Optimal Sequencing of Dynamic Multileaf Collimators

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Abstract. Dynamic multileaf collimator (DMLC)-intensity modulated radiation therapy (IMRT) is used to delivery intensity-modulated beams using an MLC, with the leaves in motion. DMLC-IMRT requires the conversion of a radiation fluence map into a leaf sequence file that controls the movement of the MLC while the beam is on. It is imperative that the fluence map delivered using the leaf sequence file be as close as possible to the fluence map generated by the dose optimization algorithm, while satisfying hardware constraints of the delivery system. Optimization of the leaf-sequencing algorithm has been the subject of several recent investigations. In this work, we present a systematic study of the optimization of leaf sequencing algorithms for dynamic multileaf collimator beam delivery and provide rigorous mathematical proofs of optimized leaf sequence settings in terms of monitor unit (MU) efficiency under most common leaf movement constraints that include leaf interdigitation constraint. Our analytical analysis shows that leaf sequencing based on unidirectional movement of the MLC leaves is as MU efficient as bi-directional movement of the MLC leaves.

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1. Introduction

The two most common methods of IMRT delivery with computer-controlled multileaf collimators (MLC) are the segmental multileaf collimator (SMLC) and dynamic multileaf collimator (DMLC). Treatment planning for both IMRT delivery techniques is usually done using the inverse planning method, where a set of optimized fluence maps are generated for a given patient's data and beam configuration. A separate software module is used to convert the optimized fluence maps into a set of leaf sequence files that control the movement of the MLC during delivery. The fundamental difference between these two delivery methods is that the leaf-sequencing algorithm defines the sub-field shapes for SMLC-IMRT and trajectories of opposing pairs of leaves for DMLC-IMRT. The purpose of the leaf sequencing algorithm is to faithfully reproduce the desired fluence map once the beam is delivered, taking into consideration any hardware and dosimetric characteristics of the delivery system. The other important goal of these algorithms is to deliver the intensity modulated pattern efficiently. Therefore, optimization of the leaf-sequencing algorithm has been the subject of numerous investigations (Bortfeld *et al* 1994a, Convery and Rosenbloom 1992, Dirkx *et al* 1998, Xia and Verhey 1998, Ma *et al* 1998, Langer *et al* 2001).

IMRT treatment delivery is not very efficient in terms of monitor units (MU). MU efficiency is defined as the ratio of dose delivered at a point in the patient with an IMRT field to the MU delivered for that field. Typical MU efficiencies of IMRT treatment plans are 3 to 10 times lower than open/wedge field-based conventional treatment plans. Therefore, total body dose due to increased leakage radiation reaching the patient in an IMRT treatment is a major concern (Intensity Modulated Radiation Therapy Collaborative Working Group 2001, Followill *et al* 1997). Low MU efficiency of IMRT delivery negatively impacts the room shielding design because of the increased workload (Intensity Modulated Radiation Therapy Collaborative Working Group 2001, Mutic *et al* 2001). The MU efficiency depends both on the degree of intensity modulation and the algorithm used to convert the intensity pattern into a leaf sequence for IMRT delivery. It is therefore important to design a leaf-sequencing algorithm that is optimal for MU efficiency to minimize total body dose to the patient.

Dynamic leaf sequencing algorithms with the leaves in motion during radiation delivery have been developed (Kallman *et al* 1988, Convery and Rosenbloom 1992, Spirou and Chui 1994, Bortfeld *et al* 1994b, Stein *et al* 1994). These DMLC leaf sequencing algorithms convert desired intensity distribution from an IMRT optimization engine into leaf trajectories, i.e., leaf motion sequences as a function of monitor units (MUs). Most of these studies did not consider any leaf movement constraints, with the exception of the maximum leaf speed constraint for dynamic delivery. Such leaf sequencing algorithms are applicable for certain types of MLC designs. For other types of MLC designs, notably the Siemens (Siemens Medical Systems Inc., Iselin, NJ) MLC design (Das *et al* 1998) and Elekta (Elekta Oncology Systems Inc., Norcross, GA) MLC design (Jordan and Williams 1994), other mechanical constraints need to be taken into consideration when designing the leaf settings for both dynamic and SMLC delivery. The minimum leaf separation constraint, for example, was recently incorporated into the design of leaf sequence (Convery and Webb 1998). Most published leaf sequencing algorithms have approximate empirical corrections to account for the various effects associated with the MLC characteristics, such as the rounded leaf tips, tongue-and-groove leaf design, leaf transmission, leaf scatter, head scatter and the finite size of the radiation source. Several recent publications (van Santvoort and Heijmen 1996, Webb *et al* 1997, Dirkx *et al* 1998, Ma *et al* 1999) have specifically addressed the issue of tongue-and-groove under dosage effects. This problem is entirely overcome by a method of leaf position synchronization due to van Santvoort and Heijmen (1996). However, their approach eliminates the tongue-and groove effect at the cost of total beam-on time. The general belief now is that this well published effect is really a non-issue in clinical practice due to patient motion (Deng *et al* 2001).

We have recently reported that leaf sequencing based on unidirectional movement of the leaves is as MU efficient as bi-directional movement of the MLC leaves for SMLC delivery (Kamath *et al* 2003). The DMLC delivery is different, the leaf positions change with respect to time, in terms of the MLC controller it is the change in position with respect to monitor units delivered that is important. The inputs required are the leaf positions at various control points, the fractional number of monitor units to be delivered at each control point, and the total number of monitor units to be delivered for that beam. In this work, we present a systematic study of the optimization of leaf sequencing algorithms for the dynamic beam delivery and provide rigorous proofs of optimized leaf sequence settings in terms of MU efficiency under various leaf movement constraints. Practical leaf movement constraints that are considered include the leaf interdigitation constraint. The question of whether bi-directional leaf movement will increase the MU efficiency when compared with unidirectional leaf movement only is also addressed.

2. Methods

2.1. Discrete Profile

The geometry and coordinate system used in this study are shown in Figure 1. We consider delivery of profiles that are piecewise continuous. Let $I(x)$ be the desired intensity profile. We first discretize the profile so that we obtain the values at sample points $x_0, x_1, x_2, \ldots, x_m$. We assume that the sample points are uniformly spaced and that $\Delta x = x_{i+1} - x_i, 0 \leq i < m$. $I(x)$ is assigned the value $I(x_i)$ for $x_i \leq x < x_{i+1}$, for each i. Now, $I(x_i)$ is our desired intensity profile. Figure 2 shows a piecewise continuous function and the corresponding discretized profile. The discretized profile is delivered either with the Segmental Multileaf Collimation (SMLC) method or with Dynamic Multileaf Collimation (DMLC). In this paper we study delivery with DMLC.

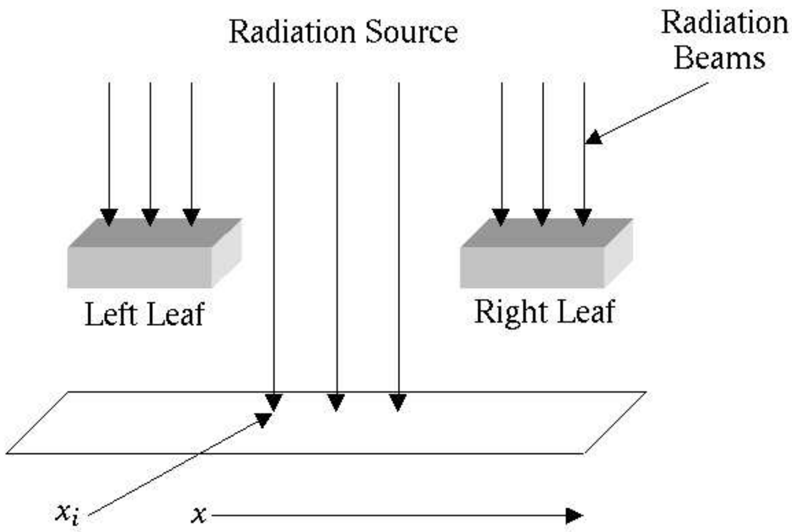

Figure 1. Geometry and coordinate system

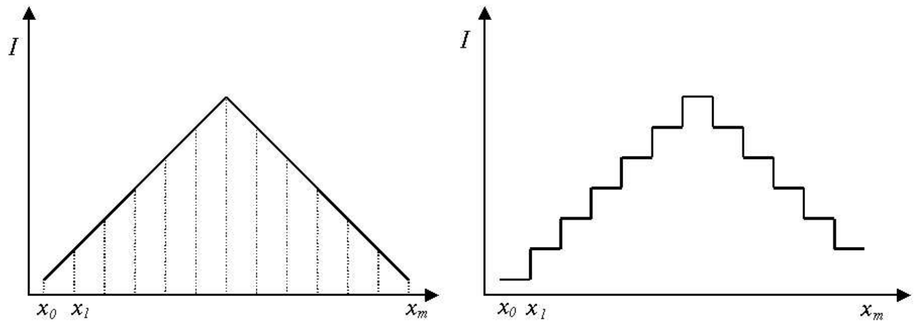

Figure 2. Discretization of profile

2.2. Movement of leaves

In our analysis we will assume that $I(x_0) > 0$ and $I(x_m) > 0$ and that when the beam delivery begins the leaves can be positioned anywhere. We also assume that the leaves can move with any velocity v, $0 \leq v \leq v_{max}$, where v_{max} is the maximum allowable velocity of the leaves and that the leaf acceleration can be infinite. Figure 3 illustrates the leaf trajectory during DMLC delivery. Let $I_l(x_i)$ and $I_r(x_i)$, respectively, denote the amount of Monitor Units (MUs) delivered when the left and right leaves leave position x_i. Consider the motion of the left leaf. The left leaf begins at x_0 and remains here until $I_l(x_0)$ MUs have been delivered. At this time the left leaf leaves x_0 and is moved to x_1, where it remains until $I_l(x_1)$ MUs have been delivered. The left leaf then moves to x_3 where it remains until $I_l(x_3)$ MUs have been delivered. At this time, the left leaf is moved to x_5, where it remains until $I_l(x_5)$ MUs have been delivered. Then it moves to x_6, where it remains until $I_l(x_6)$ MUs have been delivered. The final movement of the left leaf is to x_{10}. The left leaf arrives at x_{10} when I_{max} MUs have been delivered. At this time the machine is turned off. The total therapy time, $TT(I_l, I_r)$, is the time needed to deliver I_{max} MUs. The right leaf starts at x_0 and begins to move rightaway till it reaches x_2; leaves x_2 when $I_r(x_2)$ MUs have been delivered; leaves x_4 when $I_r(x_4)$ MUs have been delivered, and so on. Note that the machine is on throughout the treatment. All MUs that are delivered along a radiation beam along x_i before the left leaf passes x_i fall on it. Similarly, all MUs that are delivered along a radiation beam along x_i before the right leaf passes x_i, are blocked by the leaf. So the net amount of MUs delivered at a point is given by $I_l(x_i) - I_r(x_i)$, which must be the same as the desired profile $I(x_i)$.

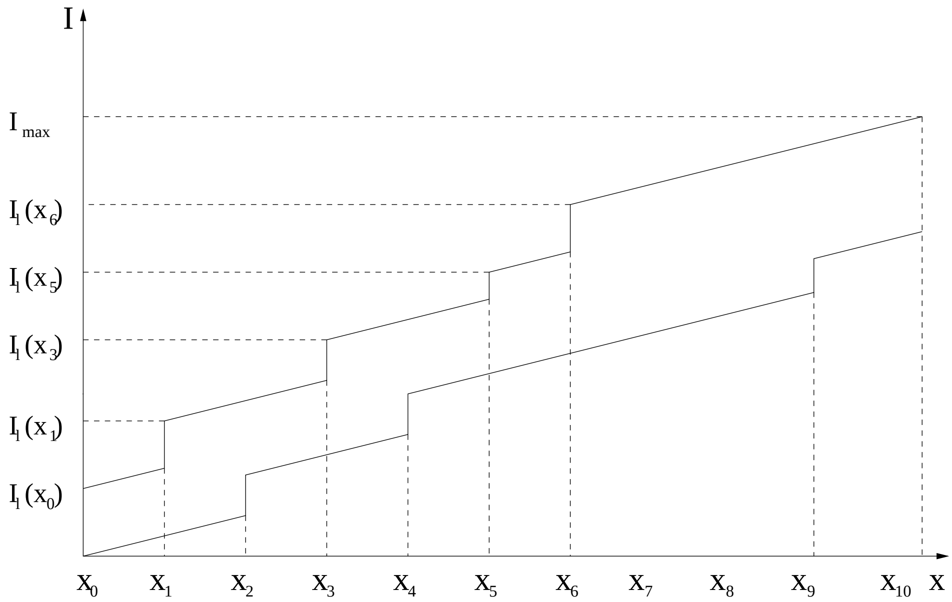

Figure 3. Leaf trajectory during DMLC delivery

Theorem 1 *The following are true of every leaf pair trajectory that delivers a discrete profile:*
(a) The left leaf must reach x_0 at some time.
(b) The right leaf must reach x_m at some time.
(c) The left leaf must reach x_m at some time.
(d) The right leaf must reach x_0 at some time.

