Parallel Computing 12 (1989) 53–69
North-Holland

VLSI architectures for back substitution *

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Received August 1987
Revised January 1989

Abstract. VLSI architectures involving unidirectional, bidirectional and broadcast chains as well as unidirectional rings are examined for the back substitution problem. We review some of the earlier designs and also develop new designs with superior performance.

Keywords. VLSI architectures, systolic systems, back substitution.

1. Introduction

VLSI architectures for a variety of problems have been proposed by several authors. A bibliography of over 150 research papers dealing with this subject appears in [5]. In this paper, we are concerned solely with the back substitution problem. The inputs to this problem are a non-singular lower triangular matrix A and a column vector b. The objective is to determine the unique column vector x with the property $Ax = b$. Throughout this paper, we assume that A is $n \times n$. So, x and b are $n \times 1$. The classical approach to obtain x is to use back substitution. [1] x is obtained using the formula

$$x_i = \left(b_i - \sum_{j=1}^{i-1} a_{ij} x_j \right) / a_{ii}, \quad 1 \leqslant i \leqslant n.$$

The x_i's are computed in the order $x_1, x_2, \ldots, x_n$. The matrix A is assumed to be dense below the diagonal. As a result, we do not develop any special methods to handle sparsity or bandedness. Furthermore, it should be noted that the factorization process generally used to obtain A may ensure that A is unit lower triangular, i.e., all diagonal elements are 1. In this case, the divisions by a_{ii}, above, may be eliminated.

VLSI architectures for the back substitution problem have been proposed earlier in [2,4,6–8]. The design of [2] employs a unidirectional chain of processors as in Fig. 1(a). The data flow is left to right. The design of Kung and Leiserson [4,7,8] employs a bidirectional chain as in Fig. 1(b). Here, data is permitted to flow both from left to right and from right to left. In [6], a ring architecture such as in Fig. 1(c) is proposed for this problem. Data can flow in only a single direction (either clockwise or counterclockwise) around the ring.

* This research was supported in part by the National Science Foundation under grant MCS-83-05567 and MIP-86-17374.

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[1] Strictly speaking, we are really doing forward substitution and not back substitution. In back substitution, A is upper triangular. However, forward and back substitution are quite similar and we prefer to use the generic term "back substitution" here.