Data Structures For One-Dimensional Packet Classification Using Most-Specific-Rule Matching *

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Abstract

We review the data structures that have been proposed for one-dimensional packet classification. Our review is limited to data structures for the case when ties among the rules that match an incoming packet are broken by selecting the matching rule that is most specific. For the case when the rule filters are destination-address prefixes or are nonintersecting ranges, this tie breaker corresponds to longest-prefix or shortest-range matching, respectively. When the rule filters are arbitrary ranges, this tie breaker resolves the tie only when the rule set is conflict free. Data structures for both static and dynamic rule tables are discussed.

Keywords: *Packet classification, packet routing, router tables, longest-prefix matching, most-specific-range matching, conflict-free ranges, static and dynamic rule tables.*

1 Introduction

An Internet router classifies incoming packets into flows[1] utilizing information contained in packet headers and a table of (classification) rules. This table is called the *rule table* (equivalently, *router table*). The packet-header information that is used to perform the classification is some subset of the source and destination addresses, the source and destination ports, the protocol, protocol flags, type of service, and so on. The specific header information used for packet classification is governed by the rules in the rule table. Each rule-table rule is a pair of the form (F, A), where F is a filter and A is an action. The action component of a rule specifies what is to be done when a packet that satisfies the rule filter is received. Sample actions are drop the packet, forward the packet along a certain output link, and reserve a specified amount of bandwidth. A rule filter F is a tuple that is comprised of one or more fields. In the simplest case of destination-based packet forwarding, F has a single field, which is a destination (address) prefix and A is the next hop for packets whose destination address has the specified prefix. For example, the rule $(01*, a)$ states that the next hop for packets whose destination address (in binary) begins with 01 is a. IP (Internet Protocol) multicasting uses rules in which F is comprised of the two fields source prefix and destination prefix; QoS routers may use five-field rule filters (source-address prefix, destination-address prefix, source-port range, destination-port range, and protocol); and firewall filters may have one or more fields.

In the d-dimensional packet classification problem, each rule has a d-field filter. In this paper, we are concerned solely with 1-dimensional packet classification. It should be noted, that data structures for multidimensional packet classification are usually built on top of data structures for 1-dimensional packet classification. Therefore, the study of data structures for 1-dimensional packet classification is fundamental to the design and development of data structures for d-dimensional, $d > 1$, packet classification.

For the 1-dimensional packet classification problem, we assume that the single field in the filter is the destination field and that the action is the next hop for the packet. With these assumptions, 1-dimensional packet classification is equivalent to the destination-based packet forwarding problem. Henceforth, we shall use the terms rule table and router table to mean tables in which the filters have a single field, which is the destination address. This single field of a filter may be specified in one of two ways:

1. *As a range.* For example, the range [35, 2096] matches all destination addresses d such that $35 \leq d \leq 2096$.

2. *As an address/mask pair.* Let x_i denote the ith bit of x. The address/mask pair a/m matches all des-

* This work was supported, in part, by the National Science Foundation under grant CCR-9912395.

[1] A **flow** is a set of packets that are to be treated similarly for routing purposes.

Prefix Name	Prefix	Range Start	Range Finish
P1	*	0	31
P2	0101*	10	11
P3	100*	16	19
P4	1001*	18	19
P5	10111	23	23

Figure 1. Prefixes and their ranges

tination addresses d for which $d_i = a_i$ for all i for which $m_i = 1$. That is, a 1 in the mask specifies a bit position in which d and a must agree, while a 0 in the mask specifies a don't care bit position. For example, the address/mask pair 101100/011101 matches the destination addresses 101100, 101110, 001100, and 001110.

When all the 1-bits of a mask are to the left of all 0-bits, the address/mask pair specifies an address prefix. For example, 101100/110000 matches all destination addresses that have the prefix 10 (i.e., all destination addresses that begin with 10). In this case, the address/mask pair is simply represented as the prefix 10*, where the * denotes a sequence of don't care bits. If W is the length, in bits, of a destination address, then the * in 10* represents all sequences of $W - 2$ bits. In IPv4 the address and mask are both 32 bits, while in IPv6 both of these are 128 bits.

Notice that every prefix may be represented as a range. For example, when $W = 6$, the prefix 10* is equivalent to the range [32, 47]. A range that may be specified as a prefix for some W is called a *prefix range*. The specification 101100/011101 may be abbreviated to ?011?0, where ? denotes a don't-care bit. This specification is not equivalent to any single range. Also, the range specification [3,6] isn't equivalent to any single address/mask specification.

Figure 1 shows a set of five prefixes together with the start and finish of the range for each. This figure assumes that W = 5. The prefix P1 = *, which matches all legal destination addresses, is called the *default* prefix.

Suppose that our router table is comprised of five rules R1–R5 and that the filters for these five rules are P1–P5, respectively. Let N1–N5, respectively, be the next hops for these five rules. The destination address 18 is matched by rules R1, R3, and R5 (equivalently, by prefixes P1, P3, and P5). So, N1, N3, and N5 are candidates for the next hop for incoming packets that are destined for address 18. Which of the matching rules (and associated action) should be selected? When more than one rule matches an incoming packet, a *tie* occurs. To select one of the many rules that may match an incoming packet, we use a *tie breaker*.

Let RS be the set of rules in a rule table and let FS be the set of filters associated with these rules. $rules(d, RS)$ (or simply $rules(d)$ when RS is implicit) is the subset of rules of RS that match/cover the destination address d. $filters(d, FS)$ and $filters(d)$ are defined similarly. A tie occurs whenever $|rules(d)| > 1$ (equivalently, $|filters(d) > 1|$).

Three popular tie breakers are:

1. *First matching rule in table.* The rule table is assumed to be a linear list ([9]) of rules with the rules indexed 1 through n for an n-rule table. The action corresponding to the first rule in the table that matches the incoming packet is used. In other words, for packets with destination address d, the rule of $rules(d)$ that has least index is selected.

   For our example router table corresponding to the five prefixes of Figure 1, rule R1 is selected for every incoming packet, because P1 matches every destination address. When using the first-matching-rule criteria, we must index the rules carefully. In our example, P1 should correspond to the last rule so that every other rule has a chance to be selected for at least one destination address.

2. *Highest-priority rule.* Each rule in the rule table is assigned a priority. From among the rules that match an incoming packet, the rule that has highest priority wins is selected. To avoid the possibility of a further tie, rules are assigned different priorities (it is actually sufficient to ensure that for every destination address d, $rules(d)$ does not have two or more highest-priority rules).

   Notice that the first-matching-rule criteria is a special case of the highest-priority criteria (simply assign each rule a priortiy equal to the negative of its index in the linear list).

3. *Most-specific-rule matching.* The filter $F1$ is **more specifi c** than the filter $F2$ iff $F2$ matches all packets matched by $F1$ plus at least one additional packet. So, for example, the range $[2, 4]$ is more specific than $[1, 6]$, and $[5, 9]$ is more specific than $[5, 12]$. Since $[2, 4]$ and $[8, 14]$ are disjoint (i.e., they have no address in common), neither is more specific than the other. Also, since $[4, 14]$ and $[6, 20]$ intersect[2], neither is more specific than the other. The prefix 110* is more specific than the prefix 11*.