Proof: (a) Suppose that the left leaf always stays to the right of x_0, then x_0 does not receive any MUs, contradicting our assumption that $I(x_0) > 0$.
(b) Similar to that of (a).
(c) If the left leaf doesn't reach x_m (i.e., it doesn't go to the right of x_{m-1}), from (b), it follows that the region between x_{m-1} and x_m receives a non-uniform distribution of MUs. Hence the discrete profile can't be accurately delivered.
(d) Similar to that of (c). ■

2.3. Maximum Velocity Constraint

As noted earlier, the velocity of leaves cannot exceed some maximum limit (say v_{max}) in practice. This implies that the leaf profile cannot be horizontal at any point. From Figure 3, observe that the time needed for a leaf to move from x_i to x_{i+1} is $\geq (x_{i+1} - x_i)/v_{max}$. If Φ is the flux density of MUs from the source, the number of MUs delivered in this time along a beam is $\geq \Phi * (x_{i+1} - x_i)/v_{max}$. So, $I_l(x_{i+1}) - I_l(x_i) \geq \Phi * (x_{i+1} - x_i)/v_{max} = \Phi * \Delta x/v_{max}$. The same is true for the right leaf profile I_r.

2.4. Optimal Unidirectional Algorithm for one Pair of Leaves

2.4.1. Unidirectional Movement. When the movement of leaves is restricted to only one direction, both the left and right leaves move along the positive x direction, from left to right (Figure 1). Once the desired intensity profile, $I(x_i)$ is known, our problem becomes that of determining the individual *intensity profiles* to be delivered by the left and right leaves, I_l and I_r such that:

$$I(x_i) = I_l(x_i) - I_r(x_i), 0 \leq i \leq m \tag{1}$$

Of course, I_l and I_r are subject to the maximum velocity constraint. We refer to (I_l, I_r) as the *treatment plan* (or simply *plan*) for I. Once we obtain the plan, we will be able to determine the movement of both left and right leaves during the therapy. For each i, the left leaf can be allowed to pass x_i when the source has delivered $I_l(x_i)$ MUs. Also, we can allow the right leaf to pass x_i when the source has delivered $I_r(x_i)$ MUs. In this manner we obtain *unidirectional leaf movement profiles* for a plan. Some simple observations about the leaf profiles are made below.

Theorem 2 *In every unidirectional plan the leaves begin at x_0 and end at x_m.*

Proof: Follows from Theorem 1 and the unidirectional constraint. ■

Lemma 1 *In the region between each pair of successive sample points, say x_i and x_{i+1}, both leaf profiles maintain the same shape, i.e., one is merely a vertical translation of the other.*

Proof: As explained previously, the input profile is discretized to a square wave I. Since the profile of I is horizontal between successive sample points and since it is equal to $I_l - I_r$, I_l and I_r must have the same shape. For example, if the left leaf moves at a constant velocity v between points x_i and x_{i+1}, so should the right leaf. ■

Lemma 2 *In an optimal plan, both leaves must move at v_{max} between every successive pair of sample points they move across.*

Proof: Suppose that in an optimal solution the leaves move between points x_i and x_{i+1} at a possibly varying velocity $v(x) \leq v_{max}$. From Lemma 1, we know that both leaf profiles are the same between x_i and x_{i+1}. Setting $v(x) = v_{max}$ results in new leaf profiles whose difference remains the same as before (which is the desired profile I) and total therapy time is lowered. This leads to a contradiction. ■

Corollary 1 *In an optimal plan, no leaf stops at an x that is not one of the x_is.*

2.4.2. Algorithm. From Equation 1, we see that one way to determine I_l and I_r from the given target profile I is to begin from x_0; set $I_l(x_0) = I(x_0)$ and $I_r(x_0) = 0$; examine the remaining x_is to the right; increase I_l at x_i whenever I increases and by the same amount (in addition to the minimum increase imposed by the maximum velocity constraint); and similarly increase I_r whenever I decreases. This can be done till we reach x_m. So the treatment begins with the leaves positioned at the leftmost sample point and ends with the leaves positioned at the rightmost sample point. Once I_l and I_r are determined the leaf movement profiles are obtained as explained earlier. Note that we move the leaves at the maximum velocity v_{max} whenever they are to be moved. The resulting algorithm is shown in Figure 4. Figure 5 shows a profile. The corresponding plan obtained using Algorithm SINGLEPAIR is in Figure 3.

Algorithm SINGLEPAIR

$I_l(x_0) = I(x_0)$
$I_r(x_0) = 0$
  For $j = 1$ to m do
    If $(I(x_j) \geq I(x_{j-1}))$
      $I_l(x_j) = I_l(x_{j-1}) + I(x_j) - I(x_{j-1}) + \Phi * \Delta x / v_{max}$
      $I_r(x_j) = I_r(x_{j-1}) + \Phi * \Delta x / v_{max}$
    Else
      $I_r(x_j) = I_r(x_{j-1}) + I(x_{j-1}) - I(x_j) + \Phi * \Delta x / v_{max}$
      $I_l(x_j) = I_l(x_{j-1}) + \Phi * \Delta x / v_{max}$
  End for

Figure 4. Obtaining a unidirectional plan

Ma *et al* (1998) shows that Algorithm SINGLEPAIR obtains plans that are optimal in therapy time. Their proof relies on the results of Boyer and Strait (1997), Spirou and Chui (1994) and Stein *et al* (1994). We provide a simpler and direct proof below.

Theorem 3 *Algorithm SINGLEPAIR obtains plans that are optimal in therapy time.*

Proof: Let $I(x_i)$ be the desired profile. Let $0 = inc0 < inc1 < \ldots < inck$ be the indices of the points at which $I(x_i)$ increases. So $x_{inc0}, x_{inc1}, \ldots, x_{inck}$ are the points at which $I(x)$ increases (i.e., $I(x_{inci}) > I(x_{inci-1})$, assume that $I(x_{-1} = 0)$). Let $\Delta i = I(x_{inci}) - I(x_{inci-1})$, $i \geq 0$.
Suppose that (I_L, I_R) is a plan for $I(x_i)$ (not necessarily the plan generated by Algorithm

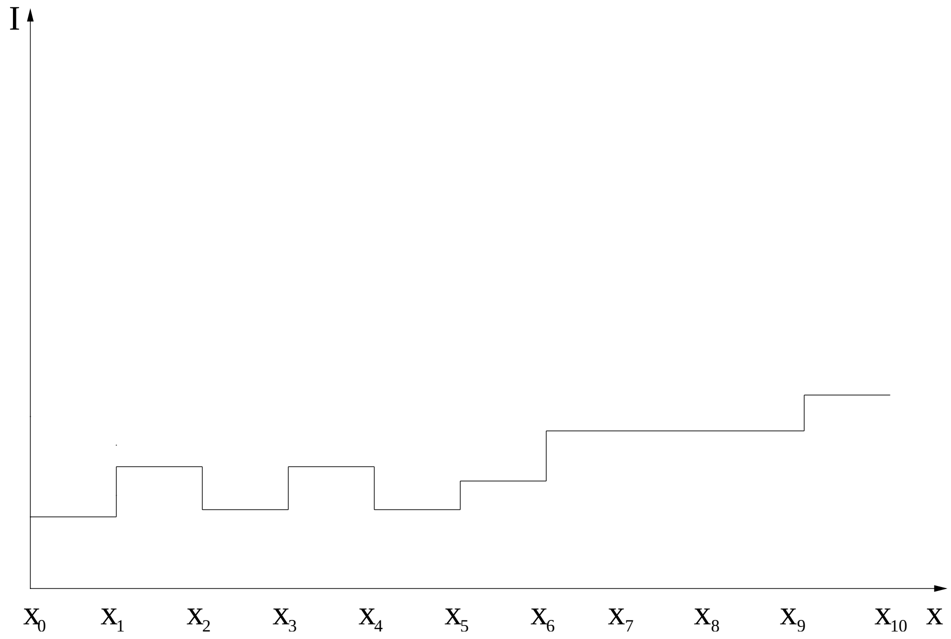

Figure 5. A profile

SINGLEPAIR). Since $I(x_i) = I_L(x_i) - I_R(x_i)$ for all i , we get

$$\Delta i = (I_L(x_{inci}) - I_R(x_{inci})) - (I_L(x_{inci-1}) - I_R(x_{inci-1}))$$
$$= (I_L(x_{inci}) - I_L(x_{inci-1})) - (I_R(x_{inci}) - I_R(x_{inci-1}))$$
$$= (I_L(x_{inci}) - I_L(x_{inci-1}) - \Phi * \Delta x/v_{max}) - (I_R(x_{inci}) - I_R(x_{inci-1}) - \Phi * \Delta x/v_{max})$$

Note that from the maximum velocity constraint $I_R(x_{inci}) - I_R(x_{inci-1}) \geq \Phi * \Delta x/v_{max}$, $i \geq 1$. So $I_R(x_{inci}) - I_R(x_{inci-1}) - \Phi * \Delta x/v_{max} \geq 0$, $i \geq 1$, and $\Delta i \leq I_L(x_{inci}) - I_L(x_{inci-1}) - \Phi * \Delta x/v_{max}$. Also, $\Delta 0 = I(x_0) - I(x_{-1}) = I(x_0) \leq I_L(x_0) - I_L(x_{-1})$, where $I_L(x_{-1}) = 0$.

Summing up Δi, we get

$\sum_{i=0}^{k}[I(x_{inci}) - I(x_{inci-1})] \leq \sum_{i=0}^{k}[I_L(x_{inci}) - I_L(x_{inci-1})] - k * \Phi * \Delta x/v_{max}$. Let $S_1 = \sum_{i=0}^{k}[I_L(x_{inci}) - I_L(x_{inci-1})]$. Then, $S_1 \geq \sum_{i=0}^{k}[I(x_{inci}) - I(x_{inci-1})] + k * \Phi * \Delta x/v_{max}$. Let $S_2 = \sum[I_L(x_j) - I_L(x_{j-1})]$, where the summation is carried out over indices j $(0 \leq j \leq m)$ such that $I(x_j) \leq I(x_{j-1})$. There are a total of $m+1$ indices of which $k+1$ do not satisfy this condition. So there are $m-k$ indices j at which $I(x_j) \leq I(x_{j-1})$. At each of these j, $I_L(x_j) \geq I_L(x_{j-1}) + \Phi * \Delta x/v_{max}$. Hence, $S_2 \geq (m-k) * \Phi * \Delta x/v_{max}$. Now, we get $S_1 + S_2 = \sum_{i=0}^{m}[I_L(x_i) - I_L(x_{i-1})] \geq \sum_{i=0}^{k}[I(x_{inci}) - I(x_{inci-1})] + m * \Phi * \Delta x/v_{max}$. Finally, $TT(I_L, I_R) = I_L(x_m) = I_L(x_m) - I_L(x_{-1}) = \sum_{i=0}^{m}[I_L(x_i) - I_L(x_{i-1})] \geq \sum_{i=0}^{k}[I(x_{inci}) - I(x_{inci-1})] + m * \Phi * \Delta x/v_{max} = TT(I_l, I_r)$. Hence, the treatment plan (I_l, I_r) generated by SINGLEPAIR is optimal in therapy time. ■

Corollary 2 *Let $I(x_i)$, $0 \leq i \leq m$ be a desired profile. Let $I_l(x_i)$ and $I_r(x_i)$, $0 \leq i \leq m$ be the left and right leaf profiles generated by Algorithm SINGLEPAIR. $I_l(x_i)$ and $I_r(x_i)$, $0 \leq i \leq m$ define optimal therapy time unidirectional left and right leaf profiles for $I(x_i)$, $0 \leq i \leq j$.*

Proof: Follows from Theorem 3 ∎

In the remainder of Section 2, (I_l, I_r) is the optimal treatment plan generated by Algorithm SINGLEPAIR for the desired profile I.

