment. Following this initial placement, the sensors relocate so as to obtain maximum coverage of the unknown environment. They communicate the information they gather to a base station outside of the environment being sensed. A distributed potential-field-based algorithm to self deploy mobile sensors under the stated assumptions is developed

in [24] and a greedy and incremental self-deployment algorithm is developed in [23]. A virtual-force algorithm to redeploy sensors so as to maximize coverage also is developed by Zou and Chakrabarty [74]. Poduri and Sukhatme [47] develop a distributed self-deployment algorithm that is based on artificial potential fields and which maximizes coverage while ensuring that each sensor has at least k other sensors within its communication range.

Kar and Banerjee [32] examine the problem of deploying the fewest number of homogeneous sensors so as to cover the plane with a connected sensor network. They assume that the sensing range equals the communication range (i.e., $r = c$). Kar and Banerjee [32] have shown that their algorithm has a sensor density that is within 2.6% of the optimal density. This algorithm may be extended to provide connected coverage for a set of finite regions [32].

Kar and Banerjee [32] have proposed an algorithm to deploy a connected sensor network so as to cover a set of points in Euclidean space. This algorithm, which assumes that $r = c$, uses at most 7.256 times the minimum number of sensors needed to cover the given point set.

Grid coverage is another version of the point coverage problem. In this version, Chakrabarty et al. [7], we are given a two- or three-dimensional grid of points that are to be sensed. Sensor locations are restricted to these grid points and each grid point is to be covered by at least q, $q \geq 1$, sensors (i.e., we seek q-coverage). For sensing, we have s sensor types available. A sensor of type i costs c_i dollars and has a sensing range r_i. At most one sensor may be placed at a grid point. In this version of the point coverage problem, the sensors do not communicate with one another and are assumed to have a communication range large enough to reach the base station from any grid position. So, network connectivity is not an issue. The objective is to find a least-cost sensor deployment that provides q-coverage.

Chakrabarty et al. [7] formulate this q-coverage deployment problem as an integer linear program (ILP) with $O(sn^2)$ variables and $O(sn^2)$ equations, where n is the number of grid points. For large n, Chakrabarty et al. [7] propose a divide-and-conquer "near-optimal" algorithm in which the base case (small number of points) is solved optimally using the ILP formulation.

3 Integer Linear Programming Formulation

We are given a set of locations to be monitored as well as a set of locations where it is feasible to place sensors. For simplicity, we model these two sets as a single set and model any differences in the two sets by ILP (integer linear program) constraints. We assume that at each location we wish to monitor a predefined subset of quantities such as temperature, sound, levels of different gases, radioactivity and so on. Each quantity to be monitored is referred to as a *modality*. The subset of modalities being monitored at different locations may be different. Let $cover(j, l)$ be the degree of monitoring coverage required at location l for modality j. For monitoring, we have available sensors of different types. Each sensor is able to monitor one or more modalities. Let $locations(i, j, l)$ be the set of locations with the property that if a sensor of type i is placed at the location then the sensor provides a unit degree of coverage for modality j at location l. Notice that with this formulation it is possible that a sensor, which is capable of monitoring modalities a and b, placed at some location may cover another location for modality a but not b (this may, for example, happen because the sensors range for modality a is different from that for modality b). Let $capacity(i, l)$ be the number of sensors of type i that may feasibly be placed at location l and let $cost(i)$ be the cost of 1 sensor of type i. The sensor deployment problem is to determine the number, $x_{i,z}$, of sensors of type i to place at each location z so as to achieve the desired degree of coverage, not violate the capacity constraint at each location, and minimize cost. The problem may be formulated as the following ILP.

$$\text{minimize } \sum_i (cost(i) \sum_l x_{i,l})$$

$$\sum_i \sum_{z \in location(i,j,l)} x_{i,z} \geq cover(j, l),\ \forall j, l$$

$$0 \leq x_{i,z} \leq capacity(i, l),\ \forall i, z$$

The total number of variables (i.e., the $x_{i,z}$s) in the above ILP is sn, where s is the number of sensor types and n is the number of locations. The total number of constraints is $(s + m)n$, where m is the number of modalities. Additional constraints

$$\sum_z x_{i,z} \leq totalType(i),\ \forall i$$

and

$$\sum_i x_{i,z} \leq totalLocation(z),\ \forall z$$

may be added to the above formulation to limit the total number of sensors of each type that may be deployed as well as the total number of sensors deployed at a particular location.

