CHAPTER 51

TESTING

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51.1 INTRODUCTION

Test data is a set of inputs that may be provided to a program to check its behavior. For example, each run of the quadratic roots program, Program 1.30 of the text, requires values for the three variables: $a$, $b$, and $c$. A possible set of inputs that can be used to test this program is $(2, 3, 4)$ (i.e., $a = 2$, $b = 3$, and $c = 4$). So, $(2, 3, 4)$ may be used as test data for this program. A test data set (often abbreviated test set) is a collection of test data. For the quadratic roots program, \{(2, 3, 4), (1, 5, −2), (3.2, 4.5, 8.2), (−2.6, −9.1, 8.4)\} is a possible test set. If this test set is used, the program will be executed four times; once with each of the four sets of inputs in the test set.

Testing is the process of executing the program code in the target environment using test data. The behavior of the program on this test data is compared with that predicted by the program specifications. Hence, for a set of inputs to be
used as test data, it is essential that the behavior of a correct program run with that set of inputs be known. Thus, to use the above test set for Program 5.3, we must know the roots of each of the four quadratic equations defined by the four sets of input data.

The number of different inputs that can be provided to a program is generally so large that no practical amount of testing can establish the correctness of the program. For the quadratic roots problem, the number of different quadratic equations that can be input is infinite in theory. In practice, this number is finite though very large. If 16 bits are used to represent each of the inputs $a$, $b$, $c$ (including sign, magnitude, and exponent), then only $2^{16}$ different values are possible for each. Hence, the number of different sets of inputs is only $2^{48}$. If our target computer can run Program 1.30 $2^{20} = 1,048,576$ times a second, it will take $2^{28}$ seconds $\approx 8.5$ years to try out all $2^{48}$ sets of inputs. It is impractical to test Program 1.30 on all possible inputs that may be provided to it. Hence, testing must often be limited to a (very small) subset of all possible inputs. Testing with this subset cannot conclusively establish the correctness of the program. As a result, the objective of testing is not to establish correctness but to expose the presence of errors. The test set must be chosen in such a way as to expose any errors that may be present in the program.

To test a program, we need the following:

1. Access to the target environment. This environment consists of both the hardware and software environment under which the program is to work.
   a. The hardware environment is clearly important. A program that runs correctly on a PC may malfunction on a workstation (or the reverse). A program may function correctly on a PC that has 32MB of memory but fail on one that has only 16MB of memory. The screen utilities may work correctly on a PC with a medium resolution monitor but incorrectly on the same computer with a high resolution monitor.
   b. The software environment includes the operating system under which the program is to run. It is quite possible for a program to function correctly under one operating system on a particular computer and fail to run under a different operating system on the same computer. For example, a program that runs correctly under Windows 95 may fail to run correctly when the operating system is changed to Windows NT. This may occur even though no change is made to the hardware environment. In fact, it is quite possible for a program to run correctly under one version of an operating system and to produce errors on another version of the same operating system. A C++ program may compile correctly using one compiler but may generate compiler errors when another compiler is used.

For the above reasons, it is essential to test the program in the environment in which it is to be eventually used.
2. When testing a program that consists of several modules (e.g., Figure 51.1), a test strategy is needed. There are essentially three different strategies we can follow to test this program. These are, respectively, called big bang testing, big bang integration testing, and incremental testing. In big bang testing, one attempts to test the entire program all at once. In big bang integration testing, the modules A-J are tested separately. When this testing is complete, all modules are put together and tested as a whole. In incremental testing, modules are integrated as they are tested.

3. Since testing involves the execution of the program (or program module) in the target environment, we need test data. Generally, the test data will consist of several sets of input data. Each set of input data is used on a different execution of the program. This data must be designed with care so as to have a high probability of exposing the errors that may exist in the program being tested. Our goal is to design as small a set of test data as needed to expose all the errors in the program. In order for an input data set to be usable as test data, we must know the behavior of a correct program on this data. This correct behavior can then be compared with the observed behavior. Any deviations from the correct behavior will signal the presence of errors in the program.