2.4.3. Properties of The Optimal Treatment Plan. The following observations are made about the optimal treatment plan (I_l, I_r) generated using Algorithm SINGLEPAIR.

Lemma 3 *At most one of the leaves stops at each* x_i*.*

Lemma 4 *Let* (I_L, I_R) *be any treatment plan for* I*.*
(a) $\Delta(x_i) = I_L(x_i) - I_l(x_i) = I_R(x_i) - I_r(x_i) \geq 0, 0 \leq i \leq m$*.*
(b) $\Delta(x_i)$ *is a non-decreasing function.*

Proof: (a) Since $I(x_i) = I_L(x_i) - I_R(x_i) = I_l(x_i) - I_r(x_i), I_L(x_i) - I_l(x_i) = I_R(x_i) - I_r(x_i)$. Further, from Corollary 2, it follows that $I_L(x_i) \geq I_l(x_i), 0 \leq i \leq m$. Therefore, $\Delta(x_i) \geq 0, 0 \leq i \leq m$.

(b) We prove this by contradiction. Suppose that $\Delta(x_n) > \Delta(x_{n+1})$ for some $n, 0 \leq n < m$. Consider the following three all encompassing cases.

Case 1: $I_l(x_{n+1}) = I_l(x_n) + \Phi * \Delta x / v_{max}$ (left leaf does not stop at x_{n+1})
Now, $I_L(x_n) = I_l(x_n) + \Delta(x_n) > I_l(x_{n+1}) - \Phi * \Delta x / v_{max} + \Delta(x_{n+1}) = I_L(x_{n+1}) - \Phi * \Delta x / v_{max}$.
This is not possible because $I_L(x_{n+1}) \geq I_L(x_n) + \Phi * \Delta x / v_{max}$ from the maximum velocity constraint.

Case 2: $I_r(x_{n+1}) = I_r(x_n) + \Phi * \Delta x / v_{max}$ (right leaf does not stop at x_{n+1})
Now, $I_R(x_n) = I_r(x_n) + \Delta(x_n) > I_r(x_{n+1}) - \Phi * \Delta x / v_{max} + \Delta(x_{n+1}) = I_R(x_{n+1}) - \Phi * \Delta x / v_{max}$.
This is not possible because $I_R(x_{n+1}) \geq I_R(x_n) + \Phi * \Delta x / v_{max}$ from the maximum velocity constraint.

Case 3: $I_l(x_n) \neq I_l(x_{n+1}) + \Phi * \Delta x / v_{max}$ and $I_r(x_n) \neq I_r(x_{n+1}) + \Phi * \Delta x / v_{max}$ (both leaves stop at x_{n+1})
From Lemma 3 it follows that this case cannot arise.

Therefore, $\Delta(x_i)$ is a non-decreasing function. ∎

Corollary 3 *Let* $\Lambda_l(x_i)$ $(\Lambda_r(x_i))$ *and* $\Lambda_L(x_i)$ $(\Lambda_R(x_i))$*, respectively, denote the amount of time for which the left (right) leaf stops at* x_i *in plans* (I_l, I_r) *and* (I_L, I_R)*. Then*
(a) $\Lambda_L(x_i) \geq \Lambda_l(x_i)$*.*
(b) $\Lambda_R(x_i) \geq \Lambda_r(x_i)$*.*

Proof: (a) Suppose that $\Lambda_L(x_i) < \Lambda_l(x_i)$. We have the following two cases:

Case 1: Both leaves move at the maximum velocity between x_{i-1} and x_i in (I_L, I_R). We get $\Delta(x_i) < \Delta(x_{i-1})$ contradicting Lemma 4(b).

Case 2: In (I_L, I_R), the leaves do not travel uniformly at the maximum velocity between x_{i-1} and x_i. In this case, transform plan (I_L, I_R) to a plan (I'_L, I'_R) as follows. Between x_{i-1} and x_i move the leaves at the maximum velocity. The leaves now arrive

at x_i earlier than they did in (I_L, I_R) by an amount δi. Propagate this difference to the right from x_i so that $I'_L(x_j) = I_L(x_j) - \delta i$ and $I'_R(x_j) = I_R(x_j) - \delta i$, $j \geq i$. Note that this transformation preserves the Λs, i.e., $\Lambda'_L(x_j) = \Lambda_L(x_j)$. Also, the resulting leaf profiles, I'_L and I'_R, still form a plan for I. Let $\Delta'(x_j) = I'_L(x_j) - I_l(x_j) = I'_R(x_j) - I_r(x_j)$. Since $\Lambda'_L(x_i) = \Lambda_L(x_i) < \Lambda_l(x_i)$ and since the leaves move at maximum velocity from x_{i-1} to x_i in (I_l, I_r) and (I'_L, I'_R), we have $\Delta'(x_i) < \Delta'(x_{i-1})$ contradicting Lemma 4(b).
(b) Similar to proof of (a). ■

2.5. Minimum Separation Constraint

Some MLCs have a minimum separation constraint that requires the left and right leaves to maintain a minimum separation S_{min} at all times during the treatment. Notice that in the plan generated by Algorithm SINGLEPAIR, the two leaves start and end at the same point. So they are in contact at x_0 and x_m. When $s_{min} > 0$, the minimum separation constraint is violated at x_0 and x_m. In order to overcome this problem we modify Algorithm SINGLEPAIR to guarantee minimum separation between the leaves in the vicinity of the end points (x_0 and x_m). In particular, we allow the left leaf to be initially positioned at a point $x_{0'} = x_0 - s_{min}$ and the right leaf to be finally positioned at $x_{m'} = x_m + S_{min}$. The only changes made relative to Algorithm SINGLEPAIR are for the movement of the left leaf from $x_{0'}$ to x_0 and for the right leaf from x_m to $x_{m'}$. We define the movement of the left leaf from $x_{0'}$ to x_0 (and a symmetric definition for the right leaf from x_m to $x_{m'}$) to be such that it maintains a distance of exactly S_{min} from the right leaf at all times. Once the left leaf reaches x_0 it follows the trajectory as before. While this modification results in the exact profile being delivered between x_0 and x_m it also results in some undesirable exposure to the intervals $(x_{0'}, x_0)$ and $(x_m, x_{m'})$. In the remainder of this section we will consider an exposure of this kind permissible, provided the exact profile is delivered between x_0 and x_m. The only difficulty arises when the number of monitor units delivered at the time the left leaf reaches x_0 in this new plan (call it (I'_l, I'_r)) is greater than $I_l(x_0)$. This would prevent us from using the old plan from x_0 to x_m, since the leaf cannot pass x_0 before it arrives there. Observe however, that if the left leaf were to arrive at x_0 any earlier, it would be too close to the right leaf. In the discussion that follows we show that in this and other cases where the original plan violates the constraint, there are no feasible solutions that deliver exactly the desired profile between x_0 and x_m, while permitting exposure outside this region. The modified algorithm, which we call MINSINGLEPAIR, is described in Figure 6. Note that the therapy time of the plan produced by MINSINGLEPAIR is the same as that of the plan produced by SINGLEPAIR. Therefore, the modified plan is optimal in therapy time.

Theorem 4 *(a)* $S_{min} * \Phi/v_{max} > I_l(x_0)$ *or (b) If the plan* (I'_l, I'_r) *generated by MINSINGLEPAIR violates the minimum separation constraint, then there is no plan for I that does not violate the minimum separation constraint.*

Proof: Suppose that (I'_l, I'_r) violates the minimum separation constraint. Assume that the first violation occurs when I_1 MUs have been delivered. Since there is no violation

Algorithm MINSINGLEPAIR

1. Apply Algorithm SINGLEPAIR
2. Modify the profile of the left leaf from $x_{0'}$ to x_0 and the right leaf from x_m to $x_{m'}$ to maintain a minimum interleaf distance of S_{min}. Call this profile (I_l', I_r').
3. If the number of MUs delivered when the left leaf arrives at x_0 is greater than $I_l(x_0)$ there is no feasible solution. End.
4. Else output (I_l', I_r'). There is a feasible solution only if (I_l', I_r') is feasible.

Figure 6. Obtaining a unidirectional plan with minimum separation constraint

when less than I_1 MUs are delivered and since the leaves are either stationary or move at the maximum velocity, at the time of the violation, it must be the case that the right leaf is stationary at a sample point (say x_k) and the left leaf is moving. The violation occurs when the left leaf passes $x' = x_k - S_{min}$. Since the left leaf is moving, $I_1 = I_l'(x') < I_r'(x_k)$. Figure 7 illustrates the situation. Suppose that there is another plan (I_l'', I_r'') that does not violate the minimum separation constraint. Let $I_l''(x') = I_l'(x') + \Delta(x')$ and let $I_r''(x_k) = I_r'(x_k) + \Delta(x_k)$. From Lemma 4(a), $\Delta(x'), \Delta(x_k) \geq 0$ and from Lemma 4(b), $\Delta(x') \leq \Delta(x_k)$. Here, we have made use of the fact that in the statement of Lemma 4, we can replace the plan (I_l, I_r) with the plan (I_l', I_r'). Now, $I_r''(x_k) - I_l''(x') = I_r'(x_k) + \Delta(x_k) - (I_l'(x') + \Delta(x')) = (I_r'(x_k) - I_l'(x')) + (\Delta(x_k) - \Delta(x'))$. Since $I_r'(x_k) > I_l'(x')$ and $\Delta(x_k) \geq \Delta(x')$, we get $I_r''(x_k) > I_l''(x')$. Therefore there is a minimum separation violation in (I_l'', I_r'') when the the left leaf passes x_j.

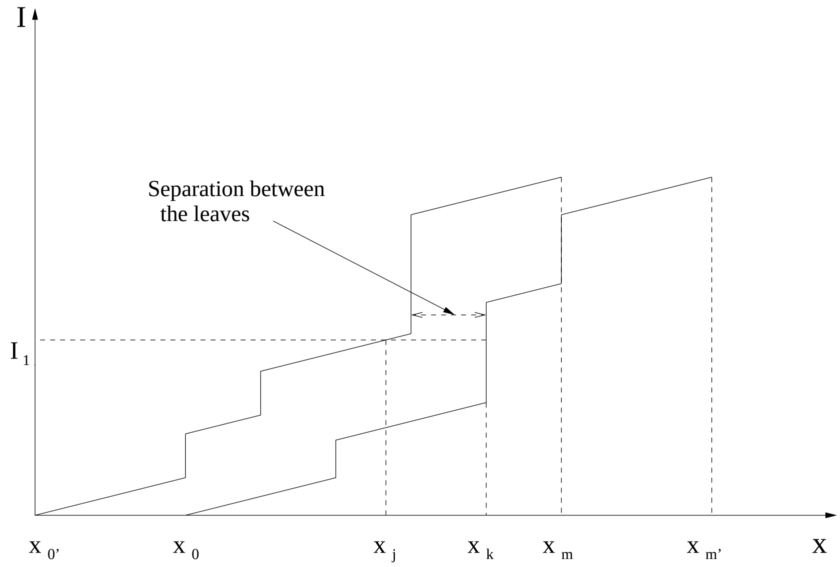

Figure 7. Minimum separation constraint violation

■

The separation between the leaves is determined by the difference in x values of the leaves when the source has delivered a certain amount of MUs. The minimum separation of the leaves is the minimum separation between the two profiles. We call this minimum separation S_{ud-min}. When the optimal plan obtained using Algorithm SINGLEPAIR is delivered, the minimum separation is $S_{ud-min(opt)}$.