   In most-specific-rule matching, ties are broken by selecting the matching rule that has the most specific filter. When the filters are destination prefixes, the most-specific-rule that matches a given destination d is the

[2]Two ranges $[u, v]$ and $[x, y]$ intersect iff $u < x \leq v < y \vee x < u \leq y < v$.

longest[3] prefix in $filters(d)$. Hence, for prefix filters, the most-specific-rule tie breaker is equivalent to the longest-matching-prefix criteria used in router tables. For our example rule set, when the destination address is 18, the longest matching-prefix is P4.

When the filters are ranges, the most-specific-rule tie breaker requires us to select the most specific range in $filters(d)$. Notice also that most-specific-range matching is a special case of the the highest-priority rule. For example, when the filters are prefixes, set the prefix priority equal to the prefix length. For the case of ranges, the range priority equals the negtive of the range size.

In a *static* rule table, the rule set does not vary in time. For these tables, we are concerned primarily with the following metrics:

1. *Time required to process an incoming packet.* This is the time required to search the rule table for the rule to use.

2. *Preprocessing time.* This is the time to create the rule-table data structure.

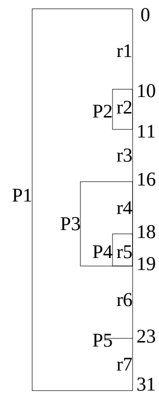
3. *Storage requirement.* That is, how much memory is required by the rule-table data structure?

In practice, rule tables are seldom truly static. At best, rules may be added to or deleted from the rule table infrequently. Typically, in a "static" rule table, inserts/deletes are batched and the rule-table data structure reconstructed as needed.

In a *dynamic* rule table, rules are added/deleted with some frequency. For such tables, inserts/deletes are not batched. Rather, they are performed in real time. For such tables, we are concerned additionally with the time required to insert/delete a rule. For a dynamic rule table, the initial rule-table data structure is constructed by starting with an empty data structures and then inserting the initial set of rules into the data structure one by one. So, typically, in the case of dynamic tables, the preprocessing metric, mentioned above, is very closely related to the insert time.

In this paper, we focus on data structures for static and dynamic router tables (1-dimensional packet classification) in which the filters are either prefixes or ranges. Although some of our data structures apply equally well to all three of the commonly used tie breakers, our focus, in this paper, is on longest-prefix matching (Section 2) and most-specific-range matching (Section 3).

[3]The length of a prefix is the number of bits in that prefix (note that the * is not used in determining prefix length). The length of P1 is 0 and that of P2 is 4.



(a)

End Point	>	=
0	P1	P1
10	P2	P2
11	P1	P2
16	P3	P3
18	P4	P4
19	P1	P4
23	P1	P5
31	-	P1

(b)

Figure 2. (a) Pictorial representation of prefixes and ranges (b) Array for binary search

2 Longest Matching-Prefix

2.1 Linear List

In this data structure, the rules of the rule table are stored as a linear list ([9]) L. Let $LMP(d)$ be the longest matching-prefix for address d. $LMP(d)$ is determined by examining the prefixes in L from left to right; for each prefix, we determine whether or not that prefix matches d; and from the set of matching prefixes, the one with longest length is selected. To insert a rule q, we first search the list L from left to right to ensure that L doesn't already have a rule with the same filter as does q. Having verified this, the new rule q is added to the end of the list. Deletion is similar. The time for each of the operations determine $LMP(d)$, insert a rule, delete a rule is $O(n)$, where n is the number of rules in L. The memory required is also $O(n)$.

Note that this data structure may be used regardless of the form of the filter (i.e., ranges, Boolean expressions, etc.) and regardless of the tie breaker in use. The time and memory complexities are unchanged.

2.2 End-Point Array

Lampson, Srinivasan, and Varghese [10] have proposed a data structure in which the end points of the ranges defined by the prefixes are stored in ascending order in an array. $LMP(d)$ is found by performaing a binary search on this ordered array of end points.

Prefixes and their ranges may be drawn as nested rectangles as in Figure 2(a), which gives the pictorial representation of the five prefixes of Figure 1.

In the data structure of Lampson et al. [10], the distinct range end-points are stored in ascending order as in

Figure 2(b). The distinct end-points (range start and finish points) for the prefixes of Figure 1 are [0, 10, 11, 16, 18, 19, 23, 31]. Let r_i, $1 \leq i \leq q \leq 2n$ be the distinct range end-points for a set of n prefixes. Let $r_{q+1} = \infty$. With each distinct range end-point, r_i, $1 \leq i \leq q$, the array stores $LMP(d)$ for d such that (a) $r_i < d < r_{i+1}$ (this is the column labeled "$>$" in Figure 2(b)) and (b) $r_i = d$ (column labeled "="). Now, $LMP(d)$, $r_1 \leq d \leq r_q$ can be determined in $O(\log n)$ time by performing a binary search to find the unique i such that $r_i \leq d < r_{i+1}$. If $r_i = d$, $LMP(d)$ is given by the "=" entry; otherwise, it is given by the "$>$" entry. For example, since $d = 20$ satisfies $19 \leq d < 23$ and since $d \neq 19$, the "$>$" entry of the end point 19 is used to determine that $LMP(20)$ is P1.

As noted by Lampson et al. [10], the range end-point table can be built in $O(n)$ time (this assumes that the end points are available in ascending order). Unfortunately, as stated in [10], updating the range end-point table following the insertion or deletion of a prefix also takes $O(n)$ time because $O(n)$ "$>$" and/or "=" entries may change. Although Lampson et al. [10] provide ways to reduce the complexity of the search for the LMP by a constant factor, these methods do not result in schemes that permit prefix insertion and deletion in $O(\log n)$ time.

It should be noted that the end-point array may be used even when ties are broken by selecting the first matching rule or the highest-priority matching rule. Further, the method applies to the case when the filters are arbitrary ranges rather than simply prefixes. The complexity of the preprocessing step (i.e., creation of the array of ordered end-points) and the search for the rule to use is unchanged. Further, the memory requirements are the same, $O(n)$ for an n-rule table, regardless of the tie breaker and whether the filters are prefixes or general ranges.