![Diagram of program modules](image-url)
51.2 MODULE TESTING STRATEGIES

51.2.1 Big Bang Testing

In big bang testing, all testing is performed on the program as a whole. When this approach is used on the program of Figure 51.1, the entire program is compiled and executed with test data. If the behavior of the program disagrees with the expected behavior on any one of the test data, then the cause of this discrepancy may be in any of the modules A-J. To detect this cause, one has to check the logic and interface of each of the modules in some systematic order. Once the cause of the error has been identified and removed, the program is recompiled and run on the test data.

Some of the disadvantages of big bang testing are:

1. One has to handle module interfacing problems as well as logic errors within a module at the same time. Module interfacing problems include such things as a discrepancy between the number and/or type of formal and actual parameters, discrepancy in the form in which a module expects data (say sorted) and the form in which it is actually provided (say unordered) by the invoking module, formal parameters that need to pass computed values back to the invoking module may not have been identified as being of type var in the module declaration, etc.

2. When the behavior of the program on a test run differs from the expected behavior, the cause of the error could be in any of the modules. Finding this cause requires the tester to trace through the entire program. Hence, debugging is expensive.

3. Big bang testing is expensive in terms of the computer time and memory required. Each time a bug is found and “fixed”, the entire program gets recompiled (unless an incremental compiler is in use). For large programs, this recompilation is very expensive. When an incremental compiler is in use, only the module (or part of the module) that has been changed needs to be recompiled. Still, to test the corrected program, the entire program is loaded into memory and run. The result may be the detection of another bug in the same module as earlier. It would be cheaper to get as many bugs as possible out of each of the modules by testing them separately and then testing the program as a whole.

51.2.2 Big Bang Integration Testing

In big bang integration, one tests each of the modules independently. Each of the 10 modules A-J of Figure 51.1 are tested independent of the others. When this testing has been completed, the modules are put together and the entire program tested. Since the individual modules have been tested before being
integrated together, one expects that all logic errors within the modules have been detected and corrected before the testing of the integrated program commences. Consequently, when the integrated program is tested, one only expects to discover problems related to the interfaces. In practice, because testing is limited to a subset of all possible inputs, logic errors within a module may be detected even when the integrated program is being tested.

The big bang integration testing strategy clearly overcomes some of the difficulties associated with big bang testing. We make the following observations with regard to this test strategy:

1. Ideally, all logic errors in a module will be detected when the module is being tested independent of the others. Debugging is easier as it is localized to the module under test.

2. Module interface problems have to be dealt with only after individual modules have been tested. In the ideal case, all bugs found when the integrated program is being tested will be related to module interface problems alone.

3. The testing of large programs using this approach is expected to require less computer time. The full memory requirements of the program are needed only when the integrated program is being tested.

4. There is significant potential for parallel testing. To test one module, we need not wait for the testing of another to complete. Given enough personnel, all modules can be tested in parallel. So, this phase of the testing can be completed quickly.

Big bang integration testing, however, has problems of its own. One of these is a leftover from big bang testing. All problems associated with integrating the modules are tackled simultaneously. Thus, while resolving the interface problems between modules B and F, the entire program is being repeatedly compiled and executed. It is cheaper to resolve these problems independent of the problems between the remaining interfaces.

A more subtle difficulty with this approach is one that doesn’t exist when the big bang approach is used. This has to do with actually testing one module independent of the others. For example, to test the module B of Figure 51.1, we need to write three modules. First, we need to write a module that invokes B. This module is called a driver. In some cases, it is sufficient for the driver module to simply contain a statement to invoke B. In other cases, the driver needs to first set up the environment that B is to work with and then when B has completed, it is necessary for the driver to output the results (if any) for comparison with the expected results.