Corollary 4 *Let $S_{ud-min(opt)}$ be the minimum leaf separation in the plan (I'_l, I'_r). Let S_{ud-min} be the minimum leaf separation in any (not necessarily optimal) given unidirectional plan. $S_{ud-min} \leq S_{ud-min(opt)}$.*

Theorem 5 *If Algorithm MINSINGLEPAIR terminates in Step 3, then there is no plan for I that does not violate the minimum separation constraint.*

Proof: Let (I''_l, I''_r) be a feasible plan (i.e., a plan that delivers I and satisfies the minimum separation constraint). Let (I'_l, I'_r) be the plan of Step 2 of MINSINGLEPAIR. From Corollary 2, it follows that $\hat{I}''_r(x_0 + S_{min}) \geq \hat{I}'_r(x_0 + S_{min}) + I''_r(x_0)$, where $\hat{I}_r(x)$ is the number of MUs delivered when the right leaf reaches x (note that $I_r(x)$ is the number of MUs delivered when the right leaf leaves x). Since (I''_l, I''_r) has no minimum separation violation, $I''_l(x_0) \geq \hat{I}''_l(x_0) \geq \hat{I}''_r(x_0 + S_{min})$. Also, because MINSINGLEPAIR terminates in Step 3, $\hat{I}'_l(x_0) > I_l(x_0)$. Hence, $I''_l(x_0) - I''_r(x_0) \geq \hat{I}'_r(x_0 + S_{min}) = \hat{I}'_l(x_0) > I_l(x_0) = I(x_0)$. So, (I''_l, I''_r) does not deliver the proper dose at x_0 and so cannot be feasible. ■

2.5.1. Comparison with SMLC. Kamath *et al* (2003) discuss the optimal therapy time algorithm for SMLC. Their algorithm generates an optimal plan that satisfies the minimum separation constraint, whenever one exists. We prove the existence of profiles for which there are feasible plans using SMLC, but no feasible plans using DMLC.

Lemma 5 *Let the minimum separation between the leaves in the optimal SMLC plan be S_{s-min}. Let the minimum leaf separation in the plan generated by algorithm MINSINGLEPAIR be S_{d-min}. Then $S_{d-min} \leq S_{s-min}$.*

Proof: Consider the delivery of a profile I by the optimal SMLC plan of Kamath *et al* (2003). Call this plan (I^s_l, I^s_r). Let $\hat{I}^s_l(x)$ be the number of MUs delivered when the left leaf arrives at x using the plan I^s_l. $\hat{I}^s_r(x)$, $\hat{I}'_l(x)$, and $\hat{I}'_r(x)$ are defined similarly. Let $\Gamma_l(x_k) = I^s_l(x_k) - \hat{I}^s_l(x_k) = I'_l(x_k) - \hat{I}'_l(x_k)$. $\Gamma_r(x_k)$ is defined similarly. Note that $\Gamma_l(x_k) > 0$ iff the left leaf stops at x_k and $\Gamma_l(x_k)$ gives the amount of MUs delivered while the left leaf is stopped at x_k. Let x_i and x_j, $j > i$, be such that $S_{s-min} = x_j - x_i$ and $\hat{I}^s_l(x_i) < I^s_r(x_j)$. Such an x_i and x_j must exist by definition of S_{s-min}.
It is easy to see that $I^s_r(x_j) - I^s_r(x_i) = \sum_{k=i+1}^{j} \Gamma_r(x_k)$. From this and $\hat{I}^s_l(x_i) < I^s_r(x_j)$, we get

$$\sum_{k=i+1}^{j} \Gamma_r(x_k) > \hat{I}^s_l(x_i) - I^s_r(x_i) \tag{2}$$

Since $I(x_i) = I^s_l(x_i) - I^s_r(x_i) = \hat{I}^s_l(x_i) + \Gamma_l(x_i) - I^s_r(x_i) = \hat{I}'_l(x_i) + \Gamma_l(x_i) - I'_r(x_i)$,

$$I'_r(x_i) = \hat{I}'_l(x_i) - (\hat{I}^s_l(x_i) - I^s_r(x_i)) \tag{3}$$

Also, we see that $I'_r(x_j) = I'_r(x_i) + \sum_{k=i+1}^{j} \Gamma_r(x_k) + (j-i) * \Phi * \Delta x / v_{max} = \hat{I}'_l(x_i) - (\hat{I}^s_l(x_i) - I^s_r(x_i)) + \sum_{k=i+1}^{j} \Gamma_r(x_j) + (j-i) * \Phi * \Delta x / v_{max}$ (from (3)) $> \hat{I}'_l(x_i) + (j-i) * \Phi * \Delta x / v_{max}$ (from (2)). So, $S_{d-min} \leq x_j - x_i = S_{s-min}$. ■

The following result immediately follows and can be easily verified.

Corollary 5 *All profiles that have feasible plans using DMLC have feasible plans using SMLC. There exist profiles for which there are feasible plans using SMLC, but no feasible plans using DMLC.*

Figure 8 shows the SMLC plan of a profile. The corresponding DMLC plan obtained using Algorithm MINSINGLEPAIR is infeasible and is shown in Figure 9.

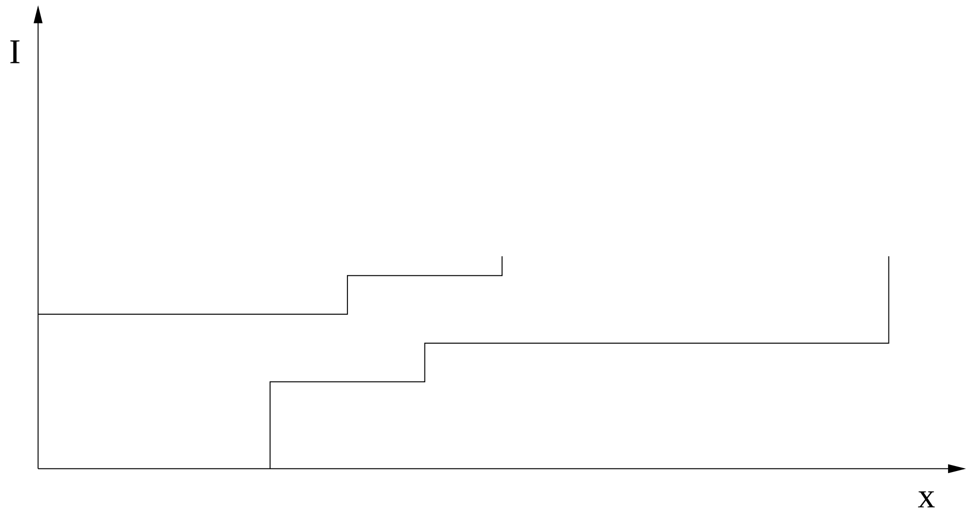

Figure 8. SMLC plan: feasible

2.6. Bi-directional Movement

In this section we study beam delivery when bi-directional movement of leaves is permitted. We explore whether relaxing the unidirectional movement constraint helps improve the efficiency of treatment plan.

2.6.1. Properties of Bi-directional Movement. For a given leaf (left or right) movement profile we classify any x-coordinate as follows. Draw a vertical line at x. If the line cuts the leaf profile exactly once we will call x a *unidirectional point* of that leaf profile. If the line cuts the profile more than once, x is a *bi-directional point* of that profile. A leaf movement profile that has at least one bi-directional point is a *bi-directional profile.* All profiles that are not bi-directional are *unidirectional profiles.* Any profile can be partitioned into segments such that each segment is a unidirectional profile. When the number of such partitions is minimal, each partition is called a *stage* of the original profile. Thus unidirectional profiles consist of exactly one stage while bi-directional profiles always have more than one stage.

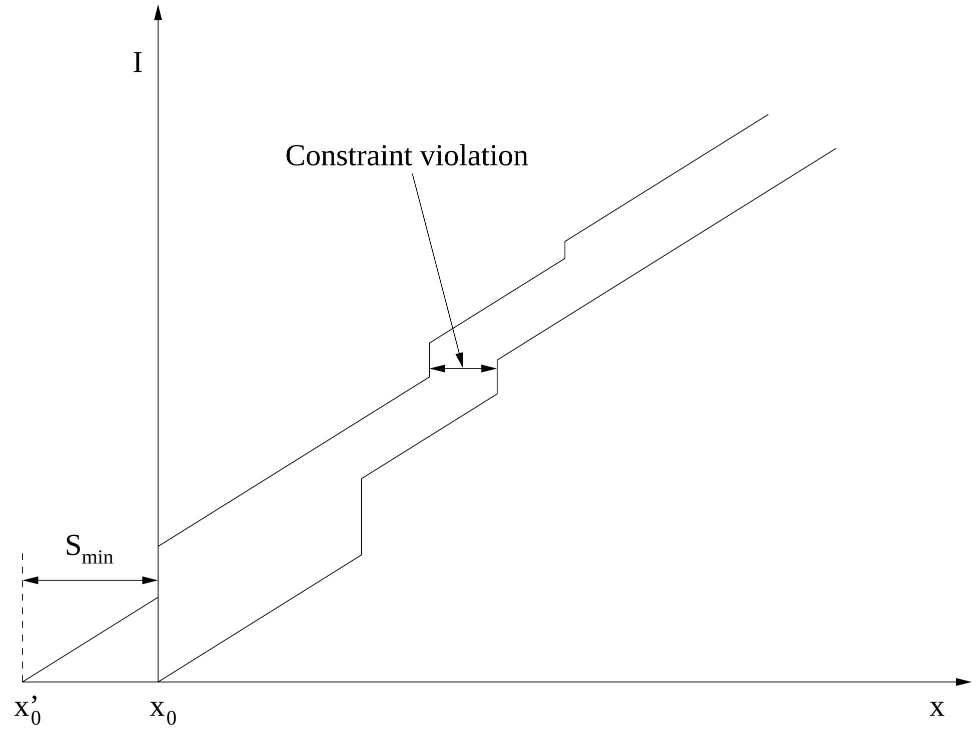

Figure 9. DMLC plan: infeasible

In Figure 10, the bi-directional leaf movement profile, B_L, shows the position of the left leaf as a function of the amount of MUs delivered by the source. The movement profile of this leaf consists of stages S_1, S_2 and S_3. In stages S_1 and S_3 the leaf moves from left to right while in stage S_2 the leaf moves from right to left. x_j is a bi-directional point of B_L. Let I_L be the intensity profile corresponding to the leaf movement profile B_L. $I_L(x)$ gives the number of MUs delivered at x using movement profile B_L. The amount of MUs delivered at x_j is $L_1 + L_2$. In stage S_1, when I_1 amount of MUs have been delivered, the leaf passes x_j. Now, no MU is delivered at x_j till the leaf passes over x_j in S_2. Further, $I_3 - I_2$ MUs are delivered to x_j in stages S_2 and S_3. Thus we have $I_l(x_j) = L_1 + L_2$, where $L_1 = I_1$ and $L_2 = I_3 - I_2$. x_k is a unidirectional point of B_L. The MUs delivered at x_k are $L_3 = I_4$. Note that the intensity profile I_L is different from the leaf movement profile B_L, unlike in the unidirectional case.

Lemma 6 *Let I_L and I_R be the intensity profiles corresponding to the bi-directional leaf movement profile pair (B_L, B_R) (i.e., B_L and B_R are, respectively, the left and right leaf movement profiles). Let $I(x_i) = I_L(x_i) - I_R(x_i)$, $0 \leq i \leq m$, be the intensity profile delivered by (B_L, B_R). Then*
(a) $I_L(x_{i+1}) \geq I_L(x_i) + \Phi * \Delta x / v_{max}$.
(b) $I_R(x_{i+1}) \geq I_R(x_i) + \Phi * \Delta x / v_{max}$.