2.3 Sets of Equal-Length Prefixes

Waldvogel et al. [21] have proposed a data structure to determine $LMP(d)$ by performing a binary search on prefix length. In this data structure, the prefixes in the router table T are partitioned into the sets S_0, S_1, ... such that S_i contains all prefixes of T whose length is i. For simplicity, we assume that T contains the default prefix *. So, $S_0 = \{*\}$. Next, each S_i is augmented with markers that represent prefixes in S_j such that $j > i$ and i is on the binary search path to S_j. For example, suppose that the length of the longest prefix of T is 32 and that the length of $LMP(d)$ is 22. To find $LMP(d)$ by a binary search on length, we will first search S_{16} for an entry that matches the first 16 bits of d. This search[4] will need to be successful for us to proceed to a larger length. The next search will be in S_{24}. This search will need to fail. Then, we will search S_{20} followed by S_{22}. So, the path followed by a binary search on length to get to S_{22} is S_{16}, S_{24}, S_{20}, S_{22}. For this to be followed, the searches in S_{16}, S_{20}, and S_{22} must succeed while that in S_{24} must fail. Since the length of $LMP(d)$ is 22, T has no matching prefix whose length is more than 22. So, the search in S_{24} is guaranteed to fail. Similarly, the search in S_{22} is guaranteed to succeed. However, the searches in S_{16} and S_{20} will succeed iff T has matching prefixes of length 16 and 20. To ensure success, every length 22 prefix P places a *marker* in S_{16} and S_{20}, the marker in S_{16} is the first 16 bits of P and that in S_{20} is the first 20 bits in P. Note that a marker M is placed in S_i only if S_i doesn't contain a prefix equal to M. Notice also that for each i, the binary search path to S_i has $O(\log l_{max}) = O(\log W)$, where l_{max} is the length of the longest prefix in T, S_js on it. So, each prefix creates $O(\log W)$ markers. With each marker M in S_i, we record the longest prefix of T that matches M (the length of this longest matching-prefix is neccessarily smaller than i).

To determine $LMP(d)$, we begin by setting $leftEnd = 0$ and $rightEnd = l_{max}$. The repetitive step of the binary search requires us to search for an entry in S_m, where $m = \lfloor (leftEnd + rightEnd)/2 \rfloor$, that equals the first m bits of d. If S_m does not have such an entry, set $rightEnd = m - 1$. Otherwise, if the matching entry is the prefix P, P becomes the longest matching-prefix found so far. If the matching entry is the marker M, the prefix recorded with M is the longest matching-prefix found so far. In either case, set $leftEnd = m + 1$. The binary search terminates when $leftEnd > rightEnd$.

One may easily establish the correctness of the described binary search. Since, each prefix creates $O(\log W)$ markers, the memory requirement of the scheme is $O(n \log W)$. When each set S_i is represented as a hash table, the data structure is called SELPH (sets of equal length prefixes using hash tables). The expected time to find $LMP(d)$ is $O(\log W)$ when the router table is represented as an SELPH. When inserting a prefix, $O(\log W)$ markers must also be inserted. With each marker, we must record a longest-matching prefix. The expected time to find these longest matching-prefixes is $O(\log^2 W)$. In addition, we may need to update the longest-matching prefix information stored with the $O(n \log W)$ markers at lengths greater than the length of the newly inserted prefix. This takes $O(n \log^2 W)$ time. So, the expected insert time is $O(n \log^2 W)$. When deleting a prefix P, we must search all hash tables for markers M that have P recorded with them and then update the recorded prefix for each of these markers. For hash tables with a bounded loading density, the expected time for a delete (including marker-prefix updates) is $O(n \log^2 W)$. Waldvogel et al. [21] have shown that by inserting the prefixes in ascending order of length,

[4] When searching S_i, only the first i bits of d are used, because all prefixes in S_i have exactly i bits.

an n-prefix SELPH may be constructed in $O(n \log^2 W)$ time.

When each set is represented as a balanced search tree, the data structure is called SELPT. In an SELPT, the time to find $LMP(d)$ is $O(\log n \log W)$; the insert time is $O(n \log n \log^2 W)$; the delete time is $O(n \log n \log^2 W)$; and the time to construct the data structure for n prefixes is $O(W + n \log n \log^2 W)$.

In the full version of [21], Waldvogel et al. show that by using a technique called marker partitioning, the SELPH data structure may be modified to have a search time of $O(\alpha + \log W)$ and an insert/delete time of $O(\alpha \sqrt[\alpha]{n}W \log W)$, for any $\alpha > 1$.

Because of the excessive insert and delete times, the sets of equal-length prefixes data structure is suitable only for static router tables. By using the prefix expansion method described in Section 2.4.2, we can limit the number of distinct lengths in the prefix set and so reduce the run time by a constant factor [21].

2.4 Tries

2.4.1 1-Bit Tries

A *1-bit trie* is a tree-like structure in which each node has a left child, left data, right child, and right data field. Nodes at level[5] $l-1$ of the trie store prefixes whose length is l. If the rightmost bit in a prefix whose length is l is 0, the prefix is stored in the left data field of a node that is at level $l-1$; otherwise, the prefix is stored in the right data field of a node that is at level $l-1$. At level i of a trie, branching is done by examining bit i (bits are numbered from left to right beginning with the number 0) of a prefix or destination address. When bit i is 0, we move into the left subtree; when the bit is 1, we move into the right subtree. Figure 3(a) gives the prefixes in the 8-prefix example of [19], and Figure 3(b) shows the corresponding 1-bit trie. The prefixes in Figure 3(a) are numbered and ordered as in [19].

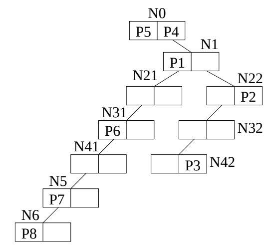
The 1-bit tries described here are an extension of the 1-bit tries described in [9]. The primary difference being that the 1-bit tries of [9] are for the case when all keys (prefixes) have the same length. Note that the height of a 1-bit trie is $O(W)$.

For any destination address d, all prefixes that match d lie on the search path determined by the bits of d. By following this search path, we may determine the longest matching-prefix, the first prefix in the table that matches d, as well as the highest-priority matching-prefix in $O(W)$ time. Further, prefixes may be inserted/deleted in $O(W)$ time. The memory required by the 1-bit trie is $O(nW)$.

[5]Level numbers are assigned beginning with 0 for the root level.

Original prefixes
P5=0*
P1=10*
P2=111*
P3=11001*
P4=1*
P6=1000*
P7=100000*
P8=1000000*

(a) 8-prefix example of [19]



(b) Corresponding 1-bit trie

Figure 3. Prefixes and corresponding 1-bit trie

IPv4 backbone routers may have more than 100 thousand prefixes. Even though the prefixes in a backbone router may have any length between 0 and W, there is a concentration of prefixes at lengths 16 and 24, because in the early days of the Internet, Internet address assignment was done by classes. All addresses in a class B network have the same first 16 bits, while addresses in the same class C network agree on the first 24 bits. Addresses in class A networks agree on their first 8 bits. However, there can be at most 256 class A networks (equivalently, there can be at most 256 8-bit prefixes in a router table). For our backbone routers that occur in practice [15], the number of nodes in a 1-bit trie is between $2n$ and $3n$. Hence, in practice, the memory required by the 1-bit-trie representation is $O(n)$.

2.4.2 Fixed-Stride Tries

Since the trie of Figure 3(b) has a height of 6, a search into this trie may make up to 7 memory accesses, one access for each node on the path from the root to a node at level 6 of the trie. The total memory required for the 1-bit trie of Figure 3(b) is 20 units (each node requires 2 units, one for each pair of (child, data) fields).