In addition to the driver for B, we need modules that simulate the modules F and G. These modules are called stubs. The stub modules need to provide the results that would otherwise have been provided by modules F and G.
To underscore the need for stubs that faithfully simulate their respective modules, consider the module fragment:

```plaintext
ModuleA(x,y);
z = x + y * y;
ModuleB(x/z, r);
p = q / r;
ModuleC(p, q, r, x/y);
```

If the stub for `ModuleA` simply proclaims that it has been reached, an error such as `x` (or `y`) undefined may be obtained from line 2. This error may be spurious in that it might be `ModuleA`'s function to appropriately define these variables. Similarly, a division by zero error that may be produced at line 3 may be the result of not simulating `ModuleA` properly. Errors involved with invoking `ModuleC` from line 5 may be caused solely by deficiencies in the stubs for `ModuleA` and `ModuleB`. Drivers and modules are to be written in such a way that all runtime errors that are produced (whether they be errors that cause abnormal termination of the program or whether they result in incorrect output) be attributable to causes other than the drivers and stubs.

### 51.2.3 Incremental Testing

In *incremental testing*, module integration is carried out in parallel with module testing. Further, individual modules are not tested independent of all other modules. Rather, when testing a particular module we make use of already tested modules that may either invoke or be invoked by the module currently being tested. So, for example, if modules A and B (Figure 51.1) have been tested and integrated and modules H and I tested by the time we are ready to test module D, then we do not need a driver module for module D. Rather, the tested module A is used to invoke D (in case the output modules haven’t yet been tested and integrated, it may be necessary to add some code to A to output results). Stubs for C and E may be needed but not for H and I. In place of stubs for H and I, we use the tested modules H and I. During the testing of D, we shall be concerned primarily with logic errors in D as well as with integration problems between A and D, D and H, and D and I.

Incremental testing has some advantages over big bang integration testing. Using this approach, the problems related to some of the module interfaces are detected earlier and so are easier to correct. Further, modules that get tested early are included in all further tests. Hence, these modules get exercised more than they would otherwise. Consequently, the degree of testing is greater for these modules. A possible disadvantage of incremental testing is some loss in ability to test modules in parallel. This is more than compensated for by the advantages stated earlier. Incremental testing is generally to be preferred over the other two test strategies.
When carrying out an incremental test of a program, one has to determine the order in which the modules are to be tested and integrated. Two popular strategies for this are: top down and bottom up. These are discussed below:

**Top Down Incremental Testing**

In *top down incremental* testing, one begins by testing the root module. To do this for the program of Figure 51.1, one needs stubs for modules B, C, D, and E. These stubs replace the corresponding modules in Figure 51.1, and the program of Figure 51.2 is tested. Once the program of Figure 51.2 has been tested, we may replace any one of the stubs B, C, D, and E by its corresponding module and test the resulting program. If we choose to replace stub D, then we test the program shown in Figure 51.3. Notice that this test requires us to write two additional stubs (those for H and I). When the program of Figure 51.3 has been tested, we may replace any of the stubs in this program by the corresponding module and perform tests on the resulting program. This process is continued until all stubs have been replaced by modules and the fully integrated program tested.

![Figure 51.2 Testing module A in the top down approach](image)

Notice that in the top down approach, no drivers are to be written. Only stubs are needed. When deciding which of several stubs to replace by its corresponding module, one may use the following guidelines:

1. Choose a stub replacement sequence that allows you to bring in the I/O (input/output) modules as soon as possible. This will make it easier to input further test cases and also to see the results of these tests.
2. Once the I/O modules have been included, choose a stub replacement sequence that allows you to bring in modules that have the highest probability of containing errors or those that may have the errors that are most difficult to fix as early as possible. This results in the most error prone modules being tested more thoroughly than the less error prone ones. Additionally, the probability of detecting serious errors early in the testing
Figure 51.3 Testing module D in the top down approach

phase is enhanced.