Proof: (a)Between the time, t_1, the left leaf moves rightward from x_i for the last time (since the left leaf ends at x_m, such a last right move must occur) and the least time t_2, $t_2 > t_1$, that the left leaf reaches x_{i+1} (again, since the left leaf ends at x_m,

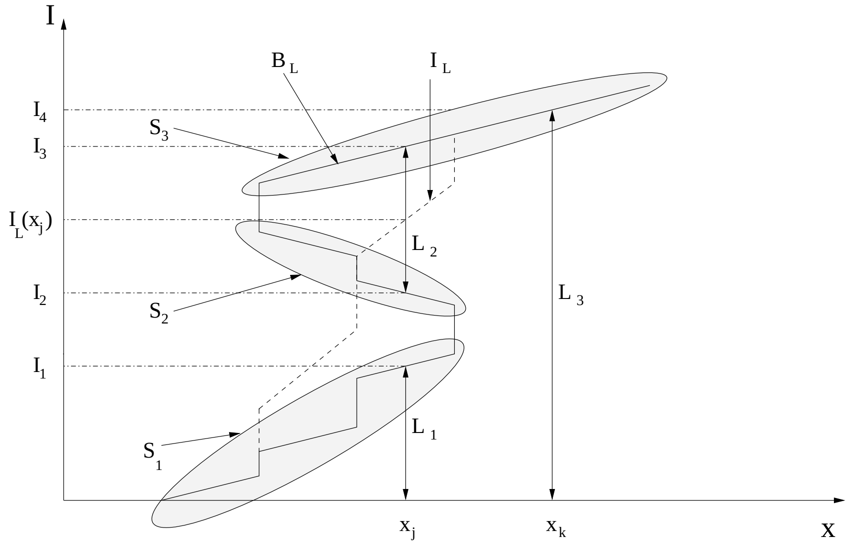

Figure 10. Bi-directional movement

such a t_2 exists), x_{i+1} receives at least $\Phi * \Delta x/v_{max}$ MUs that are not delivered to x_i. At all other times during the therapy, whenever the left leaf doesn't cover x_i, it also doesn't cover x_{i+1}. Hence, outside the time interval $[t_1, t_2]$, the number of MUs delivered to x_{i+1} is atleast as many as delivered to x_i. Therefore, for the entire therapy, $I_L(x_{i+1}) \geq I_L(x_i) + \Phi * \Delta x/v_{max}$.
(b)The proof is similar to that of (a). ■

From Lemma 6 we note that every bi-directional leaf movement profile (B_L, B_R) delivers an intensity profile (I_L, I_R) that satisfies the maximum velocity constraint. Hence, (I_L, I_R) is deliverable using a unidirectional leaf movement profile (Section 2.4.1). We will call this profile the *unidirectional leaf movement profile that corresponds to the bi-directional profile.* Thus every bi-directional leaf movement profile has a corresponding unidirectional leaf profile that delivers the same amount of MUs at each x_i as does the bi-directional profile.

Theorem 6 *The unidirectional treatment plan constructed by Algorithm SINGLEPAIR is optimal in therapy time even when bi-directional leaf movement is permitted.*

Proof: Let B_L and B_R be bidirectional leaf movement profiles that deliver a desired intensity profile I. Let I_L and I_R, respectively, be the corresponding intensity profiles for B_L and B_R. From Lemma 6, we know that I_L and I_R are deliverable by unidirectional leaf movement profiles. Also, $I_L(x_i) - I_R(x_i) = I(x_i), 1 \leq i \leq m$. From the proof of Theorem 3, it follows that the therapy time for the unidirectional plan (I_l, I_r) generated by Algorithm SINGLEPAIR is no more than that of (I_L, I_R). ■

2.6.2. Incorporating Minimum Separation Constraint. Let U_l and U_r be unidirectional leaf movement profiles that deliver the desired profile $I(x_i)$. Let B_l and B_r be a set of bi-directional left and right leaf profiles such that U_l and U_r correspond to B_l and

B_r respectively, i.e., (B_l, B_r) delivers the same plan as (U_l, U_r). We call the minimum separation of leaves in this bi-directional plan (B_l, B_r) S_{bd-min}. S_{ud-min} is the minimum separation of leaves in (U_l, U_r).

Theorem 7 $S_{bd-min} \leq S_{ud-min}$ *for every bi-directional leaf movement profile pair* (B_l, B_r) *and its corresponding unidirectional profile* (U_l, U_r).

Proof: Suppose that the minimum separation S_{ud-min} occurs when I_{ms} MUs are delivered. At this time, the left leaf arrives at x_j and the right leaf is positioned at x_k. Let B'_l and U'_l respectively, be the profiles obtained when B_l and U_l are shifted right by S_{ud-min}. Since U'_l is U_l shifted right by S_{ud-min} and since the distance between U_l and U_r is S_{ud-min} when I_{ms} MUs have been delivered, U'_l and U_r touch when I_{ms} units have been delivered. Therefore, the total MUs delivered by (U'_l, U_r) at x_k is zero. So the total MUs delivered by (B'_l, B_r) at x_k is also zero (recall that U'_l and U_r, respectively, are corresponding profiles for B'_l and B_r). This isn't possible if B_r is always to the right of B'_l (for example, in the situation of Figure 11, the MUs delivered by (B'_l, B_r) at x_k are $(L_1 + L_2) - (L'_1 + L'_2) > 0$). Therefore B'_l and B_r must touch (or cross) at least once. So $S_{bd-min} \leq S_{ud-min}$. ■

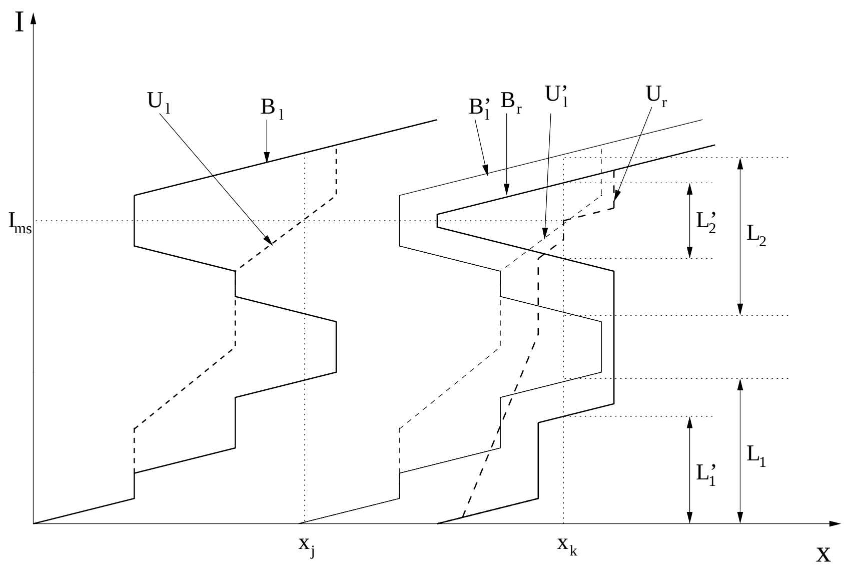

Figure 11. Bi-directional movement under minimum separation constraint

Theorem 8 *If the optimal unidirectional plan* (I'_l, I'_r) *violates the minimum separation constraint, then there is no bi-directional movement plan that does not violate the minimum separation constraint.*

Proof: Let B_l and B_r be bi-directional leaf movements that deliver the required profile. Let the minimum separation between the leaves be S_{bd-min}. Let the corresponding unidirectional leaf movements be U_l and U_r and let S_{ud-min} be the minimum separation between U_l and U_r. Also, let S_{min} be the minimum allowable separation between the leaves. From Corollary 4 and Theorem 7, we get $S_{bd-min} \leq S_{ud-min} \leq S_{ud-min(opt)} < S_{min}$. ■

2.6.3. Incorporating Maximum Separation Constraint. Let U_l and U_r be unidirectional leaf movement profiles that deliver the desired profile I. Let S_{ud-max} be the maximum leaf separation using the profiles U_l and U_r and let $S_{ud-max(opt)}$ be the maximum leaf separation for the plan (I_l, I_r) generated by Algorithm SINGLEPAIR. Let B_l and B_r be a set of bi-directional left and right leaf profiles such that U_l and U_r correspond to B_l and B_r, respectively. Let S_{bd-max} be the maximum separation between the leaves when these bi-directional movement profiles are used.

Theorem 9 $S_{bd-max} \geq S_{ud-max}$ *for every bi-directional leaf movement profile and its corresponding unidirectional movement profile.*

Proof: Suppose that the maximum separation S_{ud-max} occurs when I_{ms} MUs are delivered. At this time, the left leaf is positioned at x_j and the right leaf arrives at x_k. Let B_l' and U_l' respectively, be the profiles obtained when B_l and U_l are shifted right by S_{ud-max}. Since U_l' is U_l shifted right by S_{ud-max} and since the distance between U_l and U_r is S_{ud-max} when I_{ms} MUs have been delivered, U_l' and U_r touch when I_{ms} units have been delivered. Therefore, the total MUs delivered by (U_r, U_l') at x_k is zero. So the total MUs delivered by (B_r, B_l') at x_k is also zero (recall that U_l' and U_r, respectively, are corresponding profiles for B_l' and B_r). This isn't possible if B_r is always to the left of B_l' (for example, in the situation of Figure 12, the MUs delivered by (B_r, B_l') at x_k are $(L_1' + L_2') - (L_1 + L_2) > 0$). Therefore B_l' and B_r must touch (or cross) at least once. So $S_{bd-max} \geq S_{ud-max}$. ■

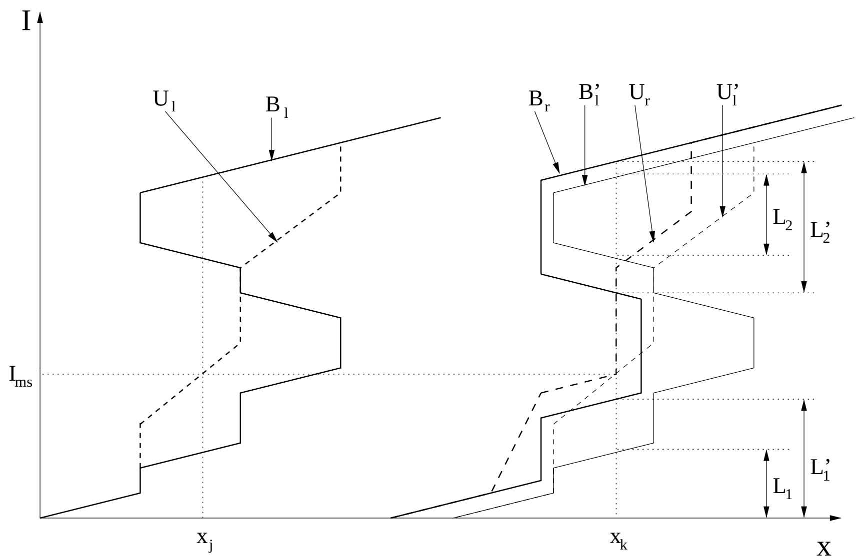

Figure 12. Bi-directional movement under maximum separation constraint

2.7. Optimal leaf Movement Algorithm Under Maximum Separation Constraint Condition

In this section we present an algorithm that generates an optimal treatment plan under the maximum separation constraint. Recall that Algorithm SINGLEPAIR generates the optimal plan without considering this constraint. We modify Algorithm SINGLEPAIR so that all instances of violation of maximum separation (that may possibly exist) are eliminated. We know (Theorem 9) that bi-directional leaf profiles do not help eliminate the constraint. So we consider only unidirectional profiles.

2.7.1. Algorithm. The algorithm is described in Figure 13.

Algorithm MAXSEPARATION

(i) Apply Algorithm SINGLEPAIR to obtain the optimal plan (I_l, I_r).

(ii) Find the least value of intensity, I_1, such that the leaf separation in (I_l, I_r) when I_1 MUs are delivered is S_{max}, where S_{max} is the maximum allowed separation between the leaves and the leaf separation when $I_1 + \epsilon$ MUs are delivered is $> S_{max}$, for some positive constant ϵ. If there is no such I_1, (I_l, I_r) is the optimal plan; end.

(iii) From Lemma 2 it follows that when I_1 MUs are delivered, the left leaf is stopped at some x_j. Let x' be the position of the right leaf at this time (see Figure 14). Note that x' may not be one of the sample points x_i, $j \leq i \leq m$. Let $\Delta I = I_l(x_j) - I_1 = I_2 - I_1$. Move the profile of I_r, which follows x', up by ΔI along I direction. To maintain $I(x) = I_l(x) - I_r(x)$ for every x, move the profile of I_l, which follows x', up by ΔI along I direction.
Goto Step 2.