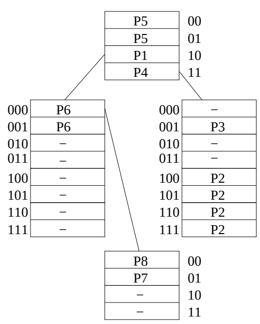
When 1-bit tries are used to represent IPv4 router tables, the trie height may be as much as 31. A lookup in such a trie takes up to 32 memory accesses.

Degermark et al. [3] and Srinivasan and Varghese [19] have proposed the use of fixed-stride tries to enable fast identification of the longest matching prefix in a router table. The *stride* of a node is defined to be the number of bits used at that node to determine which branch to take. A node whose stride is s has 2^s child fields (corresponding to the 2^s possible values for the s bits that are used) and 2^s data fields. Such a node requires 2^s memory units. In a *fixed-stride trie* (FST), all nodes at the same level have the same stride; nodes at different levels may have different strides.

Suppose we wish to represent the prefixes of Figure 3(a) using an FST that has three levels. Assume that the strides are 2, 3, and 2. The root of the trie stores prefixes whose length is 2; the level one nodes store prefixes whose length is 5 (2 + 3); and level three nodes store prefixes whose length is 7 (2 + 3 + 2). This poses a problem for the prefixes of our example, because the length of some of these prefixes is different from the storeable lengths. For instance, the length of P5 is 1. To get around this problem, a prefix with a nonpermissible length is expanded to the next permissible length. For example, P5 = 0* is expanded to P5a = 00* and P5b = 01*. If one of the newly created prefixes is a duplicate, natural dominance rules are used to eliminate all but one occurrence of the prefix. For instance, P4 = 1* is expanded to P4a = 10* and P4b = 11*. However, P1 = 10* is to be chosen over P4a = 10*, because P1 is a longer match than P4. So, P4a is eliminated. Because of the elimination of duplicate prefixes from the expanded prefix set, all prefixes are distinct. Figure 4(a) shows the prefixes that result when we expand the prefixes of Figure 3 to lengths 2, 5, and 7. Figure 4(b) shows the corresponding FST whose height is 2 and whose strides are 2, 3, and 2.

Expanded prefixes (3 levels)
00* (P5a)
01* (P5b)
10* (P1)
11* (P4)
11100* (P2a)
11101* (P2b)
11110* (P2c)
11111* (P2d)
11001* (P3)
10000* (P6a)
10001* (P6b)
1000001* (P7)
1000000* (P8)

(a) Expanded prefixes



(b) Corresponding fixed stride trie

Figure 4. Prefix expansion and fixed-stride trie

Since the trie of Figure 4(b) can be searched with at most 3 memory references, it represents a time performance improvement over the 1-bit trie of Figure 3(b), which requires up to 7 memory references to perform a search. However, the space requirements of the FST of Figure 4(b) are more than that of the corresponding 1-bit trie. For the root of the FST, we need 8 fields or 4 units; the two level 1 nodes require 8 units each; and the level 3 node requires 4 units. The total is 24 memory units.

We may represent the prefixes of Figure 3(a) using a one-level trie whose root has a stride of 7. Using such a trie, searches could be performed making a single memory access. However, the one-level trie would require $2^7 = 128$ memory units.

For IPv4 prefix sets, Degermark et al. [3] propose the use of a three-level trie in which the strides are 16, 8, and 8. They propose encoding the nodes in this trie using bit vectors to reduce memory requirements. The resulting data structure requires at most 12 memory accesses. However, inserts and deletes are quite expensive. For example, the insertion of the prefix 1* changes up to 2^{15} entries in the trie's root node. All of these changes may propogate into the compacted storage scheme of [3].

Lampson et al. [10] have proposed the use of hybrid data structures comprised of a stride-16 root and an auxiliary data structure for each of the subtries of the stride-16 root. This auxiliary data structure could be the end-point array of Section 2.2 (since each subtrie is expected to contain only a small number of prefixes, the number of end points in each end-point array is also expected to be quite small). An alternative auxiliary data structure suggested by Lampson et al. [10] is a 6-way search tree for Ipv4 router tables. In the case of these 6-way trees, the keys are the remaining up to 16 bits of the prefix (recall that the stride-16 root consumes the first 16 bits of a prefix). For IPv6 prefixes, a multicolumn scheme is suggested [10]. None of these proposed structures is suitable for a dynamic table.

In the *fixed-stride trie optimization* (FSTO) problem, we are given a set P of prefixes and an integer k. We are to select the strides for a k-level FST in such a manner that the k-level FST for the given prefixes uses the smallest amount of memory.

For some P, a k-level FST may actually require more space than a $(k-1)$-level FST. For example, when P = 00*, 01*, 10*, 11*, the unique 1-level FST for P requires 4 memory units while the unique 2-level FST (which is actually the 1-bit trie for P) requires 6 memory units. Since the search time for a $(k-1)$-level FST is less than that for a k-level tree, we would actually prefer $(k-1)$-level FSTs that take less (or even equal) memory over k-level FSTs. Therefore, in practice, we are really interested in determining the best FST that uses at most k levels (rather than exactly k levels). The *modified* MSTO problem (MFSTO) is to determine the best FST that uses at most k levels for the given prefix set P.

Let O be the 1-bit trie for the given set of prefixes, and let F be any k-level FST for this prefix set. Let s_0, ..., s_{k-1} be the strides for F. We shall say that level 0 of F covers levels $0, ..., s_0 - 1$ of O, and that level j, $0 < j < k$ of F covers levels $a, ..., b$ of O, where $a = \sum_0^{j-1} s_q$ and $b = \sum_0^{j} s_q - 1$. So, level 0 of the FST of Figure 4(b) covers levels 0 and 1 of the 1-bit trie of Figure 3(b). Level 1 of this

FST covers levels 2, 3, and 4 of the 1-bit trie of Figure 3(b); and level 2 of this FST covers levels 5 and 6 of the 1-bit trie. We shall refer to levels $e_u = \sum_0^u s_q, 0 \leq u < k$ as the *expansion levels* of O. The expansion levels defined by the FST of Figure 4(b) are 0, 2, and 5.

Let $nodes(i)$ be the number of nodes at level i of the 1-bit trie O. For the 1-bit trie of Figure 3(a), $nodes(0:6) = [1,1,2,2,2,1,1]$. The memory required by F is $\sum_0^{k-1} nodes(e_q) * 2^{s_q}$. For example, the memory required by the FST of Figure 4(b) is $nodes(0) * 2^2 + nodes(2) * 2^3 + nodes(5) * 2^2$ = 24.

Let $C(j,r)$ be the cost of the best FST that uses *at most* r expansion levels. A simple dynamic programming recurrence for C is [15]:

$$C(j,r) = \min_{m \in \{-1..j-1\}} \{C(m,r-1) + nodes(m+1) * 2^{j-m}\}, j \geq 0, r > 1 \quad (1)$$

$$C(-1,r) = 0 \text{ and } C(j,1) = 2^{j+1}, j \geq 0 \quad (2)$$

Let $M(j,r)$, $r > 1$, be the smallest m that minimizes

$$C(m,r-1) + nodes(m+1) * 2^{j-m},$$

in Equation 1.