Our guidelines favor the early testing of I/O modules as once these have been tested and integrated, the testing of the other modules becomes easier.

Bottom Up Incremental Testing

This approach is the reverse of the top down approach. Here, we begin with the leaf modules. Any of the modules F-J of Figure 51.1 may be tested first. In fact, if parallel testing of modules is possible, then all of these may be tested in parallel (by possibly different persons). To test a leaf module, a driver module is needed. However, no stub is required.

In the bottom up approach, a module can be tested only after all its children modules have been tested. Some of the possible sequences in which the modules of Figure 51.1 may be tested using the bottom up approach are:

F, G, H, I, J, B, C, D, E, A  
H, I, D, F, G, B, J, C, E, A  
G, F, C, B, H, I, D, J, E, A  

etc.

When any of the above sequences is used, we need to write driver modules but no stub modules. Like the top down approach, the bottom up approach requires us to choose the next module to be tested. The guidelines for this choice are the same as those for the top down approach. I.e., test the I/O modules, the modules that have the highest probability of containing errors, and those modules that may contain the most difficult to fix errors as early as possible.
In comparing the two incremental testing approaches, we note that stubs are generally harder to write than drivers (especially if the I/O modules get tested first in the bottom up approach). This is a strong reason to favor the bottom up approach over the top down approach. When most errors are expected to be in the higher level modules, one may favor the top down approach. The top down approach has the advantage that as each module gets tested and integrated, more of the program becomes available for use. In the bottom up approach, the root module is tested last. As a result, the tested portions of the program cannot be released for use until all testing is complete.

Using the bottom up approach, a good module test sequence for the rat in a maze program (Program 7.1) is: first test the screen utilities, then test the remaining modules in the order: welcome, InputMaze, OutputPath, FindPath. This order results from the guidelines provided above. We have favored the I/O modules over the most error prone one (FindPath) because testing FindPath is much easier when we have our I/O modules working.

**EXERCISES**

1. Describe two of the ways in which a top down incremental testing of the program of Figure 51.1 may proceed. In each case, state which stubs and drivers are needed.

2. Do the previous exercise for bottom up incremental testing.

### 51.3 GENERATION OF TEST DATA

#### 51.3.1 Introduction

When developing test data, one should keep in mind that the objective of testing is to expose the presence of errors. This is so as no practical amount of testing can assure us of their absence. If data designed to expose errors fails to expose any errors, then we may have confidence in the correctness of the program. In order to be able to tell whether or not a program malfunctions on a given test data, we must know what the correct response to this data is. Hence, we may evaluate any candidate test data on the following criteria:

3. What is this data’s potential to expose errors?

4. Do we know what the correct response to this data is?

The techniques available for test data generation fall into two categories: black box methods, and white box methods. In a black box method, test data is developed by considering only the function served by the program (or program module) to be tested. The development of the test data is done without regard to the actual code that realizes the program (or module). Hence, test data that results from a black box method is obtained from a functional analysis of the program (or module).
One can make a distinction between a requirements based black box method and a design based one. In the former, the test data is arrived at by analyzing the problem specifications alone. In the latter, the resulting design (but not the code) is analyzed. We shall not make this distinction here.

The most popular black box methods are: defensive programming methods, I/O partitioning, and cause - effect graphing. These will be studied in detail in subsequent sections.

In a white box method, a structural analysis of the program (or module) is performed. The code is examined and an attempt is made to develop test data whose execution results in a ‘‘good’’ coverage of the program instructions. What we mean by good coverage will be elaborated on in a subsequent section.

51.3.2 Black Box Methods

51.3.2.1 Defensive Programming Methods

In the chapter on defensive programming, we pointed out the most common causes of failure of a program known to be generally healthy. These are: input errors, numerical errors, and boundary errors. Our test set should include input that will expose program failure in the presence of errors of each of these types.

