Generalized Field Splitting Algorithms for Optimal IMRT Delivery Efficiency

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Abstract. Intensity modulated radiation therapy (IMRT) uses radiation beams of varying intensities to deliver varying doses of radiation to different areas of the tissue. The use of IMRT has allowed the delivery of higher doses of radiation to the tumor and lower doses to the surrounding healthy tissue. It is not uncommon for head and neck tumors, for example, to have large treatment widths that are not deliverable using a single field. In such cases, the intensity matrix generated by the optimizer needs to be split into two or three matrices, each of which may be delivered using a single field. Existing field splitting algorithms used pre-specified arbitrary split line or region where the intensity matrix is split along a column, i.e., all rows of the matrix are split along the same column (with or without overlapping of split fields, i.e., feathering). If three fields result, then the two splits are along the same two columns for all rows. In this paper we study the problem of splitting a large field into two or three subfields with field width as the only constraint, allowing for arbitrary overlap of the split fields, so that the total MU efficiency of delivering the split fields is maximized. Proof of optimality is provided for the proposed algorithm. An average decrease of 18.8% is found in total MUs when compared to the split generated by a commercial treatment planning system.

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1. Introduction

Intensity modulated radiation therapy (IMRT) delivered with conventional multileaf collimators (MLC) is increasingly used to treat large volumes because of its ability to deliver more conformal dose distribution while sparing the surrounding normal tissue (Hong et. al. 2002, Mundt et. al. 2003, Ahamad et. al. 2003, Forster et. al. 2003, Dogan et. al. 2003). The MLC systems vary in design and each one of them has certain mechanical limitations. Maximum leaf over-travel is one such limitation, which necessitates a large field to be split into two or more adjacent abutting subfields. The subfields are then delivered as separate treatment fields. All field-splitting algorithms proposed so far split the field either near the middle of the field along an arbitrarily-chosen straight line (e.g., CORVUS v5.0, NOMOS Corp., Sewickley, PA), or with a pre-defined overlap region for feathering (Wu et. al. 2000, Hong et. al. 2002, Kamath et. al. 2004a) without any consideration of delivery efficiency. Due to concerns of increased whole body dose in IMRT delivery, the problem of MU efficiency in field splitting should be addressed.

The field split problem can be formulated into the following general statement: given an intensity pattern (matrix) which exceeds a pre-defined maximum allowable field width w, find two or three subfields, each with field width $\leq w$, that would minimize the total delivered MU. In this generalized model, the field width restriction is the only constraint on the split. There are no pre-defined split lines or regions, and subfields are allowed to overlap, i.e., there may be bixels that receive parts of their desired intensity from two different fields. Width of a matrix is loosely defined as the number of columns over which non-zero intensity values span. We will use the terms *field* and (intensity) *matrix* interchangeably, since there is a one to one correspondence between them, i.e., a field has an associated intensity matrix and conversely an intensity matrix defines a field.

The field splitting of large intensity modulated fields has been studies by several authors. Wu et. al. (2000) proposed a dynamic feathering technique where the subfields overlap each other by a small amount, and the intensity in the overlap region gradually decreases for one subfield and increases for the other. The sum of intensities remains the same as for the original field. Dogan et. al. (2003) employed the methods of shifting the isocenter position along the target width and the introduction of a "pseudo target" to modify the split line positions, thereby reducing the magnitude of hot/cold spots. None of the above studies considered the delivery efficiency when generating the subfields. Kamath et. al. (2004a) studied the problem of splitting large treatment fields such that the total therapy time is minimized. However, it is assumed in that study that the intensity matrix is split along a column, i.e., all rows of the matrix are split along the same column (with or without feathering). If three fields result, then the two splits are along the same two columns for all rows. This can be considered a restricted version of the general field splitting problem. In this paper we study the most general field splitting problem. The matrices resulting from the split are each required to have a width $\leq w$ sample points, where w is the maximum allowable field width. Figure 1 shows an intensity matrix and fields resulting from a generalized split with $w = 5$.

$$\begin{array}{|cccccc|} 3 & 4 & 6 & 1 & 5 & 9 \\ 8 & 6 & 0 & 3 & 7 & 1 \\ 0 & 4 & 9 & 2 & 5 & 7 \end{array} \longrightarrow \begin{array}{|ccccc|c|} 3 & 4 & 4 & 0 & 0 & 0 \\ 8 & 6 & 0 & 2 & 1 & 0 \\ 0 & 4 & 8 & 1 & 2 & 0 \end{array} + \begin{array}{|cc|cccc|} 0 & 0 & 2 & 1 & 5 & 9 \\ 0 & 0 & 0 & 1 & 6 & 1 \\ 0 & 0 & 0 & 1 & 3 & 7 \end{array}$$

Figure 1. The matrix on the left is split into the two matrices on the right ($w = 5$). In the first row there is feathering along column 3. The second row feathers along columns 4 and 5 and the third row feathers along columns 3, 4 and 5. The widths of the fields resulting from the split are shaded.

2. Preliminaries

2.1. Single leaf pair profile

2.1.1. Intensity map. We consider delivery of intensity map produced by the optimizer. It is important to note that the intensity map from the optimizer is always a discrete matrix. The spatial resolution of this matrix is similar to the smallest beamlet size. The beamlet size typically ranges from 5-10 mm. Let $I(x)$ be the desired intensity profile along x axis. The discretized profile from the optimizer gives the intensity values at sample points $x_1, x_2, \ldots, x_m$. We assume that the sample points are uniformly spaced and that $\Delta x = x_{i+1} - x_i, 1 \leq i < m$. $I(x)$ is assigned the value $I(x_i)$ for $x_i \leq x < x_i + \Delta x$, $1 \leq i \leq m$. Now, $I(x_i)$ is our desired intensity profile, i.e., $I(x_i)$ is a measure of the number of MUs for which x_i (equivalently the interval $[x_i, x_i + \Delta x)$), $1 \leq i \leq m$, needs to be exposed. In the remainder of this paper, we will refer to a profile $I(x_i)$ simply as profile I. Figure 2 shows a profile, which is the output from the optimizer at discrete sample points $x_1, x_2, \ldots, x_m$. This profile is delivered either with the Segmental Multileaf Collimation (SMLC) method or with Dynamic Multileaf Collimation (DMLC). In this paper we study delivery with SMLC.

The notations used here are the same as those used in Kamath et. al. (2003) and Kamath et. al. (2004), and the reader is referred to these papers for fundamental definitions and algorithms. Additional notations and definitions have been introduced as needed.