Figure 13. Obtaining a plan under maximum separation constraint

Theorem 10 *Algorithm MAXSEPARATION obtains plans that are optimal in therapy time, under the maximum separation constraint.*

Proof: We use induction to prove the theorem.
The statement we prove, $S(n)$, is the following:
After Step 3 of the algorithm is applied n times, the resulting plan, (I_{ln}, I_{rn}), satisfies

(a) It has no maximum separation violation when $I < I_2(n)$ MUs are delivered, where $I_2(n)$ is the value of I_2 during the nth iteration of Algorithm MAXSEPARATION.

(b) For plans that satisfy (a), (I_{ln}, I_{rn}) is optimal in therapy time.

(i) Consider the base case, $n = 1$.
Let (I_l, I_r) be the plan generated by Algorithm SINGLEPAIR. After Step 3 is applied once, the resulting plan (I_{l1}, I_{r1}) meets the requirement that there is no maximum separation violation when $I < I_2(1)$ MUs are delivered by the radiation

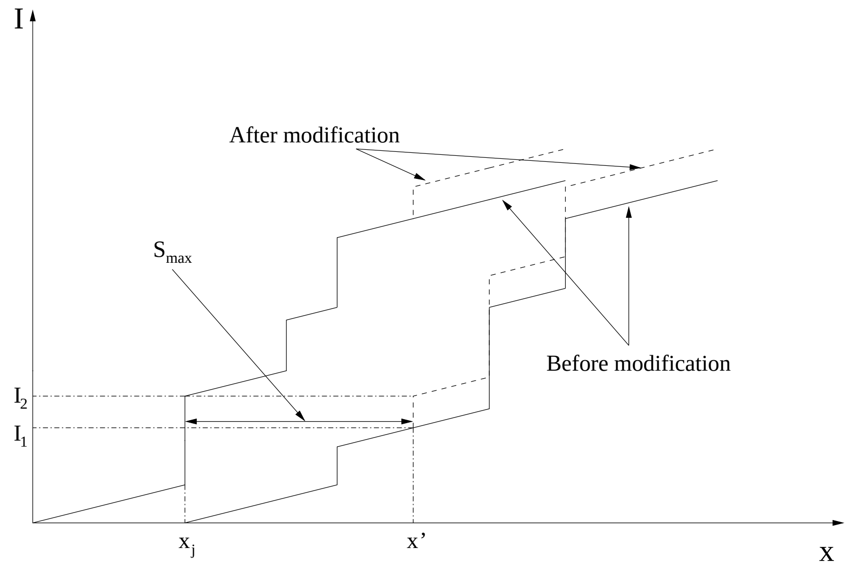

Figure 14. Maximum separation constraint violation

source. The therapy time increases by ΔI, i.e., $TT(I_{l1}, I_{r1}) = TT(I_l, I_r) + \Delta I$. Assume that there is another plan, (I'_{l1}, I'_{r1}), which satisfies condition (a) of $S(1)$ and $TT(I'_{l1}, I'_{r1}) < TT(I_{l1}, I_{r1})$. We show this assumption leads to a contradiction and so there is no such plan (I'_{l1}, I'_{r1}).

Let x_j and x' be as in Algorithm MAXSEPARATION. We consider three cases for the relationship between $I'_{l1}(x_j)$ and $I_{l1}(x_j)$.

(a) $I'_{l1}(x_j) = I_{l1}(x_j) = I_2(1)$

Since there is no maximum separation violation when $I < I_2(1)$ MUs are delivered, $I'_{r1}(x') \geq I'_{l1}(x_j) = I_{l1}(x_j) = I_{r1}(x')$. Since $I(x') = I'_{l1}(x') - I'_{r1}(x') = I_{l1}(x') - I_{r1}(x')$, we have $I'_{l1}(x') \geq I_{l1}(x')$. We now construct a plan (I''_{l1}, I''_{r1}) as follows:

$$I''_{l1}(x) = \begin{cases} I_l(x) & 0 \leq x < x' \\ I'_{l1}(x) - \Delta I & x \geq x' \end{cases}$$

$$I''_{r1}(x) = \begin{cases} I_r(x) & 0 \leq x < x' \\ I'_{r1}(x) - \Delta I & x \geq x' \end{cases}$$

Clearly $I''_{l1}(x) - I''_{r1}(x) = I(x), 0 \leq x \leq x_m$. Also, I''_{l1} is non-decreasing and satisfies the maximum velocity constraint $(I''_{l1}(x') = I'_{l1}(x') - \Delta I \geq I_{l1}(x') - \Delta I = I_l(x') \geq I_l(x' - \Delta x) + \Phi * \Delta x / v_{max} = I''_{l1}(x' - \Delta x) + \Phi * \Delta x / v_{max})$. Similarly I''_{r1} is non-decreasing and satisfies the maximum velocity constraint. So (I''_{l1}, I''_{r1}) is a plan for $I(x_i)$.

Also, $TT(I''_{l1}, I''_{r1}) = TT(I'_{l1}, I'_{r1}) - \Delta I < TT(I_{l1}, I_{r1}) - \Delta I = TT(I_l, I_r)$.

This contradicts our knowledge that (I_l, I_r) is the optimal unconstrained plan.

(b) $I'_{l1}(x_j) > I_{l1}(x_j)$

This leads to a contradiction as in the previous case.

(c) $I'_{l1}(x_j) < I_{l1}(x_j)$
In this case, $I'_{l1}(x_j) < I_{l1}(x_j) = I_l(x_j)$. This violates Corollary 2. So this case cannot arise.

Therefore $S(1)$ is true.

(ii) Induction step
Assume $S(n)$ is true. If there are no more maximum separation violations in the resulting plan, (I_{ln}, I_{rn}), then it is the optimal plan. If there are more violations, we find the next violation. Apply Step 3 of the algorithm to get a new plan. Assume that there is another plan, which costs less time than the plan generated by Algorithm MAXSEPARATION. We consider three cases as in the base case and show by contradiction that there is no such plan. Therefore $S(n+1)$ is true whenever $S(n)$ is true.
Since the number of iterations of Steps 2 and 3 of the algorithm is finite (for each iteration, the left leaf must be stationary at x_j and there can be at most one iteration for each such x_j), all maximum separation violations will eventually be eliminated.

■

When the minimum separation constraint is also applicable, we can use Algorithm MINSINGLEPAIR in place of Algorithm SINGLEPAIR in Step 1 of Algorithm MAXSEPARATION. Note that in this case the minimum leaf separation of the plan constructed by Algorithm MAXSEPARATION is $min\{S_{ud-min(opt)}, S_{max}\}$. From Theorem 10, it follows that Algorithm MAXSEPARATION constructs an optimal plan that satisfies both the minimum and maximum separation constraints provided that $S_{ud-min(opt)} \geq S_{min}$. Note that when $S_{ud-min(opt)} < S_{min}$, there is no plan that satisfies the minimum separation constraint.

2.8. Generation of Optimal Leaf Movement Under Interdigitation Constraint

2.8.1. Introduction. We use a single pair of leaves to deliver intensity profiles defined along the axis of the pair of leaves. However, in a real application, we need to deliver intensity profiles defined over a 2-D region. We use Multi-Leaf Collimators (MLCs) to deliver such profiles. An MLC is composed of multiple pairs of leaves with parallel axes. Figure 15 shows an MLC that has three pairs of leaves - $(L1, R1), (L2, R2)$ and $(L3, R3)$. $L1, L2, L3$ are left leaves and $R1, R2, R3$ are right leaves. Each pair of leaves is controlled independently. If there are no constraints on the leaf movements, we divide the desired profile into a set of parallel profiles defined along the axes of the leaf pairs. Each leaf pair i then delivers the plan for the corresponding intensity profile $I_i(x)$. The set of plans of all leaf pairs forms the solution set. We refer to this set as the *treatment schedule* (or simply *schedule*).

In practical situations, however, there are some constraints on the movement of the leaves. As we have seen in Section 2.4.3, the minimum separation constraint requires

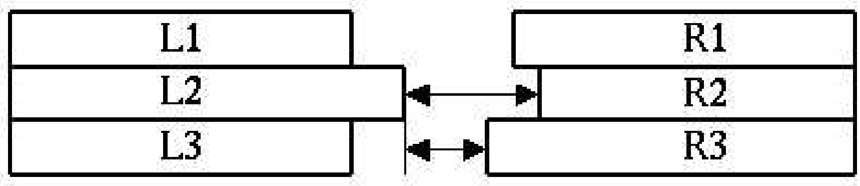

Figure 15. Inter-pair minimum separation constraint

that opposing pairs of leaves be separated by atleast some distance (S_{min}) at all times during beam delivery. In MLCs this constraint is applied not only to opposing pairs of leaves, but also to opposing leaves of neighboring pairs. For example, in Figure 15, $L1$ and $R1$, $L2$ and $R2$, $L3$ and $R3$, $L1$ and $R2$, $L2$ and $R1$, $L2$ and $R3$, $L3$ and $R2$ are pairwise subject to the constraint. We use the term *intra-pair minimum separation constraint* to refer to the constraint imposed on an opposing pair of leaves and *inter-pair minimum separation constraint* to refer to the constraint imposed on opposing leaves of neighboring pairs. The *inter-pair minimum separation constraint* with $S_{min} = 0$ is of special interest and is refered to as the *interdigitation constraint*. Recall that, in Section 2.4.3, we proved that for a single pair of leaves, if the optimal plan does not satisfy the minimum separation constraint, then no plan satisfies the constraint. In this section we present an algorithm to generate the optimal schedule for the desired profile defined over a 2-D region. We then modify the algorithm to generate schedules that satisfy the interdigitation constraint. Note that in our discussion on single pair of leaves (Section 2.2), we assumed that $I(x_0) > 0$ and that $I(x_m) > 0$. However, with multiple leaf pairs, the first and last sample points with non-zero intensity levels could be different for different pairs. Hence we will no longer make this assumption.

2.8.2. Optimal Schedule Without The Interdigitation Constraint. Assume we have n pairs of leaves. For each pair, we have m sample points. The input is represented as a matrix with n rows and m columns, where the ith row represents the desired intensity profile to be delivered by the ith pair of leaves. We apply Algorithm SINGLEPAIR to determine the optimal plan for each of the n leaf pairs. This method of generating schedules is described in Algorithm MULTIPAIR (Figure 16). Note that since x_0, x_m are not necessarily non-zero for any row, we replace x_0 by x_l and x_m by x_g in Algorithm SINGLEPAIR for each row, where x_l and x_g, respectively, denote the first and last non-zero sample points of that row. Also, for rows that contain only zeroes, the plan simply places the corresponding leaves at the rightmost point in the field (call it x_{m+1}).

Lemma 7 *Algorithm MULTIPAIR generates schedules that are optimal in therapy time.*

Proof: Treatment is completed when all leaf pairs (which are independent) deliver

Algorithm MULTIPAIR
For($i = 1; i \leq n; i++$)

Apply Algorithm SINGLEPAIR to the ith pair of leaves to obtain plan (I_{il}, I_{ir}) that delivers the intensity profile $I_i(x)$.

End For

Figure 16. Obtaining a schedule

their respective plans. The therapy time of the schedule generated by Algorithm MULTIPAIR is $max\{TT(I_{1l}, I_{1r}), TT(I_{2l}, I_{2r}), \ldots, TT(I_{nl}, I_{nr})\}$. From Theorem 3, it follows that this therapy time is optimal. ■

2.8.3. Optimal Algorithm With Interdigitation Constraint. The schedule generated by Algorithm MULTIPAIR may violate the interdigitation constraint. Note that no intra-pair constraint violations can occur for $S_{min} = 0$. So the interdigitation constraint is essentially an inter-pair constraint. If the schedule has no interdigitation constraint violations, it is the desired optimal schedule. If there are violations in the schedule, we eliminate all violations of the interdigitation constraint starting from the left end, i.e., from x_0. To eliminate the violations, we modify those plans of the schedule that cause the violations. We scan the schedule from x_0 along the positive x direction looking for the least x_v at which is positioned a right leaf (say R_u) that violates the inter-pair separation constraint. After rectifying the violation at x_v with respect to R_u we look for other violations. Since the process of eliminating a violation at x_v, may at times, lead to new violations involving right leaves positioned at x_v, we need to search afresh from x_v every time a modification is made to the schedule. We now continue the scanning and modification process until no interdigitation violations exist. Algorithm INTERDIGITATION (Figure 17) outlines the procedure.