Some examples of testing for program correctness in the face of input errors are:

1. If a program expects to input \( n \) data items, see what happens when fewer or more items are made available.
2. If the program expects input in a particular order, see what happens when input is provided in a different order.
3. The test set for the quadratic roots program (Program 1.30) should include at least one test data in which \( a = 0 \). Even though this corresponds to invalid input (as per the specifications), the program should terminate gracefully. Unfortunately, Program 1.30 does not do this. A division by zero exception results rather than a message such as ‘‘This program does not handle the case \( a = 0 \)’’.

Remember that the objective of testing a program with intentional input errors is to ensure that it either terminates gracefully or it successfully recovers from these errors. One can be sure that at some time or other, incorrect data will be provided to the program. It is essential that the program not crash when this happens. Even more important, the program should not behave as if all is well and produce results that appear correct but which, in fact, are not.

Testing for the presence of numerical errors is to be done whenever the program uses real-valued data. For this, the general rules are:
1. Use data with an inexact representation in the target computer.
2. Use data with a wide range of magnitude and also with sign changes.

Often, a program works correctly so long as it is not asked to operate on a problem boundary. To expose errors resulting from program operation at a boundary, one must specifically design test data to exercise the program at the boundary. Some examples of this are:

1. A procedure to insert \( x \) into a nondecreasing sequence \( a[1] \leq a[2] \leq \ldots \leq a[n] \) should be tested at the following boundaries:
   a. \( n = 0 \). I.e., insertion into an empty sequence.
   b. \( n = \) maximum size of list. I.e., insertion into a full sequence.
   c. \( x < a[1] \). I.e., insertion at the left end.
   d. \( x > a[n] \). I.e., insertion at the right end.

2. A procedure to find the maximum of \( n \) elements should be tested when \( n = 0 \) and 1.
3. A procedure to delete from a list should be tested on an empty list and also on a list with one element. The latter test will verify that deletion from a one element list actually leaves behind a proper empty list.

### 51.3.2.2 I/O Partitioning

The total number of different inputs that can be given to a program is usually too large to permit an exhaustive testing of the program. To arrive at a reasonably small number of test cases, one can partition the input domain into a set of classes with the following properties:

1. Every member of the input domain is in some class.
2. The classes are disjoint. I.e., no member of the input domain is in two or more classes.
3. If an error is detected by one member of a class, then the same error will be detected by all other members of that class.

From criterion (3), we conclude that a test set need include only one member from each of the created input classes. In practice, it is impossible to ensure criterion (3) without a careful and expensive examination of the code. Since we are using I/O partitioning as a black box method, examination of the code is not permitted. Consequently, we relax criterion (3) to:

(3') If an error is detected by one member of a class, then there is a good likelihood that it will be detected by any other member of the class.
Intuitively, criterion (3’) calls for us to group together members of the input domain that we expect will be handled in more or less the same way by the program. Implicit in this is the requirement that inputs that produce materially different output be placed in different classes. Hence, input partitioning also results in a partitioning of the output space. In fact, the partitioning of the input domain is often obtained by first partitioning the output domain. We shall soon see several examples of this.

Because of the relaxation (3’), we cannot be sure that a program that works correctly on one member of each partition will work correctly on all inputs. So, we change our requirement on the test set from exactly one member from each partition to at least one member from each partition.

Several examples of I/O partitioning are given below. As we shall see, I/O partitioning often results in a separate partition for the boundary data. In situations where this is the case, there is an overlap between the effort expended in developing test data when using the defensive programming methods and the I/O partitioning method. In other cases, the test data developed when identifying problem boundaries may actually correspond to data on the boundary of one or more partitions rather than data in a separate partition.