2.1.2. Delivering a profile using one field. When the left leaf is placed so that it shields exactly the points $x_1, x_2, \ldots, x_i$, we will say that the left leaf is positioned at x_{i+1}. In particular, the point x_{i+1} is not shielded by it. When the right leaf is placed so that it shields exactly $x_{i+1}, x_{i+2}, \ldots, x_m$, we say that the right leaf is positioned at x_{i+1}. Here x_m is the right most sample point. When the right leaf is positioned at x_{i+1}, it does not shield x_i. Note that according to these definitions, when both leaves are positioned

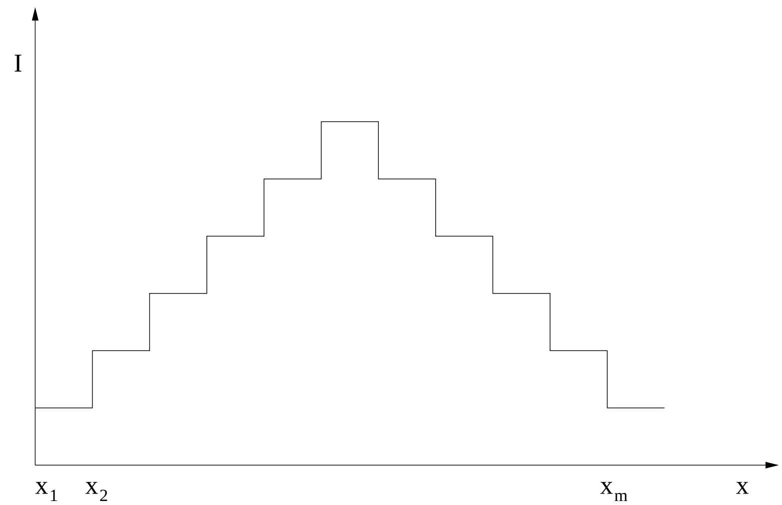

Figure 2. Profile generated by the optimizer

at the same point, all sample points are shielded. In this section we define the field splitting model and problem under study.

Let I be the desired intensity profile for a single leaf pair. The problem of delivering the exact profile I using a single field has been extensively studied. Ma et. al. (1998) provide an $O(m)$ algorithm for the problem such that the therapy time of the solution is minimized, where m is the number of sample points. Kamath et. al. (2003) also describe the algorithm (Algorithm SINGLEPAIR) and give an alternate proof that it obtains a plan (I_l, I_r) with optimal therapy time for I. The optimal therapy time for I is given by Lemma 1.

Lemma 1 *Let $inc1, inc2, \ldots, incq$ be the indices of the points at which $I(x_i)$ increases, i.e., $I(x_{inci}) > I(x_{inci-1})$. The therapy time for the plan (I_l, I_r) generated by Algorithm SINGLEPAIR is $\sum_{i=1}^{q}[I(x_{inci}) - I(x_{inci-1})]$, where $I(x_{inc1-1}) = 0$.*

Algorithm SINGLEPAIR can be directly used to obtain plans when I is deliverable using a single field. Let l be the least index such that $I(x_l) > 0$ and let g be the greatest index such that $I(x_g) > 0$. We will assume without loss of generality that $l = 1$. So the width of the profile is g sample points, where g can vary for different profiles. Assuming that the maximum allowable field width is w sample points, I is deliverable using one field if $g \leq w$; I requires at least two fields for $g > w$; I requires at least three fields for $g > 2w$. The case where $g > 3w$ is not studied as it never arises in clinical cases. The objective of field splitting is to split a profile so that each of the resulting profiles is deliverable using a single field. Further, it is desirable that the total therapy time is minimized, i.e., the sum of optimal therapy times of the resulting profiles is minimized. The problem of splitting the profile I of a single leaf pair into 2 profiles (without feathering) each of which is deliverable using one field, such that the sum of their optimal therapy times is minimized is the $S2$ (single pair 2 field split) problem. The sum of the optimal therapy times of the two resulting profiles is denoted by $S2(I)$. $S3$ and $S3(I)$ are defined similarly for splits into 3 profiles. Kamath et. al. (2004a) provide solutions for $S2$ and $S3$. They also observe that the problem $S1$ is trivial, since the input profile need not be split and is to be delivered using a single field. Note that $S1(I)$ is the optimal therapy time for delivering the profile I in a single field. From Lemma 1 and the fact that the plan generated using Algorithm SINGLEPAIR is optimal in therapy time, $S1(I) = \sum_{i=1}^{q}[I(x_{inci}) - I(x_{inci-1})]$. Let $dec1, dec2, \ldots, decr$ be the indices of the points at which $I(x_i)$ decreases, i.e., $I(x_{deci}) < I(x_{deci-1})$. Clearly, $\sum_{i=1}^{q}[I(x_{inci}) - I(x_{inci-1})] = -\sum_{i=1}^{r}[I(x_{deci}) - I(x_{deci-1})]$. Let $\delta_i = I(x_i) - I(x_{i-1})$, where $I(x_0) = 0$ and $I(x_{m+1}) = 0$.

Lemma 2 $S1(I) = \sum_{i=1}^{q} \delta_{inci} = -\sum_{i=1}^{r} \delta_{deci}$.

We will show that using the generalized field split model, it is possible to reduce the optimal therapy time for single leaf pair profiles as compared to the models of Kamath et. al. (2004a). Let $S2G$ (single pair 2 field generalized split) be the problem of splitting the profile I of a single leaf pair into 2 profiles in the generalized model, each of which is deliverable using one field such that the sum of their optimal therapy times is minimized. The definitions of $S3G$, $S2G(I)$ and $S3G(I)$ are obvious extensions of the above definitions under the generalized model. Note that $S1G$ is the same as $S1$.

2.2. Multiple leaf pair profiles

The input intensity matrix (say I) for the leaf sequencing problem is obtained using the inverse planning technique. The matrix I consists of n rows and m columns. Each row of the matrix specifies the number of monitor units (MUs) that need to be delivered using one leaf pair. Denote the rows of I by $I_1, I_2, \ldots, I_n$. For the case where I is deliverable using one field, the leaf sequencing problem has been well studied in the past. The algorithm that generates optimal therapy time schedules for multiple leaf pairs (Algorithm MULTIPAIR, Kamath et. al. (2003)) applies algorithm SINGLEPAIR independently to each row I_i of I. Without loss of generality assume that the least column index containing a non zero element in I is 1 and the largest column index containing a non zero element in I is g. If $g > w$, the profile will need to be split. Kamath et. al. (2004a) define problems $M1$, $M2$ and $M3$ for multiple leaf pairs as being analogous to $S1$, $S2$ and $S3$ for single leaf pair. The optimal therapy times $M1(I)$, $M2(I)$ and $M3(I)$ are also defined similarly. They also provide solutions to problems $M2$ and $M3$ (note $M1$ is solved using Algorithm MULTIPAIR, Kamath et. al. (2003)). These definitions are extended for generalized field splitting ($M1G$, $M2G$, $M3G$ and $M1G(I)$, $M2G(I)$, $M3G(I)$).