Theorem 1 *[Sahni and Kim [15]]* $\forall (j \geq 0, k > 2)[M(j,k) \geq max\{M(j-1,k), M(j,k-1)\}]$.

Theorem 1 results in an algorithm to compute $C(W-1,k)$ in $O(kW^2)$. Using the computed M values, the strides for the OFST that uses at most k expansion levels may be determined in an additional $O(k)$ time. Although the resulting algorithm has the same asymptotic complexity as does the optimization algorithm of Srinivasan and Varghese [19], experiments conducted by Sahni and Kim [15] using real IPv4 prefix-data-sets indicate that the algorithm based on Theorem 1 runs 2 to 4 times as fast.

2.4.3 Variable-Stride Tries

In a *variable-stride trie* (VST) [19], nodes at the same level may have different strides. Figure 5 shows a two-level VST for the 1-bit trie of Figure 3. The stride for the root is 2; that for the left child of the root is 5; and that for the root's right child is 3. The memory requirement of this VST is 4 (root) + 32 (left child of root) + 8 (right child of root) = 44.

Since FSTs are a special case of VSTs, the memory required by the best VST for a given prefix set P and number of expansion levels k is less than or equal to that required by the best FST for P and k. Despite this, FSTs may be preferred in certain router applications "because of their simplicity and slightly faster search time" [19].

Let r-VST be a VST that has at most r levels. Let $Opt(N,r)$ be the cost (i.e., memory requirement) of the

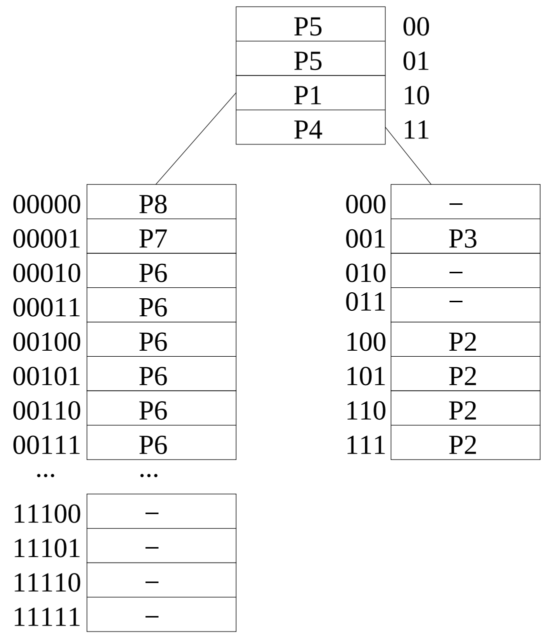


Figure 5. Two-level VST for prefixes of Figure 3(a)

best r-VST for a 1-bit trie whose root is N. Nilsson and Karlsson [14] propose a greedy heuristic to construct optimal VSTs. They call the resulting VSTs LC-tries (level-compressed tries). An LC-tries obtained from a 1-bit trie by replacing full subtries of the 1-bit trie by single multibit nodes. This replacement is done by examining the 1-bit trie top to bottom (i.e., from root to leaves). Srinivasan and Varghese [19], have obtained the following dynamic programming recurrence for $Opt(N,r)$.

$$Opt(N,r) = \min_{s \in \{1...1+height(N)\}} \{2^s + \sum_{M \in D_s(N)} Opt(M,r-1)\}, \quad r > 1 \quad (3)$$

where $D_s(N)$ is the set of all descendents of N that are at level s of N. For example, $D_1(N)$ is the set of children of N and $D_2(N)$ is the set of grandchildren of N. $height(N)$ is the maximum level at which the trie rooted at N has a node. For example, in Figure 3(b), the height of the trie rooted at N1 is 5. When $r = 1$,

$$Opt(N,1) = 2^{1+height(N)} \quad (4)$$

Srinivasan and Varghese [19], describe a way to determine $Opt(R,k)$ using Equations 3 and 4. The complexity of their algorithm is $O(n * W^2 * k)$, where n is the number of prefixes in P and W is the length of the longest prefix. By modifying the equations of Srinivasan and Varghese [19] slightly, Sahni and Kim [16] are able to compute $Opt(R,k)$ in $O(mWk)$ time, where m is the number of nodes in the 1-bit trie. Since $m = O(n)$ for realistic router prefix sets, the complexity of our algorithm is $O(nWk)$. Let

$$Opt(N,s,r) = \sum_{M \in D_s(N)} Opt(M,r),\ s > 0,\ r > 1,$$

and let $Opt(N,0,r) = Opt(N,r)$. From Equations 3 and 4, it follows that:

$$\begin{aligned} Opt(N,0,r) &= min_{s \in \{1 \ldots 1+height(N)\}}\{2^s + \\ &\quad Opt(N,s,r-1)\},\ r > 1 \end{aligned} \quad (5)$$

and

$$Opt(N,0,1) = 2^{1+height(N)}. \quad (6)$$

For $s > 0$ and $r > 1$, we get

$$\begin{aligned} Opt(N,s,r) &= \sum_{M \in D_s(N)} Opt(M,r) \\ &= Opt(LeftChild(N), s-1, r) \\ &+ Opt(RightChild(N), s-1, r). \end{aligned} \quad (7)$$

For Equation 7, we need the following initial condition:

$$Opt(null,*,*) = 0 \quad (8)$$

With the assumption that the number of nodes in the 1-bit trie is $O(n)$, we see that the number of $Opt(*,*,*)$ values is $O(nWk)$. Each $Opt(*,*,*)$ value may be computed in $O(1)$ time using Equations 5 through 8 provided the Opt values are computed in postorder. Therefore, we may compute $Opt(R,k) = Opt(R,0,k)$ in $O(nWk)$ time. Our algorithm requires $O(W^2k)$ memory for the $Opt(*,*,*)$ values. To see this, notice that there can be at most $W+1$ nodes N whose $Opt(N,*,*)$ values must be retained at any given time, and for each of these at most $W+1$ nodes, $O(Wk)$ $Opt(N,*,*)$ values must be retained. To determine the optimal strides, each node of the 1-bit trie must store the stride s that minimizes the right side of Equation 5 for each value of r. For this purpose, each 1-bit trie node needs $O(k)$ space. Since the 1-bit trie has $O(n)$ nodes in practice, the memory requirements of the 1-bit trie are $O(nk)$. The total memory required is, therefore, $O(nk + W^2k)$.

In practice, we may prefer an implementation that uses considerably more memory. If we associate a cost array with each of the $O(n)$ nodes of the 1-bit trie, the memory requirement increases to $O(nWk)$. The advantage of this increased memory implementation is that the optimal strides can be recomputed in $O(W^2k)$ time (rather than $O(nWk)$) following each insert or delete of a prefix. This is so because, the $Opt(N,*,*)$ values need be recomputed only for nodes along the insert/delete path of the 1-bit trie. There are $O(W)$ such nodes.

Faster algorithms to determine optimal 2- and 3-VSTs also are developed in [16].