Let $M = ((I_{1l}, I_{1r}), (I_{2l}, I_{2r}), \ldots, (I_{nl}, I_{nr}))$ be the schedule generated by Algorithm MULTIPAIR for the desired intensity profile.
Let $N(p) = ((I_{1lp}, I_{1rp}), (I_{2lp}, I_{2rp}), \ldots, (I_{nlp}, I_{nrp}))$ be the schedule obtained after Step iv of Algorithm INTERDIGITATION is applied p times to the input schedule M. Note that $M = N(0)$.

To illustrate the modification process we use examples. There are two types of violations that may occur. Call them Type1 and Type2 violations and call the corresponding modifications Type1 and Type2 modifications. To make things easier, we only show two neighboring pairs of leaves. Suppose that the $(p+1)$th violation occurs between the right leaf of pair u, which is positioned at x_v, and the left leaf of pair $t, t \in \{u-1, u+1\}$.

In a Type1 violation, the left leaf of pair t starts its sweep at a point $xStart(t, p) > x_v$ (see Figure 18). To remove this interdigitation violation, modify (I_{tlp}, I_{trp}) to $(I_{tl(p+1)}, I_{tr(p+1)})$ as follows. We let the leaves of pair t start at x_v and move them

Algorithm INTERDIGITATION

(i) $x = x_0$

(ii) While (there is an interdigitation violation) do

(iii) Find the least x_v, $x_v \geq x$, such that a right leaf is positioned at x_v and this right leaf has an interdigitation violation with one or both of its neighboring left leaves. Let u be the least integer such that the right leaf R_u is positioned at x_v and R_u has an interdigitation violation. Let L_t denote the left leaf with which R_u has an interdigitation violation. Note that $t \in \{u-1, u+1\}$. In case R_u has violations with two adjacent left leaves, we let $t = u - 1$.

(iv) Modify the schedule to eliminate the violation between R_u and L_t.

(v) $x = x_v$

(vi) End While

Figure 17. Obtaining a schedule under the constraint

at the maximum velocity v_{max} towards the right, till they reach $xStart(t,p)$. Let the number of MUs delivered when they reach $xStart(t,p)$ be I_1. Raise the profiles $I_{tlp}(x)$ and $I_{trp}(x)$, $x \geq xStart(t,p)$, by an amount $I_1 = \Phi * (xStart(t,p) - x_v)/v_{max}$. We get,

$$I_{tl(p+1)}(x) = \begin{cases} \Phi * (x - x_v)/v_{max} & x_v \leq x < xStart(t,p) \\ I_{tlp}(x) + I_1 & x \geq xStart(t,p) \end{cases}$$

$I_{tr(p+1)}(x) = I_{tl(p+1)}(x) - I_t(x)$, where $I_t(x)$ is the target profile to be delivered by the leaf pair t.

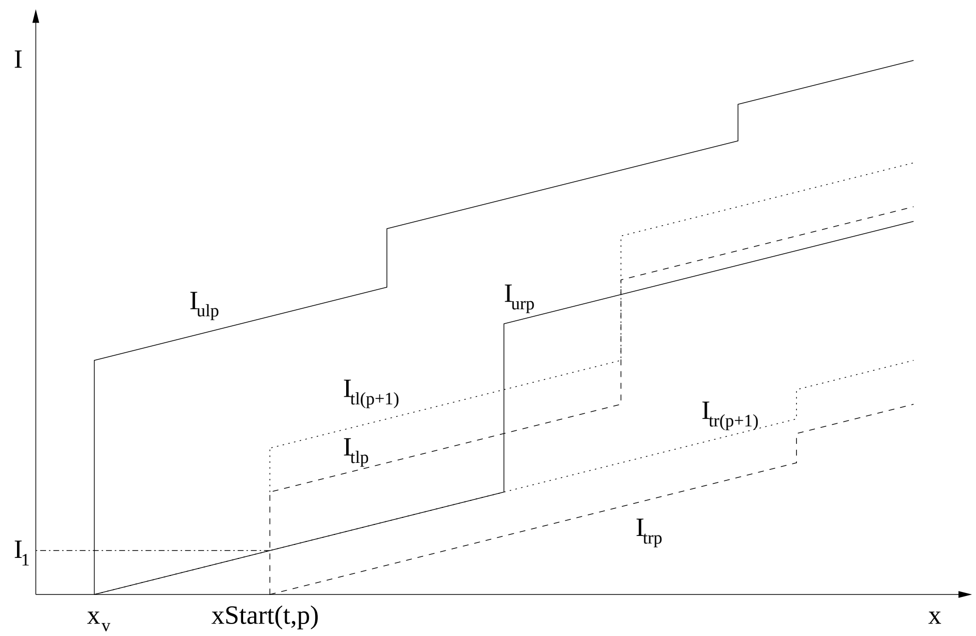

Figure 18. Eliminating a Type1 violation

A Type2 violation occurs when the left leaf of pair t, which starts its sweep from $x \leq x_v$, passes x_v before the right leaf of pair u passes x_v (Figure 19). In this case, $I_{tl(p+1)}$ is as defined below.

$$I_{tl(p+1)}(x) = \begin{cases} I_{tlp}(x) & x < x_v \\ I_{tlp}(x) + \Delta I & x \geq x_v \end{cases}$$

where $\Delta I = I_{urp}(x_v) - I_{tlp}(x_v) = I_3 - I_2$. Once again, $I_{tr(p+1)}(x) = I_{tl(p+1)}(x) - I_t(x)$, where $I_t(x)$ is the target profile to be delivered by the leaf pair t.

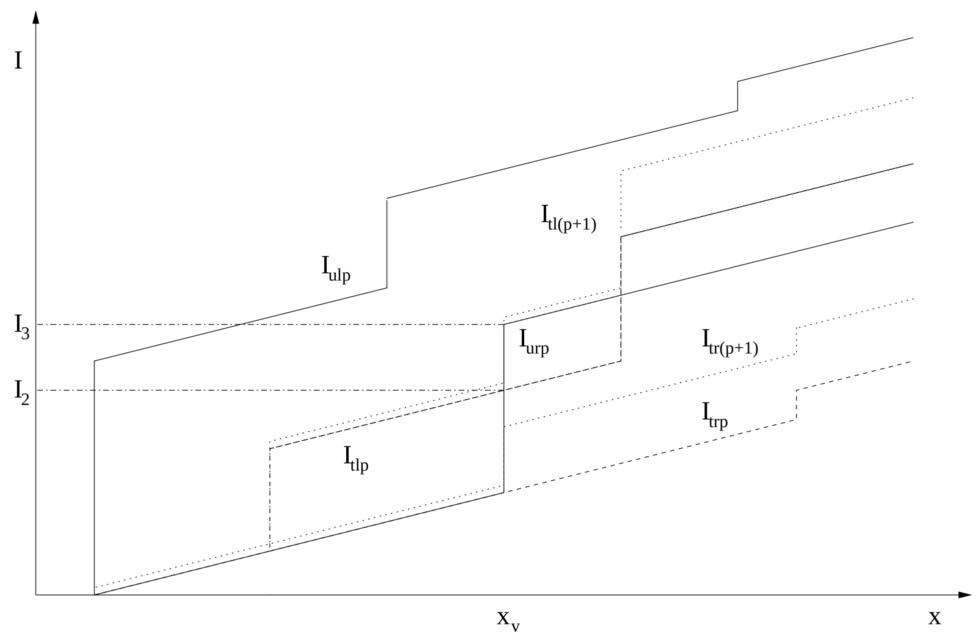

Figure 19. Eliminating a Type2 violation (close parallel dotted and solid line segments overlap, they have been drawn with a small separation to enhance readability)

In both Type1 and Type2 modifications, the other profiles of $N(p)$ are not modified. Since $I_{tr(p+1)}$ differs from I_{trp} for $x \geq x_v$ there is a possibility that $N(p+1)$ has inter-pair separation violations for right leaf positions $x \geq x_v$. Since none of the other right leaf profiles are changed from those of $N(p)$ and since the change in I_{tl} only delays the rightward movement of the left leaf of pair t, no interdigitation violations are possible in $N(p+1)$ for $x < x_v$. One may also verify that since I_{tl0} and I_{tr0} are feasible plans that satisfy the maximum velocity constrains, so also are I_{tlp} and I_{trp}, $p > 0$.

Lemma 8 $I_{jrp}(xStart(j,p)) = 0$, $1 \leq j \leq m$, $p \geq 0$.

Proof: The proof is by induction on p. Let $T(p)$ be the following statement: $I_{jrp}(xStart(j,p)) = 0$.

- For the base case, $p = 0$. (I_{jl0}, I_{jr0}) is the plan generated by Algorithm SINGLEPAIR and it satisfies the stated property.
- Assume that $T(p)$ is true. For the $(p+1)$th violation, we have the following two cases:

- The $(p+1)$th violation is a Type1 violation.
  A Type1 modification is applied. Such a modification always results in changing the start position of the leaves of pair t (as defined in Algorithm INTERDIGITATION) to x_v (which becomes $xStart(t, p{+}1)$) and $I_{tr(p+1)}(x_v) = 0$. For $j \neq t$, $I_{jr(p+1)}(xStart(j, p+1)) = I_{jrp}(xStart(j, p)) = 0$ by induction hypothesis.
- The $(p+1)$th violation is a Type2 violation.
  A Type2 modification is applied. Let t be as in Algorithm INTERDIGITATION. Suppose that $xStart(t, p) < x_v$. Since a Type2 modification does not alter the plan for $x < x_v$, $I_{tr(p+1)}(xStart(t, p+1)) = I_{tr(p+1)}(xStart(t, p)) = 0$. If $xStart(t, p) = x_v$, it must be the case that $xStart(u, p) = xStart(t, p)$ (as otherwise, there is a Type1 violation at $xStart(u, p) < x_v$). So the right leaf of pair u is not stopped at x_v. Hence, there is no interdigitation violation at x_v. So, the case $xStart(t, p) = x_v$ cannot arise. For $j \neq t$, the plan is unchanged. So, $I_{jr(p+1)}(xStart(j, p+1)) = I_{jrp}(xStart(j, p)) = 0$ by induction hypothesis.

■

Corollary 6 *A Type2 violation in which $I_{tlp}(x_v) = 0$ cannot occur.*

Proof: From the proof of Lemma 8, it follows that whenever there is a Type2 violation, $xStart(t, p) < x_v$. Hence, $I_{tlp}(x_v) > 0$. ■

Lemma 9 *In case of a Type1 violation, (I_{tlp}, I_{trp}) is the same as (I_{tl0}, I_{tr0}).*

Proof: Let p be such that there is a Type1 violation. Let t, u and v be as in Algorithm INTERDIGITATION. If (I_{tlp}, I_{trp}) is different from (I_{tl0}, I_{tr0}), leaf pair t was modified in an earlier iteration (say iteration $q < p$) of the while loop of Algorithm INTERDIGITATION. Let $v(q)$ be the v value in iteration q. If iteration q was a Type1 violation, then $xStart(t, p) \leq xStart(t, q+1) = x_{v(q)} \leq x_v$. So, iteration p cannot be a Type1 violation. If iteration q was a Type2 violation, $xStart(t, p) \leq xStart(t, q) \leq x_{v(q)} \leq x_v$. Again, iteration p cannot be a Type1 violation. Hence, there is no prior iteration q, $q < p$, when the profiles (I_{tl}, I_{tr}) were modified. ■

Lemma 10 *(a) A Type1 modification eliminates a Type1 violation.*
(b) A Type2 modification eliminates a Type2 violation.