3. Optimal generalized split for single leaf pair

In this section we present algorithms for problems $S2G$ and $S3G$. In both cases the field splits will not be explicitly determined. Rather, leaf movement trajectories will be obtained for the split fields and the fields will be deduced from these trajectories.

3.1. Splitting a profile into two

Consider a fluence profile I (Fig. 3a). (I_l, I_r) is its optimal plan generated using Algorithm SINGLEPAIR. Suppose that $w < g \leq 2w$, so that the field needs to be split into two. Call the fields resulting from a split as the left and the right fields, labeling them to denote their respective positions. It is possible that the left field stretches as far to the right and including sample point x_w and that the right field stretches as far to the left and including x_{g-w+1}. Examine the unidirectional leaf trajectories in the plan (I_l, I_r) (Fig. 3b). Clearly, the plan cannot be delivered as such because of the field width constraint. Our strategy is to follow the plan (I_l, I_r) to the maximum extent possible while delivering it using two fields. There are two cases. In the first case, $I_l(x_{g-w}) \leq I_r(x_{w+1})$ (as is the case in Fig. 3b), the left field can be treated using the plan (I_l, I_r) for $I_l(x_{g-w})$ MUs. At the end of this time, the left leaf will be at $x_i \leq x_{g-w}$ (and can immediately move to x_{g-w+1}) and the right leaf will immediately be positioned in the range $[x_{g-w+1}, x_{w+1}]$ (since $I_r(x_{w+1}) \geq I_l(x_{g-w})$). Since both resulting leaf positions are within the range permissible for the right field ($[x_{g-w+1}, x_g]$), we stop treatment using the plan (I_l, I_r) in the left field, move to the right field and continue the treatment in the right field. No MU increase is needed due to field splitting.

In the second case, $I_l(x_{g-w}) > I_r(x_{w+1})$. Fig. 3d shows a profile I and Fig. 3e shows the plan (I_l, I_r) for which this is the case. The left field can be treated using the plan (I_l, I_r) for $I_l(x_{g-w})$ MUs. The left leaf can move to $x \geq x_{g-w+1}$ at this time and the remainder of the plan can be delivered using the right field as in the first case. However, the right leaf will have to cross the right end of the left field (x_{w+1}) when $I_r(x_{w+1}) < I_l(x_{g-w})$ MUs have been delivered in the plan (I_l, I_r). Since this is not possible, the right leaf is stopped at the point x_{w+1} till $I_l(x_{g-w})$ MUs are delivered in the left field. As a result of this, the right leaf profile will be raised by $I_l(x_{g-w}) - I_r(x_{w+1})$ MUs for $x \geq x_{w+1}$. To maintain constant difference between the profiles, the left leaf profile is also raised by $I_l(x_{g-w}) - I_r(x_{w+1})$ MUs for $x \geq x_{w+1}$. Call the modified plan (I'_l, I'_r). When $I_l(x_{g-w})$ MUs are delivered, the left leaf can move to the right field and the remainder of the plan (I'_l, I'_r) is delivered using the right field. The plan (I'_l, I'_r), has an increase in total therapy time by $I_l(x_{g-w}) - I_r(x_{w+1})$ compared to (I_l, I_r). In Fig. 3e, the horizontal dotted line at $I_l(x_{g-w})$ corresponds to the time at which the transition is made from the left to the right field. Fig. 3f shows the fluence profiles delivered in the left and right fields as a result of this split. Algorithm $S2G$ summarizes the general method.

Algorithm $S2G$

(1) Find plan (I_l, I_r) for I using Algorithm SINGLEPAIR, ignoring the field width constraints.

(2) If $I_l(x_{g-w}) > I_r(x_{w+1})$, raise the left and right leaf profiles by $I_l(x_{g-w}) - I_r(x_{w+1})$ for $x > x_w$. Otherwise, do not modify the plan. Call the resulting plan (I'_l, I'_r).

(3) Treat the left field using the plan (I'_l, I'_r) for $I'_l(x_{g-w})$ MUs. Then switch to the right field and continue treatment with (I'_l, I'_r).

