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Computing Hough Transforms on Hypercube Multicomputers*

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Abstract. Efficient algorithms to compute the Hough transform on MIMD and SIMD hypercube multicomputers are developed. Our algorithms can compute p angles of the Hough transform of an $N \times N$ image, $p \leq N$, in $O(p + \log N)$ time on both MIMD and SIMD hypercubes. These algorithms require $O(N^2)$ processors. We also consider the computation of the Hough transform on MIMD hypercubes with a fixed number of processors. Experimental results on an NCUBE/7 hypercube are presented.

Key words: Hough transform, MIMD and SIMD hypercube multicomputers, complexity.

1. Introduction

The Hough transform is used to transform edges to another space, called the Hough space, so that the desired group of edges forms a cluster in the transformed space. Let $I[0\ldots N-1, 0\ldots N-1]$ be an $N \times N$ image such that $I[x, y] = 1$ iff the image point $[x, y]$ is a possible edge point. $I[x, y] = 0$ otherwise. The p angle Hough transform of I to detect straight lines in an image is the array H such that

$$H[i, j] = |\{(x, y) | i = \lfloor x \cos \theta_j + y \sin \theta_j \rfloor, \theta_j = \frac{\pi}{p}(j + 1) \text{ and } I[x, y] = 1\}|. \quad (1)$$

j takes on the integer values $0, 1, \ldots, p-1$. These correspond to the p angles $\theta_j = \frac{\pi}{p}(j+1)$, $0 \leq j < p$. Hence $0 < \theta_j \leq \pi$. For θ_j in this range and x and y in the range $0\ldots N-1$, $\lfloor x \cos \theta_j + y \sin \theta_j \rfloor$ is in the range $-\sqrt{2}N\ldots\sqrt{2}N$. Hence H is at most a $2\sqrt{2}N \times p$ matrix.

The general equation of a straight line can be given by the parametric equation

$$x \cos \theta + y \sin \theta = r, \quad (2)$$

where θ is the angle that the normal, to the line given by Equation (2), makes with the x axis and r is the length of the normal. Any edge point (x_i, y_i) on this line satisfies the equation

$$x_i \cos \theta + y_i \sin \theta = r. \quad (3)$$

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