1. Suppose that we are to test a program to find \( \max\{x, y, z\} \) where \( x, y, \) and \( z \) are distinct integers. In case \( x, y, \) and \( z \) are not distinct, an error is to be generated. From the problem specifications, the following partitioning of the output space is obtained:
   a. \( x \)
   b. \( y \)
   c. \( z \)
   d. Error

   We can reasonably expect any program that solves this problem to handle inputs that result in outputs that are in different output partitions differently. Hence, the input domain may be partitioned as below:
   a. \{all distinct integers \( x, y, \) and \( z \) such that \( x \) is the maximum\}
   b. \{all distinct integers \( x, y, \) and \( z \) such that \( y \) is the maximum\}
   c. \{all distinct integers \( x, y, \) and \( z \) such that \( z \) is the maximum\}
   d. \{all integers \( x, y, \) and \( z \) such that at least two are the same\}

   Hence, our test set should include at least one member of each of these four classes. For instance, we could try the four data sets: \( (8, 2, 3), (1, 7, 2), (2, 1, 3), (8, 1, 8) \). In case we expect some input instances in any one of the above partitions to be handled differently from others in the same partition, then this partition should be further partitioned.
2. We wish to test a menu that has the five options: A, B, C, D, and E. The input domain is partitioned into the six partitions: \{A\}, \{B\}, \{C\}, \{D\}, \{E\}, \{all other inputs\}. The last partition contains all the invalid inputs that might be provided. If there is reason to suspect that the menu module handles members of the invalid partition differently, then this partition must be further partitioned.

3. Consider the root finding program (Program 1.30). From the problem specification, we see that there are three possible different outcomes from the resulting program:
   a. The quadratic has only one distinct real root ($b^2 - 4ac = 0$).
   b. There are two distinct real roots ($b^2 - 4ac > 0$).
   c. There are two distinct complex roots ($b^2 - 4ac < 0$).

   This results in a partitioning of the inputs into three partitions; one for each of three possible outcomes. In addition, we should add one partition for invalid inputs. This partition consists of all inputs $(a, b, c)$ with $a = 0$. Our test set should contain at least one member of each of these four partitions. Notice that a division by zero error is produced when Program 1.30 is run with any member of the invalid set as input.

4. When partitioning the I/O domain for a list insertion module, we get the two partitions: full list, and nonfull list. We expect the module to handle additions to all nonfull lists in the same way. So, we need to test the add module with a full list and with at least one nonfull list.

5. Suppose that a sort module has been written to sort $n$ numbers for $n$ in the range $[1, 1000]$. The input domain is naturally partitioned into the partitions: $n < 1$, $1 \leq n \leq 1000$, and $n > 1000$. Notice that the valid inputs are in one partition and the invalid inputs have been partitioned into two. If we suspect that the program may handle the cases $n = 1$ and $n = 1000$ differently, then the partition for the valid inputs should be further partitioned into three. Observe that the cases $n = 1$ and $n = 1000$ correspond to the boundaries of the partition $1 \leq n \leq 1000$ and will be isolated for testing by one of the defensive programming test methods. Our test set should include at least one member from each of the partitions.

51.3.2.3 Cause-Effect Graphing

Cause-effect graphing is a systematic way to arrive at a test set that has a good chance of revealing errors. It is quite similar to I/O partitioning and may be regarded as a formal approach to this. It is particularly useful in arriving at test data that incorporates a combination of input conditions. Cause-effect graphing generally results in a finer partitioning of the input domain than obtained by ad
hoc methods.

For the program or module to be tested, we need to first identify the following:

1. A set of causes. This is a set of Boolean expressions involving input values. The truth of these expressions in some way affects the working of the program.

2. A set of effects. This is a set of states that the program moves into based upon a combination of causes. This set may include both output states as well as internal states.

3. A relationship between the causes and effects.

The relationship between causes and effects is represented in terms of a diagram (formally called a graph). This diagram has one node for each of the causes and effects. Additional nodes as needed may be added to the diagram. The cause-effect relationships are represented by edges (or lines) that join a pair of nodes.