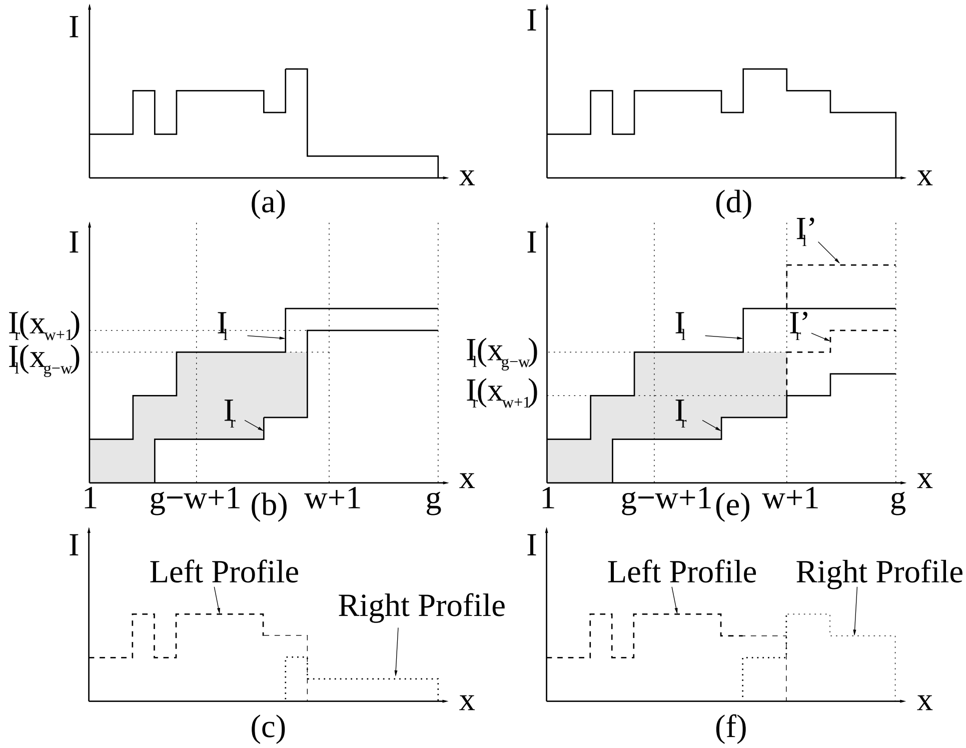

Figure 3. (a) A single pair profile. (b) Plan (I_l, I_r), for profile I of (a), is obtained using Algorithm SINGLEPAIR and is constructed without taking field width constraints into account. It can be delivered for $I_l(x_{g-w})$ MUs in the left field (shaded) and the remainder in the right field. (c) The left and right profiles resulting from the split generated by Algorithm $S2G$. This split is delivered in optimal time using the plan (I_l, I_r). (d) A single pair profile. (e) Plan (I_l, I_r), for profile I of (d), is obtained using Algorithm SINGLEPAIR and is constructed without taking field width constraints into account. (I'_l, I'_r) is the modified plan obtained using Algorithm $S2G$ and can be delivered for $I_l(x_{g-w})$ MUs in the left field (shaded) and the remainder in the right field. (f) The left and right profiles resulting from the split generated by Algorithm $S2G$. This split is delivered in optimal time using the plan (I'_l, I'_r).

From the above discussion, we make the following observation.

Lemma 3 *The optimal total therapy time of the split generated by Algorithm $S2G$ is $S1(I) + \max\{0, I_l(x_{g-w}) - I_r(x_{w+1})\}$, where $S1(I)$ is found by ignoring the field width constraints.*

Below we show that Algorithm $S2G$ generates generalized field splits that are optimal in total therapy time.

Lemma 4 *The following is true of all treatment plans delivered using one or more fields:*

(a) If $I(x_{i-1}) > I(x_i)$, the right leaf must be positioned at x_i for at least $I(x_{i-1}) - I(x_i) = -\delta_i$ MUs in every plan for I.

(b) If $I(x_{i-1}) < I(x_i)$, the left leaf must be positioned at x_i for at least $I(x_i) - I(x_{i-1}) = \delta_i$ MUs in every plan for I.

Proof: (a) It is not possible to expose $I(x_{i-1})$ and shield $I(x_i)$ at the same time, except by positioning the right leaf at x_{i+1}. This must be done for at least $-\delta_i$ MUs, when $I(x_{i-1}) > I(x_i)$.

(b) Similar to that of (a).

■

Lemma 5 *$S2G(I) \geq S1(I) + \max\{0, I_l(x_{g-w}) - I_r(x_{w+1})\}$, where $S1(I)$ is found by ignoring the field width constraints.*

Proof: There are two cases.

Case 1: $I_l(x_{g-w}) - I_r(x_{w+1}) \leq 0$.
In this case $\max\{0, I_l(x_{g-w}) - I_r(x_{w+1})\} = 0$. Every treatment plan that can be delivered using two fields can be delivered using one field by ignoring the field width constraints. Therefore, $S2G(I) \geq S1(I)$.

Case 2: $I_l(x_{g-w}) - I_r(x_{w+1}) > 0$.
Let L and R, respectively, denote the left and right profiles resulting from a generalized split. Let $\delta 1_i = L(x_i) - L(x_{i-1})$ and let $\delta 2_i = R(x_i) - R(x_{i-1})$. From Lemma 2, the optimal total therapy time of the split is $S1(L) + S1(R) = -(\sum_{\delta 1_i<0} \delta 1_i + \sum_{\delta 2_i<0} \delta 2_i)$. Due to the field width constraint, the points $x_1, x_2, \ldots, x_{g-w}$, can only be exposed in L. So, $L(x_i) = I(x_i)$, $1 \leq i \leq g-w$, and therefore, $\delta 1_i = \delta_i$, $1 \leq i \leq g-w$. Similarly, $\delta 2_i = \delta_i$, $w+2 \leq i \leq g$. Let the number of MUs for which the left leaf stops at point x_i in optimal plans for L and R, respectively, be $\hat{L}_l(x_i)$ and $\hat{R}_l(x_i)$ and the number of MUs for which the right leaf stops at point x_i in optimal plans for L and R, respectively, be $\hat{L}_r(x_i)$ and $\hat{R}_r(x_i)$.
Since the points $1, 2, \ldots, g-w$, can only be exposed in L, $S1(L) \geq \sum_{1\leq i\leq g-w} \hat{L}_l(x_i) \geq \sum_{\delta 1_i>0, 1\leq i\leq g-w} \delta 1_i$ (Lemma 4) $= \sum_{\delta_i>0,1\leq i\leq g-w} \delta_i = \sum_{\delta_i>0,1\leq i\leq g-w} \delta_i - \sum_{\delta_i<0,i\leq w+1} \delta_i + \sum_{\delta_i<0,i\leq w+1} \delta_i = I_l(x_{g-w}) - I_r(x_{w+1}) - \sum_{\delta_i<0,i\leq w+1} \delta_i$. Similarly, from Lemma 2 and the fact that the points $w+1, w+2, \ldots, g$, can only be exposed in R, $S1(R) \geq \sum_{i>w+1} \hat{R}_r(x_i) \geq \sum_{\delta 2_i<0,i>w+1}(-\delta 2_i)$ (Lemma 4) $= \sum_{\delta_i<0,i>w+1}(-\delta_i)$. Adding, $S1(L) + S1(R) \geq I_l(x_{g-w}) - I_r(x_{w+1}) + \sum_{\delta_i<0,i\leq w+1} (-\delta_i) + \sum_{\delta_i<0,i>w+1} (-\delta_i) = I_l(x_{g-w}) - I_r(x_{w+1}) + S1(I)$. Therefore, $S2G(I) \geq S1(I) + I_l(x_{g-w}) - I_r(x_{w+1})$.

■

Theorem 1 *Algorithm S2G generates generalized field splits that are optimal in total therapy time.*

Proof: Follows from Lemmas 3 and 5. ■

Example 1 *Consider the profile of Figure 4. It has $g = 5$ and $w = 3$. It is seen that the profile can be delivered using 20 MUs with feathering, whereas it will require at least 30 MUs using any split that does not use feathering. For the profile of Figure 5, however, splits with and without the feathering option will require 20 MUs. So allowing feathering will not reduce total MUs.*

10	10	20	10	10

=

10	10	10		

+

		10	10	10

Figure 4. Feathering reduces total MUs ($g = 5$ and $w = 3$)

10	10	0	10	10

Figure 5. Feathering cannot reduce total MUs($g = 5$ and $w = 3$)

3.2. Splitting a profile into three

Consider the problem of splitting a single leaf pair profile I (Figure 6) into three fields. In the discussion below, the indices are calculated assuming that $2w < g \leq 3w$. However, the method can easily be used for $g \leq 2w$ with some modifications. The method we describe is an extension of the method used for splits into two. Denote the three fields resulting from the split as left, middle and right fields. As in case of the split into two, the left field can extend over the points $x_1, x_2, \ldots, x_w$, and the right field can extend over $x_{g-w+1}, x_{g-w+2}, \ldots, x_g$. For the position of the middle profile, there are several possibilities within a range. We examine each one of these and select the best.