Proof: (a) From Lemma 8, $I_{urp}(x_v) = 0$. By changing the start position of leaf pair t to x_v, we eliminate this violation.
(b) Follows from the construction of $(I_{tl(p+1)}, I_{tr(p+1)})$. ■

Note that $I_{ilp}(x)$ and $I_{trp}(x)$ are defined only for $x \geq xStart(t, p)$. In the sequel, we adopt the convention that $z \geq I_{ilp}(x)$ ($z \geq I_{irp}(x)$) is true whenever $I_{ilp}(x)$ ($I_{irp}(x)$) is undefined, irrespective of whether z is defined or not.

Lemma 11 *Let $F = ((I'_{1l}, I'_{1r}), (I'_{2l}, I'_{2r}), \ldots, (I'_{nl}, I'_{nr}))$ be any feasible schedule for the desired profile, i.e., a schedule that does not violate the interdigitation constraint. Let $S(p)$, be the following assertions.*

(a) $I'_{il}(x) \geq I_{ilp}(x)$*,* $0 \leq i \leq n, x_0 \leq x \leq x_m$

(b) $I'_{ir}(x) \geq I_{irp}(x)$*,* $0 \leq i \leq n, x_0 \leq x \leq x_m$

$S(p)$ *is true for* $p \geq 0$*.*

Proof: The proof is by induction on p.

(i) Consider the base case, $p = 0$. From Corollary 2 and the fact that the plans $(I_{il0}, I_{ir0}), 0 \leq i \leq n$, are generated using Algorithm SINGLEPAIR, it follows that $S(0)$ is true.

(ii) Assume $S(p)$ is true. Suppose Algorithm INTERDIGITATION finds a next violation and modifies the schedule $N(p)$ to $N(p+1)$. Suppose that the next violation occurs between the right leaf of pair u, positioned at x_v, and the left leaf of pair t. We modify pair t's plan for $x \geq x_v$, to eliminate the violation. All other plans in the schedule remain unaltered. Therefore, to establish $S(p+1)$ it suffices to prove that

$$I'_{tl}(x) \geq I_{tl(p+1)}(x), x \geq x_v \tag{4}$$

$$I'_{tr}(x) \geq I_{tr(p+1)}(x), x \geq x_v \tag{5}$$

We need prove only one of these two relationships since $I'_{tl}(x) - I'_{tr}(x) = I_{tl(p+1)}(x) - I_{tr(p+1)}(x), x_0 \leq x \leq x_m$. We now consider pair t's plan for $x \geq x_v$. Note that the $(p+1)$th violation may either be a Type1 or Type2 violation. We show that Equation 4 is true in both cases. This, in turn, implies that $S(p+1)$ is true whenever $S(p)$ is true and hence completes the proof. Note that in $(I_{tl(p+1)}, I_{tr(p+1)})$, the leaves move at maximum speed between adjacent sample points. So, it is sufficient to show Equation 4 for sample points $\geq x_v$.

(a) The $(p+1)$th violation is a Type1 violation.
From $S(p)$ it follows that $I'_{ur}(x_v) \geq I_{urp}(x_v)$. So, the right leaf of pair u leaves x_v no earlier in I'_{ur} than it does in I_{urp}. From this and the fact that F satisfies the interdigitation constraint, we conclude that leaf pair t cannot begin its sweep at the right of x_v. This observation together with the fact that in $(I_{tl(p+1)}, I_{tr(p+1)})$ the leaves move at the maximum velocity from x_v to $x' = xStart(t,p)$ implies that $\hat{I}'_{tl}(x') \geq \hat{I}_{tl(p+1)}(x')$ and $\hat{I}'_{tr}(x') \geq \hat{I}_{tr(p+1)}(x')$, where $\hat{I}$ denotes an arrival time. Now, from Lemma 8, we get $I'_{tr}(x') \geq \hat{I}'_{tr}(x') \geq \hat{I}_{tr(p+1)}(x') = I_{tr(p+1)}(x')$. So $I'_{tl}(x') = I'_{tr}(x') + I_t(x') \geq I_{tr(p+1)}(x') + I_t(x') = I_{tl(p+1)}(x')$. From this and the fact that the left leaf of pair t moves at the maximum velocity between x_v and x', it follows that Equation 4 holds for all x between x_v and x'. To prove that Equation 4 holds for all sample points to the right of x' (and so holds for all x between x_0 and x_m), consider a sample point x_w that is to the right of x'. Let $\Delta I' = I'_{tl}(x') - I_{tl(p+1)}(x') \geq 0$ and let I_1 be as in Algorithm INTERDIGITATION. Define a new plan (I''_{tl}, I''_{tr}) for leaf pair t as below

$$I''_{tl}(x) = \begin{cases} undefined & x < x' \\ I'_{tl}(x) - \Delta I' - I_1 & x \geq x' \end{cases}$$

$$I''_{tr}(x) = \begin{cases} undefined & x < x' \\ I'_{tr}(x) - \Delta I' - I_1 & x \geq x' \end{cases}$$

Note that $I''_{tl}(x') = I'_{tl}(x') - \Delta I' - I_1 = I_{tl(p+1)}(x') - I_1 = I_{tlp}(x') \geq 0$. Similarly, $I''_{tr}(x') \geq 0$. Hence (I''_{tl}, I''_{tr}) is a plan for I_t. Also, $I''_{tl}(x_w) = I'_{tl}(x_w) - \Delta I' - I_1 \leq I'_{tl}(x_w) - I_1$. If $I'_{tl}(x_w) < I_{tl(p+1)}(x_w)$, $I''_{tl}(x_w) < I_{tl(p+1)}(x_w) - I_1 = I_{tlp}(x_w) = I_{tl0}(x_w)$ (Lemma 9). This contradicts Corollary 2. Hence, $I'_{tl}(x_w) \geq I_{tl(p+1)}(x_w)$.

(b) The $(p+1)$th violation is a Type2 violation.
The situation is illustrated in Figure 19. Since F satisfies the interdigitation constraint, the left leaf of pair t does not pass x_v before the right leaf of pair u passes x_v. So,

$$I'_{tl}(x_v) \geq I'_{ur}(x_v) \tag{6}$$

From $S(p)$ and the definition of a Type2 modification, we get,

$$I'_{ur}(x_v) \geq I_{urp}(x_v) = I_{tl(p+1)}(x_v) \tag{7}$$

Equations 6 and 7 yield

$$I'_{tl}(x_v) - I_{tl(p+1)}(x_v) \geq 0 \tag{8}$$

Lemma 4b implies,

$$I'_{tl}(x) - I_{tl}(x) \geq I'_{tl}(x_v) - I_{tl}(x_v), x \geq x_v \tag{9}$$

(Lemma 4b yields Equation 9 only for $x \geq x_v$ and x is a sample point. From this and the fact that the left leaf moves at maximum velocity in I_{tl} between adjacent sample points, we get Equation 9 for all x, $x \geq x_v$.)
From Equation 9, we get

$$I'_{tl}(x) - I_{tl(p+1)}(x) \geq I'_{tl}(x_v) - I_{tl}(x_v) + I_{tl}(x) - I_{tl(p+1)}(x), x \geq x_v \tag{10}$$

From the definitions of Type1 and Type2 modifications and the working of Algorithm INTERDIGITATION, it follows that

$$I_{tl(p+1)}(x) - I_{tl}(x) = I_{tl(p+1)}(x_v) - I_{tl}(x_v), x \geq x_v \tag{11}$$

From Equations 10, 11 and 8, we get

$$I'_{tl}(x) - I_{tl(p+1)}(x) \geq I'_{tl}(x_v) - I_{tl(p+1)}(x_v) \geq 0, x \geq x_v \tag{12}$$

Therefore,

$$I'_{tl}(x) \geq I_{tl(p+1)}(x), x \geq x_v \tag{13}$$

■

Lemma 12 *For the execution of Algorithm INTERDIGITATION*
(a) $O(n)$ Type1 violations can occur.
(b) $O(n^2m)$ Type2 violations can occur.
(c) Let T_{max} be the optimal therapy time for the input matrix. The time complexity is $O(mn + n * min\{nm, T_{max}\})$.

Proof: (a) It follows from Lemma 9 that each leaf pair can be involved in at most one Type1 violation as pair t, i.e, the pair whose profile is modified. Hence, the number of Type1 violations is $\leq n$.
(b) We first obtain a bound on the number of Type2 violations at a fixed x_v. Let u, t be as in Algorithm INTERDIGITATION. Note that u is chosen to be the least possible index. Let u_i be the value of u in the ith iteration of Algorithm INTERDIGITATION at x_v. t_i is defined similarly. Let $u_i^{max} = max_{j \leq i}\{u_j\}$. If $t_i = u_i - 1$, it is possible that $u_{i+1} = t_i = u_i - 1$ and $t_{i+1} = u_i - 2$. Note that in this case, $t_{i+1} \neq u_i = u_{i+1} + 1$. Next, it is possible that $u_{i+2} = u_i - 2$ and $t_{i+2} = u_{i-3}$ (again $t_{i+2} \neq u_i - 1 = u_{i+2} + 1$). In general, one may verify that $t_i = u_i + 1$ is possible only if $u_i^{max} = u_i$. If $t_i = u_i + 1$, then $u_{i+1} \geq t_i = u_i + 1$, since the violation between u_i and t_i has been eliminated and no profiles with an index less than t_i have been changed during iteration i at x_v. It is also easy to verify that $t_i = 1, u_i = 2 \Rightarrow u_{i+1} \geq u_i^{max}, u_{i+2}^{max} > u_i^{max}$. From this and $t_i \in \{u_i + 1, u_i - 1\}$ it follows that $u_{i+u_i^{max}}^{max} > u_i^{max}$. We know that $u_1^{max} \geq 1$. It follows that $u_2^{max} \geq 2$, $u_4^{max} \geq 3$, $u_7^{max} \geq 4$ and in general, $u_{(i(i+1)/2)+1}^{max} \geq i + 1$. Clearly, for the last violation (say jth) at x_v, $u_j^{max} \leq n$ and for this to be true, $j = O(n^2)$. So the number of Type2 violations at x_v is $O(n^2)$. Since x_v has to be a sample point, there are m possible choices for it. Hence, the total number of Type2 violations is $O(n^2m)$.
(c) Since the input matrix contains only integer intensity values, each violation modification raises the profile for one pair of leaves by at least one unit. Hence, if T_{max} is the optimal therapy time, no profile can be raised more than T_{max} times. Therefore, the total number of violations that Algorithm INTERDIGITATION needs to repair is at most nT_{max}. Combining this bound with those of (a) and (b), we get $O(min\{n^2m, nT_{max}\})$ as a bound on the total number of violations repaired by Algorithm INTERDIGITATION. By proper choice of data structures and programming methods it is possible to implement Algorithm INTERDIGITATION so as to run in $O(mn + n * min\{nm, T_{max}\})$ time. ∎

Note that Lemma 12 provides two upper bounds of on the complexity of Algorithm INTERDIGITATION: $O(n^2m)$ and $O(n * max\{m, T_{max}\})$. In most practical situations, $T_{max} < nm$ and so $O(n * max\{m, T_{max}\})$ can be considered a tighter bound.

Theorem 11 *The following are true of Algorithm INTERDIGITATION:*

(a) The algorithm terminates.

(b) The schedule generated is feasible and is optimal in therapy time.

Proof: (a) Lemma 12 provides a polynomial upper bound ($O(n^2m)$) on the complexity of Algorithm INTERDIGITATION. The result follows from this.

(b) When the algorithm terminates, no interdigitation violations remain and the final schedule is feasible. From Lemma 11, it follows that the final schedule is optimal in therapy time.

∎

3. Conclusion

DMLC-IMRT has the advantage that it can be delivered in shorter times compared to SMLC-IMRT especially for complex intensity modulated field. We have presented mathematical formalisms and rigorous proofs of leaf sequencing algorithms for dynamic multileaf collimation that maximize MU efficiency. These leaf sequencing algorithms explicitly account for leaf interdigitation constraint. We have shown that our algorithms obtain feasible solutions that are optimal in treatment MUs. Furthermore, our analysis shows that unidirectional leaf movement is at least as efficient as bi-directional movement. Thus these algorithms are well suited for common use in DMLC beam delivery.

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