Once this cause-effect graph has been obtained, a systematic procedure can be used to obtain all the different cause combinations that result in different effect combinations. For each of these, one can generate a set of inputs to be used as test data. Hence, the test set has at least as many test data as the number of cause combinations generated from the cause-effect graph.

Let us consider an example. Suppose we wish to test a program module that finds \( \max\{x, y, z\} \) when \( x, y, \) and \( z \) are distinct integers. Further, suppose that this module is to put out an error message whenever two or more of \( \{x, y, z\} \) are the same. For this module, we can use the cause set:

\[
C_1 \quad x < y \\
C_2 \quad x < z \\
C_3 \quad y < x \\
C_4 \quad y < z \\
C_5 \quad z < x \\
C_6 \quad z < y \\
C_7 \quad x = y \\
C_8 \quad x = z \\
C_9 \quad y = z
\]

This cause set represents all possible relationships between pairs of inputs. The effect set consists of the four possible outputs from the modules. These are:

\[
E_1 \quad x
\]
The cause-effect graph for the ‘‘max’’ problem defined above is shown in Figure 51.4. Before examining this graph, we state the following conventions for drawing cause-effect graphs:

1. Nodes that represent causes are drawn in one column at the left end of the graph.
2. Nodes that represent effects are drawn in one column at the right end.
3. Intermediate nodes (nodes labeled I1-I12 in Figure 51.4), as needed, are drawn in between the cause and effect nodes. These are arranged in columns.
4. Each node (whether cause, intermediate, or effect) can be in one of two states: true and false. (Later, we shall introduce a third state. For now, these two states will suffice.) For example, a cause node is true iff the Boolean expression it represents is true. In our example, C1 is true iff \( x < y \). An effect node is true iff the effect it denotes does, in fact, occur. Node E3 is true iff the output \( z \) is generated by the program.
5. Every edge connects a node in one column to a node in some column to its right.
6. The collection of edges that enters any node must be related in exactly one of the ways shown in Figure 51.5. The ‘‘identity’’ relation expressed by Figure 51.5(a) has the significance that node B is in the same state as node A. So, B is true iff A is true. The ‘‘negation’’ relation of Figure 51.5(b) means that the state of A is different from that of B. If A is true, then B is false. If A is false, then B is true. The ‘‘or’’ relation of Figure 51.5(c) indicates that node Q is true iff at least one of A, B, ..., Z is true. Finally, the ‘‘and’’ relation of Figure 51.5(d) has the property that Q is true iff every one of A, B, ..., Z is true.
7. If a single edge comes into a node, then that node is either an identity or a negation node. Node ‘‘B’’ of Figure 51.5(a) is an identity node while node ‘‘B’’ of Figure 51.5(b) is a negation node. When all entering edges are related by the ‘‘or’’ (‘‘and’’) relation, the node is an ‘‘or’’ (‘‘and’’) node. Node ‘‘Q’’ of Figure 51.5(c) is an ‘‘or’’ node while in Figure 51.5(d), node Q is an ‘‘and’’ node.

Returning to our example and Figure 51.4, we see that I1 is true iff C3, C5, and I12 are true. But, I12 is true iff C9 is false. So, I1 is true iff \( y < x \) and \( z < x \) and \( y \neq z \). I2 is true iff C3 and C6 are true. I.e., iff \( y < x \) and \( z < y \). I3 is true iff C4 and C5 are true. I.e., iff \( y < z \) and \( z < x \). Hence, when any one of I1, I2, and
Figure 51.4 Cause-effect graph for "max" problem.
I3 is true, $x = \max\{x, y, z\}$. Furthermore, the conditions under which I1, I2 and I3 are true cover all conditions under which the program for the “max” problem is to output $x$. Since E1 is an “or” node, the graph of Figure 51.4 implies that the effect E1 occurs whenever any one (or more) of I1, I2, and I3 is true. Hence, the output $x$ is produced exactly when $x$ is the maximum and $x$, $y$, and $z$ are distinct.