The left most sample point that can be exposed in the middle field is x_{g-2w+1}. When x_{g-2w+1} is included in the middle field, the middle field can extend over $x_{g-2w+1}, x_{g-2w+2}, \ldots, x_{g-w}$. In this case, the w points $x_{g-w+1}, x_{g-w+2}, \ldots, x_g$, are treated by the right profile and so the middle profile cannot be any further to the left, without leaving at least the point x_{g-w} not treated. Shifting the left boundary of the middle field one sample point to the right, the middle profile can extend over $x_{g-2w+2}, x_{g-2w+3}, \ldots, x_{g-w+1}$. Proceeding in this manner, it is clear that the left most position included in the middle profile has to be one of the following: $x_{g-w+1}, x_{g-w+2}, \ldots, x_{w+1}$. Algorithm $S3G$ determines the optimal total therapy time separately assuming each one of these is necessarily the left most in the middle profile. The global optimum will be the least among these times. Next we explain how we find the optimal therapy time when a point x_j is the left most point in the middle profile.

Suppose that the left most point of the middle profile is x_j, i.e., x_j is necessarily part of the middle profile. First construct the trajectories for the left and right leaves assuming I is treated using one field (Figure 7). Next, examine this plan (I_l, I_r) and determine whether $I_l(x_{j-1}) < I_r(x_{w+1})$. If this is the case, start treatment of the left field with both leaves at the extreme left and move to the middle field when the left leaf reaches x_j in the left to right sweep. Otherwise, stop the right leaf at x_{w+1} till the left leaf reaches x_j. As a result of this action, the right leaf profile gets raised by an amount $I_l(x_{j-1}) - I_r(x_{w+1})$ for $x \geq x_{w+1}$. Raise the left leaf profile by the same amount for $x \geq x_{w+1}$ to account for the difference between the profiles. Call this modified plan (I'_l, I'_r). In the plan (I'_l, I'_r) see if $I'_l(x_{g-w}) \leq I'_r(x_{j+w})$. If this is the case, stop treating the middle field and move to the right field when the left leaf reaches x_{g-w+1} during the sweep. Otherwise, stop the right leaf at sample point x_{j+w} till the left leaf reaches x_{g-w+1}. The right leaf profile I'_r gets raised by an amount $I'_l(x_{g-w}) - I'_r(x_{j+w})$ for $x \geq x_{j+w}$. The left leaf profile I'_l is also raised by $I'_l(x_{g-w}) - I'_r(x_{j+w-1})$ for $x \geq x_{j+w}$. The resulting plan is denoted by (I''_l, I''_r). We show that the split generated as a result of this method is optimal in total therapy time for all cases where the middle profile has x_j as its left most point. The split generated for the profile of Figure 6 with the x_j as in Figure 7 is shown in Figure 8.

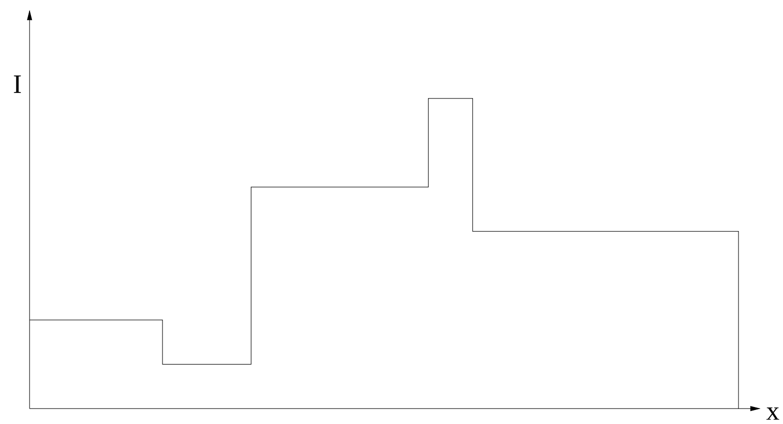

Figure 6. A single pair profile

Algorithm *S3G*

(1) Find plan (I_l, I_r) for I using Algorithm SINGLEPAIR, ignoring the field width constraints.

(2) For $j = g - 2w + 1$ to $w + 1$ do steps 3 through 5.

(3) If $I_l(x_{j-1}) > I_r(x_{w+1})$, raise the left and right leaf profiles by $I_l(x_{j-1}) - I_r(x_{w+1})$ for $x > x_w$. Otherwise, do not modify the plan. Call the resulting plan (I'_l, I'_r).

(4) If $I'_l(x_{g-w}) > I'_r(x_{j+w})$, raise the left and right leaf profiles by $I'_l(x_{g-w}) - I'_r(x_{j+w})$ for $x \geq x_{j+w}$. Otherwise, do not modify the plan. Call the resulting plan (I''_l, I''_r).

(5) If $TT(I''_l, I''_r)$ is the least among all j so far, set $j' = j$.

(6) Treat the profile using the plan (I''_l, I''_r) obtained using $j = j'$. Treat the left field for the first $I''_l(x_{j-1})$ MUs; then move to the middle field; finally, switch to the right field when $I''_l(x_{g-w})$ MUs have been delivered.

Example 2 *Consider the profile of Figure 9. It has $g = 7$ and $w = 3$. It is seen that the profile can be delivered using 30 MUs with feathering, whereas it will require at least 50*

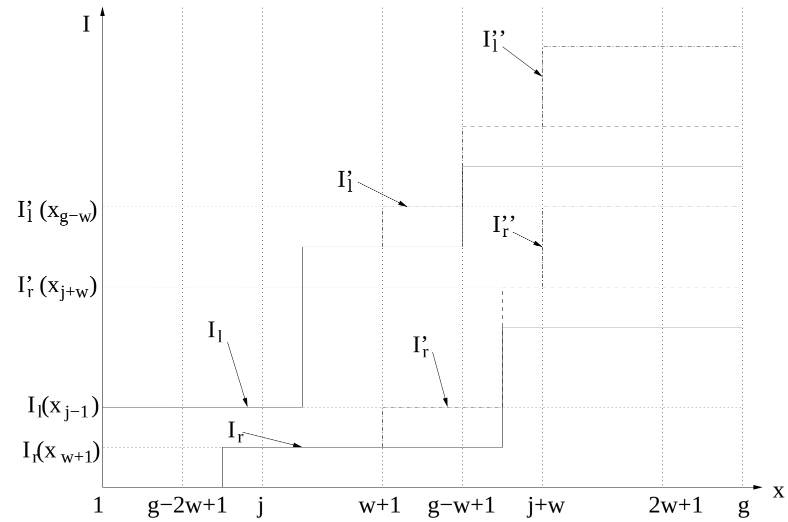

Figure 7. Plan (I_l, I_r), for profile I of Figure 6, is obtained using Algorithm SINGLEPAIR and is constructed without taking field width constraints into account. (I_l'', I_r'') is the modified plan obtained during an iteration of Algorithm $S3G$ with x_j as shown.