2.5 Binary Search Trees

Sahni and Kim [17] propose the use of a collection of red-black trees to determine $LMP(d)$. The CRBT comprises a front-end data structure that is called the *binary interval tree* (BIT) and a back-end data structure called a *collection of prefix trees* (CPT). For any destination address d, define the *matching basic interval* to be a basic interval with the property that $r_i \leq d \leq r_{i+1}$ (note that some ds have two matching basic intervals).

The BIT is a binary search tree that is used to search for a matching basic interval for d. The BIT comprises internal and external nodes and there is one internal node for each r_i. Since the BIT has q internal nodes, it has $q+1$ external nodes. The first and last of these, in inorder, have no significance. The remaining $q-1$ external nodes, in inorder, represent the $q-1$ basic intervals of the given prefix set. Figure 6(a) gives a possible (we say possible because, any red-black binary search tree organization for the internal nodes will suffice) BIT for our five-prefix example of Figure 2(a). Internal nodes are shown as rectangles while circles denote external nodes. Every external node has three pointers: *startPointer*, *finishPointer*, and *basicIntervalPointer*. For an external node that represents the basic interval $[r_i, r_{i+1}]$, *startPointer* (*finishPointer*) points to the header node of the prefix tree (in the back-end structure) for the prefix (if any) whose range start and finish points are r_i (r_{i+1}). Note that only prefixes whose length is W can have this property. *basicIntervalPointer* points to a prefix node in a prefix tree of the back-end structure. In Figure 6(a), the labels in the external (circular) nodes identify the represented basic interval. The external node with r1 in it, for example, has a *basicIntervalPointer* to the rectangular node labeled r1 in the prefix tree of Figure 6(b).

For each prefix and basic interval, x, define $next(x)$ to be the smallest range prefix (i.e., the longest prefix) whose range includes the range of x. For the example of Figure 2(a), the $next()$ values for the basic intervals $r1$ through $r7$ are, respectively, $P1$, $P2$, $P1$, $P3$, $P4$, $P1$, and $P1$. Notice that the next value for the range $[r_i, r_{i+1}]$ is the same as the ">" value for r_i in Figure 2(b), $1 \leq i < q$. The $next()$ values for the nontrivial prefixes $P1$ through $P4$ of Figure 2(a) are, respectively, "-", $P1$, $P1$, and $P3$.

The back-end structure, which is a collection of prefix

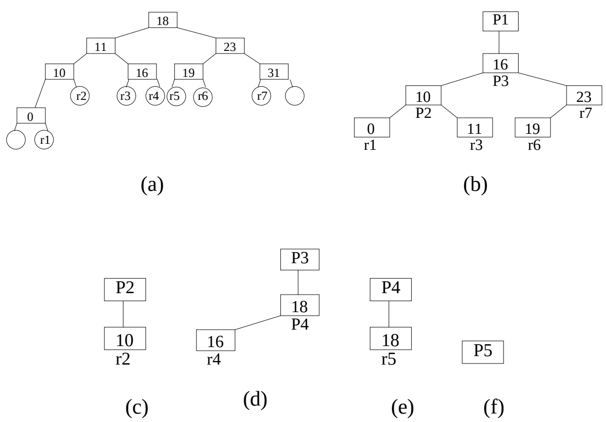


Figure 6. CBST for Figure 2(a). (a) base interval tree (b) prefix tree for $P1$ (c) prefix tree for $P2$ (d) prefix tree for $P3$ (e) prefix tree for $P4$ (f) prefix tree for $P5$

trees (CPT), has one prefix tree for each of the prefixes in the router table. Each prefix tree is a red-black tree. The prefix tree for prefix P comprises a header node plus one node, called a *prefix node*, for every nontrivial prefix or basic interval x such that $next(x) = P$. The header node identifies the prefix P for which this is the prefix tree. The prefix trees for each of the five prefixes of Figure 2(a) are shown in Figures 6(b)-(f). Notice that prefix trees do not have external nodes and that the prefix nodes of a prefix tree store the start point of the range or prefix represented by that prefix node. In the figures, the start points of the basic intervals and prefixes are shown inside the prefix nodes while the basic interval or prefix name is shown outside the node.

The search for $LMP(d)$ begins with a search of the BIT for the matching basic interval for d. Suppose that external node Q of the BIT represents this matching basic interval. When the destination address equals the left (right) end-point of the matching basic interval and *startPointer* (*finishPointer*) is not null, $LMP(d)$ is pointed to by *startPointer* (*finishPointer*). Otherwise, the back-end CPT is searched for $LMP(d)$. The search of the back-end structure begins at the node $Q.basicIntervalPointer$. By following parent pointers from $Q.basicIntervalPointer$, we reach the header node of the prefix tree that corresponds to $LMP(d)$.

When a CRBT is used, $LMP(d)$ may be found in $O(\log n)$ time. Inserts and deletes also take $O(\log n)$ time when a CRBT is used. In [18], Sahni and Kim propose an alternative BIT structure (ABIT) that has internal nodes only. Although the ABIT structure increases the memory requirements of the router table, the time to search, insert, and delete is reduced by a constant factor [18]. Suri et al. [20] have proposed a B-tree data structure for dynamic router tables. Using their structure, we may find $LMP(d)$ in $O(\log n)$ time. However, inserts/deletes take $O(W \log n)$. When W bits fit in $O(1)$ words (as is the case for IPv4 and IPv6 prefixes) logical operations on W-bit vectors can be done in $O(1)$ time each. In this case, the scheme of [20] takes $O(W + \log n)$ time for an update.

Several researchers ([1, 8, 5, 18], for example), have investigated router table data structures that account for bias in access patterns. Gupta, Prabhakar, and Boyd [8], for example, propose the use of ranges. They assume that access frequencies for the ranges are known, and they construct a bounded-height binary search tree of ranges. This binary search tree accounts for the known range access frequencies to obtain near-optimal IP lookup. Although the scheme of [8] performs IP lookup in near-optimal time, changes in the access frequencies, or the insertion or removal of a prefix require us to reconstruct the data structure, a task that takes $O(n \log n)$ time.

Ergun et al. [5] use ranges to develop a biased skip list structure that performs longest prefix-matching in $O(\log n)$ expected time. Their scheme is designed to give good expected performance for bursty[6] access patterns". The biased skip list scheme of Ergun et al. [5] permits inserts and deletes in $O(\log n)$ time only in the severely restricted and impractical situation when all prefixes in the router table are of the same length. For the more general, and practical, case when the router table comprises prefixes of different length, their scheme takes $O(n)$ expected time for each insert and delete. Sahni and Kim [18] extend the biased skip lists of Ergun et al. [5] to obtain a biased skip lists structure in which longest prefix-matching as well as inserts and deletes take $O(\log n)$ expected time. They also propose a splay tree scheme for bursty access patterns. In this scheme, longest prefix-matching, insert and delete have an $O(\log n)$ amortized complexity.