The above ILP formulation may be used to model the grid coverage problem studiedin [7]. In this problem, the locations form a $\sqrt{n} \times \sqrt{n}$ grid; the number m of modalities is 1; $cover(1, z) = q$ for each location z; and $totalLocation(z) = 1$ for each location z. The formulation becomes

$$\text{minimize } \sum_i (cost(i) \sum_z x_{i,z})$$

$$\sum_i \sum_{z \in location(i,1,l)} x_{i,z} \geq q, \ \forall l$$

$$\sum_i x_{i,z} \leq 1, \ \forall z$$

$$0 \leq x_{i,z} \leq 1 \ \forall i, z$$

The grid ILP is a 0-1 ILP with sn variables and $(s+2)n$ constraints. In contrast, the 0-1 ILP formulation of [7] has $O(qn^2)$ variables and $O(qn^2)$ constraints.

4 A Greedy Algorithm

5 Conclusion

We have reviewed some of the recent advances made in the development of algorithms for wireless sensor networks. This paper has focussed on sensor deployment and coverage, routing (specifically, unicast and multicast) and sensor fusion. Both centralized and distributed localized algorithms have been considered.

References

[1] J. Aslam, Q. Li and R. Rus, Three power-aware routing algorithms for sensor netowrk, *Wireless Communications and Mobile Computing*, 3, 2003, 187-208.

[2] S. Banerjee and A. Misra, Energy efficient reliable communication for multi-hop wireless networks, *WINET*, 2001.

[3] R. Brooks and S. Iyengar, Robust distributed computing and sensing algorithm, *IEEE Computer*, June 1996, 53-60.

[4] I. Caragiannis, C. Kaklamanis, and P. Kanellopoulos, New Results for energy-efficient broadcasting in wireless networks, *Proc. of the 13th Annual International Symposium on Algorithms and Computation*, 2002.

[5] M. Cardei and D. Du, Improving wireless sensor network lifetime through power aware organization, ACM Wireless Networks, to appear.

[6] Julien Cartigny, David Simplot, and Ivan Stojmenovic, Localized minimum-energy broadcasting in ad-hoc networks, *IEEE INFOCOM*, 2003.

[7] K. Chakrabarty, S. Iyengar, H. Qi and E. Cho, Grid coverage for surveillance and target location in distributed sensor networks, *IEEE Transactions on Computers*,

[8] J. Chang and L. Tassiulas, Routing for maximum system lifetime in wireless ad-hoc networks,, *37th Annual Allerton Conference on Communication, Control, and Computing*, Monticello, IL, Sept. 1999.

[9] J. Chang, and L. Tassiluas, Energy conserving routing in wireless ad-hoc networks, *IEEE INFOCOM*, 2000.

[10] J. Chang and L. Tassiulas, Fast approximate algorithms for maximum lifetime routing in wireless ad-hoc networks, *IFIP-TC6 Networking 2000*, *LNCS*, 1815, Springer Verlag, 2000, pp. 702-713.

[11] A.E.F Clementi, P. Crescenzi, P. Penna, G. Rossi, and P. Vocca, A worst case analysis of an MST-based heuristic to construct energy-efficient broadcast trees in wireless networks, Technical Report 010 of the Univ. of Rome, 2001.

[12] D. Culler and W. Hong, Wireless sensor networks, Special Issue, *CACM*, 47, 6, 2004.

[13] Arindam K. Das, Robert J. Marks, and Mohamed El-Sharkawi, Minimum power broadcast trees for wireless networks, *IEEE International Symposium on Circuits and Systems*, May 2002.

[14] D. Dolev, The Byzantine generals strike again, *Jr. of Algorithms*, 1982, 14-30.

[15] S. Doshi, T. Brown, Minimum energy routing schemes for a wireless ad hoc network, *IEEE INFOCOM*, 2002.

[16] O. Egecioglu and T. Gonzalez, Minimum-energy broadcast in simple graph with limited node power, *IASTED International Conference on Parallel and Distributed Computing and Systems*, Anaheim, CA, August 2001, 334-338.

[17] C. Florens and R. McEliece, Scheduling algorithms for wireless ad-hoc sensor networks, *IEEE GLOBECOM*, 2002, 6-10.

[18] C. Florens and R. McEliece, Packets distribution algorithms for sensor networks, *INFOCOM* 2003.

[19] C. Florens, M. Francesdhetti and R. McEliece, Lower bounds on data collection time in sensory networks, *IEEE Jr. on Selected Areas in Communications*, 1, 11, 2004.

[20] M. Garey and D. Johnson, *Computers and intractibility: A guide to the theory of NP-completeness*, W. H. Freeman and Co., 1979.

[21] Sudipto Guha and Samir Khuller, Approximation algorithms for connected dominating sets, *Fourth Annual European Symposium on Algorithms*, 1996.