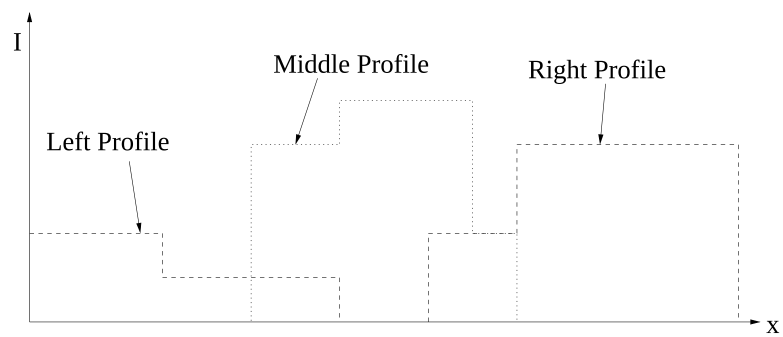

Figure 8. The left and right profiles resulting from the split generated during the iteration of Algorithm $S3G$ shown in Figure 7. This split is delivered in optimal time using the plan (I_l'', I_r'').

MUs using any split that does not use feathering. For the profile of Figure 5, however, splits with and without the feathering option will require 30 MUs. So allowing feathering will not reduce total MUs.

4. Optimal generalized split for multiple leaf pairs

4.1. Splitting a profile into two

Let $\{(I_{1l}, I_{1r}), (I_{2l}, I_{2r}), \ldots, (I_{nl}, I_{nr})\}$ be the schedule generated by Algorithm MULTIPAIR for delivering the profile I. The points $x_1, x_2, \ldots, x_w$, need to be completely treated in the left field. Let k be an index of a leaf pair for which the left leaf is slowest in reaching x_{g-w+1} during the left to right sweep, i.e., $I_{kl}(x_{g-w}) = \max_{1 \le i \le n}\{I_{il}(x_{g-w})\}$. For each leaf pair i, compare $I_{ir}(x_{w+1})$ with

10	10	20	10	20	10	10
			=			
10	10	10				
			+			
		10	10	10		
			+			
				10	10	10

Figure 9. Feathering reduces total MUs ($g = 7$ and $w = 3$)

10	10	0	10	0	10	10

Figure 10. Feathering cannot reduce total MUs ($g = 7$ and $w = 3$)

$I_{kl}(x_{g-w})$. If $I_{kl}(x_{g-w}) \leq I_{ir}(x_{w+1})$, then the profile of leaf pair i remains unaltered. If $I_{kl}(x_{g-w}) > I_{ir}(x_{w+1})$, then the right leaf of pair i will have to stop at x_{w+1} till the left leaf of pair k arrives at x_{g-w+1}. As a result, the left and right leaf profiles of pair i get raised by $I_{kl}(x_{g-w}) - I_{ir}(x_{w+1})$ for $x > x_w$. Call the resulting schedule $\{(I'_{1l}, I'_{1r}), (I'_{2l}, I'_{2r}), \ldots, (I'_{nl}, I'_{nr})\}$. When the left leaf of pair k reaches x_{g-w+1} in this schedule, stop treatment of the left field and move to the right field. The remainder of the schedule is delivered in the right field. The method is described in Algorithm *M2G*. The optimal total therapy time for the split generated by Algorithm *M2G* is $max_j\{S1(I_j) + \max\{0, I_{kl}(x_{g-w}) - I_{jr}(x_{w+1})\}\}$.

Algorithm *M2G*

(1) Find the schedule $\{(I_{1l}, I_{1r}), (I_{2l}, I_{2r}), \ldots, (I_{nl}, I_{nr})\}$ for I using Algorithm MULTIPAIR, ignoring the field width constraints.

(2) Let $I_{kl}(x_{g-w}) = \max_{1 \leq i \leq n}\{I_{il}(x_{g-w})\}$.

(3) For each leaf pair i do step 4.

(4) If $I_{kl}(x_{g-w}) > I_{ir}(x_{w+1})$, raise the left and right profiles of pair i by $I_{kl}(x_{g-w}) - I_{ir}(x_{w+1})$ for $x > x_w$. Otherwise, do not modify the plan for pair i.

(5) Call the resulting schedule $\{(I'_{1l}, I'_{1r}), (I'_{2l}, I'_{2r}), \ldots, (I'_{nl}, I'_{nr})\}$.

(6) Treat the left field using the schedule $\{(I'_{1l}, I'_{1r}), (I'_{2l}, I'_{2r}), \ldots, (I'_{nl}, I'_{nr})\}$ for $I'_{kl}(x_{g-w})$ MUs. Then switch to the right field and continue treatment with this schedule.

Lemma 6 *$M2G(I) \geq max_j\{S1(I_j) + \max\{0, I_{kl}(x_{g-w}) - I_{jr}(x_{w+1})\}\}$, where $S1(I_j)$ is found by ignoring the field width constraints and k is as in Algorithm M2G.*

Proof: For each row j, we show that $M2G(I) \geq S1(I_j) + \max\{0, I_{kl}(x_{g-w}) - I_{jr}(x_{w+1})\}$. It follows that $M2G(I) \geq max_j\{S1(I_j) + \max\{0, I_{kl}(x_{g-w}) - I_{jr}(x_{w+1})\}\}$.

Case 1: $I_{kl}(x_{g-w}) - I_{jr}(x_{w+1}) \leq 0$.
In this case $\max\{0, I_{kl}(x_{g-w}) - I_{jr}(x_{w+1})\} = 0$. It is easy to see that, $M2G(I) \geq S1(I_j)$.