2.6 Priority Search Trees

A priority-search tree (PST) [13] is a data structure that is used to represent a set of tuples of the form $(key1, key2, data)$, where $key1 \geq 0$, $key2 \geq 0$, and no two tuples have the same $key1$ value. The data structure is simultaneously a min-tree on $key2$ (i.e., the $key2$ value in each node of the tree is $\leq$ the $key2$ value in each de-

[6] In a *bursty* access pattern the number of different destination addresses in any subsequence of q packets is $<< q$. That is, if the destination of the current packet is d, there is a high probability that d is also the destination for one or more of the next few packets. The fact that Internet packets tend to be bursty has been noted in [2, 11], for example.

scendent node) and a search tree on $key1$. There are two common PST representations [13]:

1. In a **radix priority-search tree** (RPST), the underlying tree is a binary radix tree on $key1$.
2. In a **red-black priority-search tree** (RBPST), the underlying tree is a red-black tree.

McCreight [13] has suggested a PST representation of a collection of ranges with distinct finish points. This representation uses the following mapping of a range r into a PST tuple:

$$(key1, key2, data) = (finish(r), start(r), data) \quad (9)$$

where $data$ is any information (e.g., next hop) associated with the range. Each range r is, therefore mapped to a point $map1(r) = (x, y) = (key1, key2) = (finish(r), start(r))$ in 2-dimensional space.

Let $ranges(d)$ be the set of ranges that match d. McCreight [13] has observed that when the mapping of Equation 9 is used to obtain a point set $P = map1(R)$ from a range set R, then $ranges(d)$ is given by the points that lie in the rectangle (including points on the boundary) defined by $x_{left} = d$, $x_{right} = \infty$, $y_{top} = d$, and $y_{bottom} = 0$. These points are obtained using the method $enumerateRectangle(x_{left}, x_{right}, y_{top}) = enumerateRectangle(d, \infty, d)$ of a PST (y_{bottom} is implicit and is always 0).

When an RPST is used to represent the point set P, the complexity of

$$enumerateRectangle(x_{left}, x_{right}, y_{top})$$

is $O(\log maxX + s)$, where $maxX$ is the largest x value in P and s is the number of points in the query rectangle. When the point set is represented as an RBPST, this complexity becomes $O(\log n + s)$, where $n = |P|$. A point (x, y) (and hence a range $[y, x]$) may be inserted into or deleted from an RPST (RBPST) in $O(\log maxX)$ ($O(\log n)$) time [13].

Let R be a set of prefix ranges. For simplicity, assume that R includes the range that corresponds to the prefix *. With this assumption, $LMP(d)$ is defined for every d. One may verify that $LMP(d)$ is the prefix whose range is $[maxStart(ranges(d)), minFinish(ranges(d))]$ (Lu and Sahni [12] show that R must contain such range). To find this range easily, we first transform $P = map1(R)$ into a point set $transform1(P)$ so that no two points of $transform1(P)$ have the same x-value. Then, we represent $transform1(P)$ as a PST. For every $(x, y) \in P$, define $transform1(x, y) = (x', y') = (2^W x - y + 2^W - 1, y)$. Then, $transform1(P) = \{transform1(x, y) | (x, y) \in P\}$.

We see that $0 \leq x' < 2^{2W}$ for every $(x', y') \in transform1(P)$ and that no two points in $transform1(P)$ have the same x'-value. Let $PST1(P)$ be the PST for $transform1(P)$. The operation

$$enumerateRectangle(2^W d - d + 2^W - 1, \infty, d)$$

performed on $PST1$ yields $ranges(d)$. To find $LMP(d)$, we employ the

$$minXinRectangle(x_{left}, x_{right}, y_{top})$$

operation, which determines the point in the defined rectangle that has the least x-value. It is easy to see that

$$minXinRectangle(2^W d - d + 2^W - 1, \infty, d)$$

performed on $PST1$ yields $LMP(d)$.

To insert the prefix whose range in $[u, v]$, we insert $transform1(map1([u, v]))$ into $PST1$. In case this prefix is already in $PST1$, we simply update the next-hop information for this prefix. To delete the prefix whose range is $[u, v]$, we delete $transform1(map1([u, v]))$ from $PST1$. When deleting a prefix, we must take care not to delete the prefix *. Requests to delete this prefix should simply result in setting the next-hop associated with this prefix to $\emptyset$.

Since, $minXinRectangle$, insert, and delete each take $O(W)$ ($O(\log n)$) time when $PST1$ is an RPST (RBPST), $PST1$ provides a router-table representation in which longest-prefix matching, prefix insertion, and prefix deletion can be done in $O(W)$ time each when an RPST is used and in $O(\log n)$ time each when an RBPST is used.

3 Most-Specific-Range Matching

Let $msr(d)$ be the most-specific range that matches the destination address d. For static tables, we may simply represent the n ranges by the up to $2n - 1$ basic intervals they induce. For each basic interval, we determine the most-specific range that matches all points in the interval. These up to $2n - 1$ basic intervals may then be represented as up to $4n - 2$ prefixes [6] with the property that $msr(d)$ is uniquely determined by $LMP(d)$. Now, we may use any of the earlier discussed data structures for static tables in which the filters are prefixes. In this section, therefore, we discuss only those data structures that are suitable for dynamic tables.

3.1 Nonintersecting Ranges

Let R be a set of nonintersecting ranges. For simplicity, assume that R includes the range z that matches all destination addresses ($z = [0, 2^{32} -$

1] in the case of IPv4). With this assumption, $msr(d)$ is defined for every d. Similar to the case of prefixes, for nonintersecting ranges, $msr(d)$ is the range $[maxStart(ranges(d)), minFinish(ranges(d))]$ (Lu and Sahni [12] show that R must contain such a range). We may use $PST1(transform1(map1(R)))$ to find $msr(d)$ using the same method described in Section 2.6 to find $LMP(d)$.

Insertion of a range r is to be permitted only if r does not intersect any of the ranges of R. Once we have verified this, we can insert r into $PST1$ as described in Section 2.6. Range intersection may be verified by noting that there are two cases for range intersection. When inserting $r = [u, v]$, we need to determine if $\exists s = [x, y] \in R[u < x \leq v < y \vee x < u \leq y < v]$. We see that $\exists s \in R[x < u \leq y < v]$ iff $map1(R)$ has at least one point in the rectangle defined by $x_{left} = u$, $x_{right} = v - 1$, and $y_{top} = u - 1$ (recall that $y_{bottom} = 0$ by default). Hence, $\exists s \in R[x < u \leq y < v]$ iff $minXinRectangle(2^W u - (u-1) + 2^W - 1, 2^W(v-1) + 2^W - 1, u - 1)$ exists in PST1.

To verify $\exists s \in R[u < x \leq v < y]$, map the ranges of R into 2-dimensional points using the mapping, $map2(r) = (start(r), 2^W - 1 - finish(r))$. Call the resulting set of mapped points $map2(R)$. We see that $\exists s \in R[u < x \leq v < y]$ iff $map2(R)$ has at least one point in the rectangle defined by $x_{left} = u + 1$, $x_{right} = v$, and $y_{top} = (2^W - 1) - v - 1$. To verify this, we maintain a second PST, $PST2$ of points in $transform2(map2(R))$, where $transform2(x, y) = (2^W x + y, y)$ Hence, $\exists s \in R[u < x \leq v < y]$ iff $minXinRectangle(2^W(u+1), 2^W v + (2^W - 1) - v - 1, (2^W - 1) - v - 1)$ exists.