[22] W. Heinzelman, A. Chandrakasan and H. Balakrishnan, Energy-efficient communication protocol for wireless microsensor networks, *IEEE HICSS*, 2000.

[23] A. Howard, M. Mataric and G. Sukhatme, An incremental self-deployment algorithm for mobile sensor networks, *Autonomous Robots*, Special Issue on Intelligent Embedded Systems.

[24] A. Howard, M. Mataric and G. Sukhatme, Mobile sensor network deployment using potential fields: A distributed, scalable solution to the area coverage problem, *6th International Symposium on Distributed Autonomous Robotics Systems (DARS02)*, 2002.

[25] C. Huang and Y. Tseng, The coverage problem in a wireless sensor network, *WSNA*, 2003.

[26] S. Iyengar and R. Brooks, Computing and communications in distributed sensor networks, Special Issue, *Jr. of Parallel and Distributed Computing*, 64, 7, 2004.

[27] S. Iyengar and R. Brooks, *Handbook of Distributed Sensor Networks*, Chapman & Hall/CRC, 2005.

[28] R. Kannan, S.Sarangi, S.S. Iyengar and L. Ray, Sensor-centric quality of routing in sensor networks, *INFOCOM*, 2003.

[29] R. Kannan, S. Sarangi, S. Ray and S. Iyengar, Minimal sensor integrity: Computing the vulnerability of sensor grids, *Info. Proc. Letters*, 86, 1, 2003, 49-55.

[30] R. Kannan and S. S. Iyengar, Game-theoretic models for reliable, path-length and energy-constrained routing in wireless sensor networks, *IEEE Journal on Selected Areas in Communications*, 2004.

[31] R. Kannan, L. Ray, R. Kalidindi, and S. Iyengar, Energy threshold constrained geographic routing protocol for wireless sensor networks, *Signal Processing Letters*, 1, 1, 79-84, 2003.

[32] K. Kar and S. Banerjee, Node placement for connected coverage in sensor networks, *Proc WiOpt 2003: Modeling and Optimization in Mobile, Ad Hoc and Wireless Networks*, 2003.

[33] K. Kar, M. Kodialam, T. Lakshman and L. Tassiulas, Routing for network capacity maximization in energy-constrained ad-hoc networks, *IEEE INFOCOM*, 2003.

[34] L. Lamport, R. Shostak, and M. Pease, The Byzantine generals problem. *ACM Trans. Prog. Lang. Syst.*, July 1986, 499-516.

[35] X. Li, P. Wan and O. Frieder, Coverage in wireless ad-hoc sensor networks, *IEEE Transactions on Computers*, 52, 2002, 753-763.

[36] G. Lin and G. Noubir, Energy efficient broadcast with sectored antennas in wireless ad hoc networks, *IASTED/IEEE Wireless and optical Communications*, July 2002.

[37] J. Lu and S. Tatsuya, Coverage-aware self-scheduling in sensor networks, *IEEE Computer Communications Workshop (CCW 2003)*, 2003.

[38] S. Mahaney and F. Schneider, Inexact agreement: Accuracy, precision, and graceful degradation, *Fourth ACM Symp. on Principles of Distr. Computing*, 1985, 237-249.

[39] M. Maleki, K. Dantu, and M. Pedram, Power-aware source routing protocol for mobile ad hoc networks, *SLPED'02*, 2002.

[40] R. Marks, A. Das, M. El-Sharkawi, P. Arabshahi and A. Gray, Minimum power broadcast trees for wireless networks: Optimizing using the viability lemma, *IEEE ISCAS*, 2002.

[41] K. Marzullo, Tolerating failures of continuous-valued sensors, *ACM Trans. Computer Systems*, Nov. 1990, 284-304.

[42] S. Meguerdichian, F. Koushanfar, M. Potkonjak and M. Srivastava, Coverage problems in wireless ad-hoc sensor networks, *IEEE InfoCom*, 2001.

[43] S. Meguerdichian, F. Koushanfar, G. Qu and M. Potkonjak, Exposure in wireless ad hoc sensor networks, *7th Annual International Conference on Mobile Computing and Networking (MobiCom'01)*, 2001, 139-150.

[44] T. Melodia, D. Pompili, and I. Akyildiz, Optimal local topology knowledge for energy efficient geographical routing in sensor networks, *IEEE INFOCOM*, 2004.

[45] A. Misra and S. Banerjee, MRPC: maximizing network lifetime for reliable routing in wireless,, *IEEE Wireless Communications and Networking Conference (WCNC)*, 2002.

[46] J. Park and S. Sahni, Maximum lifetime broadcasting in wireless networks, submitted.