Case 2: $I_{kl}(x_{g-w}) - I_{jr}(x_{w+1}) > 0$.
Let $\delta ji = I_j(x_i) - I_j(x_{i-1})$. Let L and R, respectively, denote the left and right profiles resulting from a generalized split. Let L_j denote the jth row of L and let R_j denote the jth row of R. Let $\delta 1_{ji} = L_j(x_i) - L_j(x_{i-1})$ and let $\delta 2_{ji} = R_j(x_i) - R_j(x_{i-1})$. The optimal total therapy time of the split is $M1(L) + M1(R)$. Due to the field width constraint, the points $x_1, x_2, \ldots, x_{g-w}$, can only be exposed in L. So, $L_j(x_i) = I_j(x_i)$, $1 \leq i \leq g-w$, and therefore, $\delta 1_{ji} = \delta_{ji}$, $1 \leq i \leq g-w$. Similarly, $\delta 2_{ji} = \delta_{ji}$, $w+2 \leq i \leq g$. Let the number of MUs for which the left leaf of leaf pair j stops at point x_i in optimal schedules for L and R, respectively, be $\hat{L}_{jl}(x_i)$ and $\hat{R}_{jl}(x_i)$ and the number of MUs for which the right leaf stops at point x_i in optimal schedules for L and R, respectively, be $\hat{L}_{jr}(x_i)$ and $\hat{R}_{jr}(x_i)$.
Since the points $1, 2, \ldots, g-w$, can only be exposed in L, for each j, $M1(L) \geq \sum_{1 \leq i \leq g-w} \hat{L}_{kl}(x_i) \geq \sum_{\delta 1_{ki} > 0, 1 \leq i \leq g-w} \delta 1_{ki}$ (Lemma 4) $= \sum_{\delta_{ki} > 0, 1 \leq i \leq g-w} \delta_{ki} - \sum_{\delta_{ji} < 0, i \leq w+1} \delta_{ji} + \sum_{\delta_{ji} < 0, i \leq w+1} \delta_{ji} = I_{kl}(x_{g-w}) - I_{jr}(x_{w+1}) + \sum_{\delta_i < 0, i \leq w+1}(-\delta_{ji})$. Similarly, from Lemma 2 and the fact that the points $w+1, w+2, \ldots, g$, can only be exposed in R_j, for each j, $M1(R) \geq S1(R_j) \geq \sum_{i > w+1} \hat{R}_{jr}(x_i) \geq \sum_{\delta 2_{ji} < 0, i > w+1}(-\delta 2_{ji})$ (Lemma 4) $= \sum_{\delta_{ji} < 0, i > w+1}(-\delta_{ji})$. Adding, $M1(L) + M1(R) \geq \sum_{\delta_{ji} < 0, i > w+1}(-\delta_{ji}) + \sum_{\delta_{ji} < 0, i \leq w+1}(-\delta_{ji}) + I_{kl}(x_{g-w}) - I_{jr}(x_{w+1}) = S1(I_j) + I_{kl}(x_{g-w}) - I_{jr}(x_{w+1})$. Therefore, $M2G(I) \geq S1(I_j) + I_{kl}(x_{g-w}) - I_{jr}(x_{w+1})$.

■

4.2. Splitting a profile into three

Again, we compute the indices assuming that $2w < g \leq 3w$. In general the method can also be used for $g \leq 2w$. As in the case of single leaf pair, the left most position included in the middle profile has to be one of the following: $x_{g-2w+1}, x_{g-2w+2}, \ldots x_{w+1}$. The optimal total therapy time is separately found assuming each one of these points is necessarily the left most in the middle profile. The optimal total therapy time will be the least among these.

Assume that the left most point of the middle profile is x_j, $g - 2w + 1 \leq j \leq w + 1$. The points $x_1, x_2, \ldots, x_{j-1}$, need to be completely treated in the left field. Let k be an index of a left pair for which the left leaf is slowest in reaching x_j during the left to right sweep, i.e., $I_{kl}(x_{j-1}) = \max_{1 \leq i \leq n}\{I_{il}(x_{j-1})\}$. For each leaf pair i, compare $I_{ir}(x_{w+1})$ with $I_{kl}(x_{j-1})$. If $I_{kl}(x_{j-1}) \leq I_{ir}(x_{w+1})$, then the profile of leaf pair i remains unaltered. On the other hand, if $I_{kl}(x_{j-1}) > I_{ir}(x_{w+1})$, then the right leaf of pair i will have to stop at x_{w+1} till the left leaf of pair k arrives at x_j. As a result, the left and right leaf profiles of pair i get raised by $I_{kl}(x_{j-1}) - I_{ir}(x_{w+1})$ for $x > x_w$. Call this modified schedule $\{(I'_{1l}, I'_{1r}), (I'_{2l}, I'_{2r}), \ldots, (I'_{nl}, I'_{nr})\}$. Move to treatment of the middle field when the left leaf of pair k arrives at x_j in this schedule. Note that the middle field can extend up to x_{j+w-1} on the right and that the left most point of the right field is x_{g-w+1}. Modify the schedule $\{(I'_{1l}, I'_{1r}), (I'_{2l}, I'_{2r}), \ldots, (I'_{nl}, I'_{nr})\}$ as before so that the treatment of the right field begins when the slowest left leaf reaches x_{g-w+1}. The final schedule is $\{(I''_{1l}, I''_{1r}), (I''_{2l}, I''_{2r}), \ldots, (I''_{nl}, I''_{nr})\}$. The split generated as a result of this method is optimal in total therapy time for all cases where the middle profile has x_j as its left most point. Algorithm $M3G$ varies j over $g-2w+1, g-2w+2, \ldots, w+1$, and finds the best split.

Algorithm $M3G$

(1) Find the schedule $\{(I_{1l}, I_{1r}), (I_{2l}, I_{2r}), \ldots, (I_{nl}, I_{nr})\}$ for I using Algorithm MULTIPAIR, ignoring the field width constraints.

(2) For $j = g - 2w + 1$ to $w + 1$ do steps 3 through 10.

(3) Let $I_{kl}(x_{j-1}) = \max_{1 \le i \le n}\{I_{il}(x_{j-1})\}$.

(4) For each leaf pair i do step 5.

(5) If $I_{kl}(x_{j-1}) > I_{ir}(x_{w+1})$, raise the left and right profiles of pair i by $I_{kl}(x_{j-1}) - I_{ir}(x_{w+1})$ for $x > x_w$. Otherwise, do not modify the plan for pair i.

(6) Call the resulting schedule $\{(I'_{1l}, I'_{1r}), (I'_{2l}, I'_{2r}), \ldots, (I'_{nl}, I'_{nr})\}$.

(7) Let $I'_{yl}(x_{g-w}) = \max_{1 \le i \le n}\{I'_{il}(x_{g-w})\}$. For each leaf pair i do step 8.