To delete a range r, we must delete r from both $PST1$ and $PST2$. Deletion of a range from a PST is similar to deletion of a prefix as discussed in Section 2.6.

The complexity of the operations to find $msr(d)$, insert a range, and delete a range are the same as that for these opertions for the case when R is a set of ranges that correspond to prefixes.

3.2 Conflict-Free Ranges

The range set R has a **conflict** iff there exists a destination address d for which $ranges(d) \neq \emptyset \wedge msr(d) = \emptyset$. R is **conflict free** iff it has no conflict. Notice that sets of prefix ranges and sets of nonintersecting ranges are conflict free. The two-PST data structure of Section 3.1 may be extended to the general case when R is an arbitrary conflict-free range set. Once again, we assume that R includes the range z that matches all destination addresses. $PST1$ and $PST2$ are defined for the range set R as in Sections 2.6 and 3.1.

Sahni and Lu [12] have shown that when R is conflict free, $msr(d)$ is the range $[maxStart(ranges(d)), minFinish(ranges(d))]$. Hence, $msr(d)$ may be obtained by performing the operation

$$minXinRectangle(2^W d - d + 2^W - 1, \infty, d)$$

on PST1. Insertion and deletion are complicated by the need to verify that the addition or deleteion of a range to/from a conflict-free range set leaves behind a conflict-free range set. To perform this check efficiently, Lu and Sahni [12] augment $PST1$ and PST with a representation for the chains in the normalized range-set, $norm(R)$, that corresponds to R. This requires the use of several red-black trees. The reader is referred to [12] for a description of this augmentation.

The overall complexity of the augmented data structure of [12] is $O(\log n)$ for each operation when $RBPST$s are used for $PST1$ and $PST2$. When $RPST$s are used, the search complexity is $O(W)$ and the insert and delete complexity is $O(W + \log n) = O(W)$.

4 Conclusion

We have reviewed the various data structures that have been proposed for longest-prefix matching and most-specific-range matching in both static and dynamic router tables. Although data structures for maximum-priority matching may be used find $LMP(d)$ and $msr(d)$ by setting the rule priority equal to the prefix length (in the case when the filters are prefixes) or to the negative of the range size (in case the filters are ranges), these data structures have not been considered here, because the performance of maximum-priority data structures is inferior to that of data structures designed specifically for the longest-matching prefix and most-specific-range matching problems. Table 1 summarizes the performance characteristics of various data structures for the longest matching-prefix problem. Data structures for maximum-priority matching are developed in [6, 7].

References

[1] G. Cheung and S. McCanne, Optimal routing table design for IP address lookups under memory constraints, *IEEE INFOCOM*, 1999.

[2] K. Claffy, H. Braun, and G, Polyzos, A parameterizable methodology for internet traffic flow profiling, *IEEE Journal of Selected Areas in Communications*, 1995.

[3] M. Degermark, A. Brodnik, S. Carlsson, and S. Pink, Small forwarding tables for fast routing lookups, *ACM SIGCOMM*, 1997, 3-14.

Data Structure	Search	Update	Memory Usage
Linear List	$O(n)$	$O(n)$	$O(n)$
End-point Array	$O(\log n)$	$O(n)$	$O(n)$
Sets of Equal-Length Prefixes	$O(\alpha + \log W)$ expected	$O(\alpha \sqrt[\alpha]{n} W \log W)$ expected	$O(n \log W)$
1-bit tries	$O(W)$	$O(W)$	$O(nW)$
s-bit Tries	$O(W/s)$	-	$O(2^s nW/s)$
CRBT	$O(\log n)$	$O(\log n)$	$O(n)$
ACRBT	$O(\log n)$	$O(\log n)$	$O(n)$
BSLPT	$O(\log n)$ expected	$O(\log n)$ expected	$O(n)$
CST	$O(\log n)$ amortized	$O(\log n)$ amortized	$O(n)$
PST	$O(\log n)$	$O(\log n)$	$O(n)$

Table 1. Performance of data structures for longest matching-prefix

[4] W. Doeringer, G. Karjoth, and M. Nassehi, Routing on longest-matching prefixes, *IEEE/ACM Transactions on Networking*, 4, 1, 1996, 86-97.

[5] F. Ergun, S. Mittra, S. Sahinalp, J. Sharp, and R. Sinha, A dynamic lookup scheme for bursty access patterns, *IEEE INFOCOM*, 2001.

[6] A. Feldman and S. Muthukrishnan, Tradeoffs for packet classification, *IEEE INFOCOM* 2000.

[7] P. Gupta and N. McKeown, Dynamic algorithms with worst-case performance for packet classification, *IFIP Networking*, 2000.

[8] P. Gupta, B. Prabhakar, and S. Boyd, Near-optimal routing lookups with bounded worst case performance, *IEEE INFOCOM*, 2000.

[9] E. Horowitz, S. Sahni, and D. Mehta, Fundamentals of Data Structures in C++, W.H. Freeman, NY, 1995, 653 pages.

[10] B. Lampson, V. Srinivasan, and G. Varghese, IP lookup using multi-way and multicolumn search, *IEEE INFOCOM 98*, 1998.

[11] S. Lin and N. McKeown, A simulation study of IP switching, *IEEE INFOCOM*, 2000.

[12] H. Lu and S. Sahni, $O(\log n)$ dynamic router-tables for prefixes and ranges. Submitted.

[13] E. McCreight, Priority search trees, *SIAM Jr. on Computing*, 14, 1, 1985, 257-276.

[14] S. Nilsson and G. Karlsson, Fast address look-up for Internet routers, *IEEE Broadband Communications*, 1998.

[15] S. Sahni and K. Kim, Efficient construction of fixed-Stride multibit tries for IP lookup, *Proceedings 8th IEEE Workshop on Future Trends of Distributed Computing Systems*, 2001.

[16] S. Sahni and K. Kim, Efficient construction of variable-stride multibit tries for IP lookup, *Proceedings IEEE Symposium on Applications and the Internet (SAINT)*, 2002, 220-227.

[17] S. Sahni and K. Kim, $O(\log n)$ dynamic packet routing, submitted.

[18] S. Sahni and K. Kim, Efficient dynamic lookup for bursty access patterns, submitted.

[19] V. Srinivasan and G. Varghese, Faster IP lookups using controlled prefix expansion, *ACM Transactions on Computer Systems*, Feb:1-40, 1999.

[20] S. Suri, G. Vargjese, and P. Warkhede, Multiway range trees: Scalable IP lookup with fast updates, *GLOBECOM 2001*.

[21] M. Waldvogel, G. Varghese, J. Turner, and B. Plattner, Scalable high speed IP routing lookups, *ACM SIGCOMM*, 1997, 25-36.