[47] S. Poduri and G. Sukhatme, Constrained coverage for mobile sensor networks, *IEEE Intl. Conf. on Robotics and Automation (ICRA'04)*, 2004, 165-171.

[48] T. Rappaport, *Wireless communications: Principles and practices*, Prentice Hall, 1996.

[49] S. Sahni, Data structures, algorithms, and applications in Java, 2nd Edition, Silicon Press, NJ, 2005.

[50] S. Sahni and X. Xue, Algorithms for wireless sensor networks, *Intl. Jr. on Distr. Sensor Networks*, ¡em¿Invited Paper¡/em¿, Preview Issue, 2004, 35-56.

[51] S. Shakkottai, R. Srikant and N. Shroff, Unreliable sensor grids: coverage, connectivity and diameter, *INFOCOM*, 2003.

[52] S. Singh, C. Raghavendra, and J. Stepanek, Power-aware broadcasting in mobile ad hoc networks, *IEEE PIMRC'99*, Osaka, Japan, Sep. 1999.

[53] S. Singh, M. Woo, and C. Raghavendra, Power-aware routing in mobile ad hoc networks, *ACM/IEEE MOBICOM*, 1998.

[54] S. Slijepcevic and M. Potkonjak, Power efficient optimization of wireless sensor networks, *IEEE Intl. Conf. on Communications*, 2001.

[55] A. Spyropoulos and C. Raghavendra, Energy efficient communications in ad hoc networks using directional antenna, *IEEE INFOCOM*, 2002.

[56] I. Stojmenovic, and Xu Lin, Power-aware localized routing in wireless networks, *IEEE Transactions on Parallel and Distributed Systems*, 2000.

[57] R. Szewczyk, E. Osterweil, J. Polastre, M. Hamilton, A. Mainwaring and D. Estrin, Habitat monitoring with sensor networks, *CACM*, 47, 6, 2004, 34-40.

[58] D. Tian and N. Georganas, A coverage-preserving node scheduling scheme for large wireless sensor networks,

[59] C. K. Toh, Maximum battery life routing to support ubiquitous mobile computing in wireless ad hoc networks, *IEEE Communications Magazine*, June, 2001, 138-147.

[60] G. Veltri, Q. Huang, G. Qu and M. Potkonjak, Minimal and maximal exposure path algorithms for wireless embedded sensor networks, *SenSys'03*, 2003.

[61] P. Wan, G. Calinescu, X. Li and O. Frieder, Minimum-energy broadcast routing in static ad hoc wireless networks, *IEEE INFOCOM*, 2001.

[62] X. Wang et al., Integrated coverage and connectivity configuration in wireless sensor networks, *SenSys*, 2003.

[63] J. Wieselthier, G. Nguyen, and A. Ephremides, On the construction of energy-efficient broadcasting and multicast trees in wireless networks, *IEEE INFOCOM*, 2000.

[64] J. Wieselthier, G. Nguyen, Algorithm for energy-efficient multicasting in static ad hoc wireless networks, *Mobile Networks and Applications*, 2001, 251-261.

[65] J. Wu and F. Dai, Broadcasting in ad-hoc networks based on self pruning, IEEE INFOCOM, 2003.

[66] J. Wu, M. Gao and I. Strojmenovic, On calculating power-aware connected dominating sets for efficient routing in ad hoc wireless networks, *Jr. Communications and Networks*, 4, 1, 2002.

[67] X. Xu and S. Sahni, Optimal deployment of wireless sensors on a grid.

[68] Y. Xu, J. Heidermann and D. Estrin, Geography-informed energy conservation for ad hoc routing, *MOBICOM*, 2001.

[69] T. Yan, T. He, J. Stankovic, Differentiated surveillance for sensor networks, *First International Conference on Embedded Networked Sensor Systems*, 2003, 51 - 62.

[70] F. Ye, G. Zhong, S. Lu and L. Zhang, Peas: A robust energy conserving protocol for long-lived sensor networks, *23rd ICDCS*, 2003.

[71] F. Ye, G. Zhong, S. Lu and L. Zhang, Energy efficient robust sensing coverage in large sensor networks, Technical Report, UCLA, 2002.

[72] Y. Yu, B. Krishnamachari and V. Prasanna, Energy-latency tradeoffs for data gathering in wireless sensor networks, *INFOCOM*, 2004.

[73] H. Zhang and J. Hou, Maintaining sensing coverage and connectivity in large sensor networks, Technical Report UIUC, UIUCDCS-R-2003-2351, 2003.

[74] Y. Zou and K. Chakrabarty, Sensor deployment and target localization in distributed sensor networks, *ACM Transactions on Embedded Computing Systems*, 3, 1, 2004, 61-91.