(8) If $I'_{yl}(x_{g-w}) > I'_{ir}(x_{j+w})$, raise the left and right profiles of pair i by $I'_{yl}(x_{g-w}) - I'_{ir}(x_{j+w})$, for $x > x_{j+w}$, in the schedule $\{(I'_{1l}, I'_{1r}), (I'_{2l}, I'_{2r}), \ldots, (I'_{nl}, I'_{nr})\}$. Otherwise, do not modify the plan for pair i.

(9) Call the resulting schedule $\{(I''_{1l}, I''_{1r}), (I''_{2l}, I''_{2r}), \ldots, (I''_{nl}, I''_{nr})\}$.

(10) If $TT\{(I''_{1l}, I''_{1r}), (I''_{2l}, I''_{2r}), \ldots, (I''_{nl}, I''_{nr})\}$ is the least among all j so far, $j = j'$.

(11) Treat the left field using the schedule $\{(I''_{1l}, I''_{1r}), (I''_{2l}, I''_{2r}), \ldots, (I''_{nl}, I''_{nr})\}$, which is obtained using $j = j'$ for $I''_{kl}(x_{j-1})$ MUs. Here, k and y are as obtained for $j = j'$. Then switch to the middle field and continue treatment with this schedule till $I'_{yl}(x_{g-w})$ MUs. Finally, move to the right field and complete the treatment.

5. Results

The performance of the Algorithms $M2G$ was tested using 30 clinical fluence matrices, each of which exceeded the maximum allowable field width w. The fluence matrices were generated with a commercial inverse treatment planning system (CORVUS v5.0). The percent decrease in MUs as a result of optimal field splitting over the split generated by the commercial system were computed (Table 1). For the computation of total MUs for all the splits generated, the each split subfield was leaf sequenced using Algorithm $SINGLEPAIR$ (Kamath et. al. 2003), which sequences a subfield in minimum number of MUs. The average decrease in MUs is 18.8% for the 30 fluence matrices. The maximum decrease in MUs is 45%. Also shown are the number of MUs for the split generated by Algorithm $M2$ (Kamath et. al. 2004a). All the subfields overlap to various

Matrix (I)	Width	$C(I)$	$M2(I)$	$M2G(I)$	% MU decrease
1	18	280	240	200	28.6
2	26	560	440	400	28.6
3	16	300	260	220	26.7
4	15	280	280	280	0
5	17	380	280	250	34.2
6	16	400	300	280	30.0
7	16	350	350	320	8.6
8	16	340	310	300	11.8
9	16	390	310	240	38.5
10	16	350	320	320	8.6
11	25	360	360	340	5.5
12	22	400	360	340	15
13	20	320	320	300	6.3
14	22	320	320	280	12.5
15	24	540	480	440	18.5
16	24	540	500	480	11.1
17	24	460	420	380	17.4
18	24	520	520	520	0
19	24	560	520	480	14.3
20	15	310	310	290	6.5
21	16	400	300	260	35.0
22	16	440	350	280	36.4
23	16	320	320	300	6.3
24	16	400	300	220	45.0
25	16	330	290	290	12.1
26	15	320	280	220	31.3
27	15	300	300	300	0
28	16	280	280	280	0
29	16	360	310	230	36.1
30	17	420	360	260	38.1

Table 1. Total MUs for clinical cases

degrees, creating a natural feathering area which is clinically desirable. In summary, we have developed an algorithm that splits a large intensity-modulated field into subfields, such that the overall MU efficiency is maximized, for the most general case of field split that allows for subfield overlap.

6. Conclusion

Maximum leaf over-travel is a limitation on some MLCs, which necessitates a large field to be split into two or more adjacent abutting subfields. We have developed algorithms to split large intensity-modulated fields into two or three sub-fields allowing for arbitrary overlap of split fields. The overlap of subfields creates a natural feathering area that helps reduce the sensitivity to displacement at the field junction region due to uncertainties in setup and organ motion. Our algorithms result in field splits such that the MU efficiency for delivering the subfields is provably optimal. Application of our optimal field splitting algorithm to split a large field into two subfields on clinical data reduced total MUs by up to 45% compared to a commercial planning system.

References

Ahamad A, Stevens CW, Smythe WR, Liao Z, Vaporciyan AA, Rice D, Walsh G, Guerrero T, Chang J, Bell B, Komaki R, Forster KM 2003 Promising early local control of malignant pleural mesothelioma following postoperative intensity modulated radiotherapy (IMRT) to the chest *Cancer J.* **9** 476-84

Dogan N, Leybovich L B, Sethi A and Emami B 2003 Automatic feathering of split fields for step-and-shoot intensity modulated radiation therapy *Phys. Med. Biol.* **48** 1133-1140

Forster K M, Smythe W R, Starkschall G, Liao Z, Takanaka T, Kelly J F, Vaporciyan A, Ahamad A, Dong L, Salehpour M, Komaki R and Stevens C W 2003 Intensity-modulated radiotherapy following extrapleural pneumonectomy for the treatment of malignant mesothelioma: clinical implementation *Int. J. Radiat. Oncol. Biol. Phys.* **55** 606-16

Hong L, Kaled A, Chui C, LoSasso T, Hunt M, Spirou S, Yang J, Amols H, Ling C, Fuks Z and Leibel S 2002 IMRT of large fields: whole-abdomen irradiation *Int. J. Radiat. Oncol. Biol. Phys.* **54** 278-89

Kamath S, Sahni S, Li J, Palta J and Ranka S 2003 Leaf sequencing algorithms for segmented multileaf collimation *Phys. Med. Biol.* **48** 307-324

Kamath S, Sahni S, Palta J, Ranka S and Li J 2004 Optimal leaf sequencing with elimination of tongue-and-groove underdosage *Phys. Med. Biol.* **49** N7-N19

Kamath S, Sahni S, Ranka S, Li J and Palta J 2004a Optimal field splitting for large intensity-modulated fields *Med. Phys.* **31** 3314-23

Ma L, Boyer A, Xing L and Ma C-M 1998 An optimized leaf-setting algorithm for beam intensity modulation using dynamic multileaf collimators *Phys. Med. Biol.* **43** 1629-43

Mundt A J, Mell L K and Roeske J C 2003 Preliminary analysis of chronic gastrointestinal toxicity in gynecology patients treated with intensity-modulated whole pelvic radiation therapy *Int. J. Radiat. Oncol. Biol. Phys.* **56** 1354-60

Wu Q, Arnfield M, Tong S, Wu Y and Mohan R 2000 Dynamic splitting of large intensity-modulated fields *Phys. Med. Biol.* **45** 1731-40