A cause-effect graph is a formal representation of all the conditions under which a particular effect can occur. These conditions are represented as combinations of causes and may be extracted from the graph by working backwards from the effect nodes. To get all the conditions that can result in an effect $E_j$, we...
begin at the node Ej. This node is assigned the state: true. To determine all combinations of causes that result in a node being in a prespecified state, we use the following rules:

1. If this is an identity node (Figure 51.5(a)) that is to be in the state true (false), we determine all conditions under which the node on its left is true (false).
2. If this is a negation node that is to be in the state true, we determine all conditions under which the node on its left is false (true).
3. In case this is an ‘‘and’’ node that is to be in the state true, we determine the conditions under which all the left nodes are true. If the ‘‘and’’ node is to be in the state false, then all combinations of states of the left nodes other than the combination having all left nodes true need to be explored. If there are \( n \) left nodes, then there are \( 2^n - 1 \) combinations of states for the left nodes that will result in the ‘‘and’’ node having the state false. All combinations of causes leading to each of these \( 2^n - 1 \) combinations of states need to be determined.
4. In the case of an ‘‘or’’ node that is to be in the false state, we determine all conditions under which each of the left nodes is false. In case the ‘‘or’’ node is to be in the true state and there are \( n \) left nodes, then all cause combinations that result in any of the \( 2^n - 1 \) combinations of states of the left nodes that imply the ‘‘or’’ node is in the true state are to be determined.

Let us use these rules to determine all conditions under which the effect \( E_2 \) occurs. For clarity, Figure 51.4 has been redrawn in Figure 51.6. Only relevant nodes and edges have been retained.

\( E_2 \) is an ‘‘or’’ node. Its state is set to true. This node has three left nodes: I4, I5, I6. For \( E_2 \) to be in the true state, (I4, I5, I6) can be in any one of the seven state combinations:

1. (true, true, true)
2. (true, true, false)
3. (true, false, true)
4. (true, false, false)
5. (false, true, true)
6. (false, true, false)
7. (false, false, true)

We need to determine the conditions under which each of the above seven state combinations occurs. Let us begin with the first. I4, I5, and I6 are ‘‘and’’ nodes. Hence, for I4 to be true, C1, C6, and I11 must be true. Since I11 is a negation node, for I11 to be true, C8 must be false. For I5, C1 and C5 must be
Figure 51.6 Portion of Figure 51.5
true. For I6, C2 and C6 must be true. So, the conditions under which I4, I5, and I6 have the state combination (true, true, true) are:

\[(C1 \text{ and } C6 \text{ and } (\text{not C8})) \text{ and } (C1 \text{ and } C5) \text{ and } (C2 \text{ and } C6)\]

\[= C1 \text{ and } C2 \text{ and } C5 \text{ and } C6 \text{ and } (\text{not C8})\]

This is the only set of conditions that results in the first of the seven state combinations listed above.

It is quite possible to have several sets of conditions that result in a given state combination. This, in fact, is the case with the second state combination (true, true, false). There are three possible state combinations of the left nodes, (C2, C6), of I6 that result in I6 having the state false. These are:

1. (true, false)
2. (false, true)
3. (false, false)

These are, respectively, represented by the cause combinations:

1. C2 and (not C6)
2. (not C2) and C6
3. (not C2) and (not C6)

Combining these with the conditions under which I4 and I5 are true, we get the following three conditions under which (I4, I5, I6) are in the state (true, true, false):

\[(C1 \text{ and } C6 \text{ and } (\text{not C8})) \text{ and } (C1 \text{ and } C5) \text{ and } ((\text{not C2}) \text{ and } C6)\]

\[= (C1 \text{ and } C6 \text{ and } (\text{not C8})) \text{ and } (C1 \text{ and } C5) \text{ and } ((\text{not C2}) \text{ and } (\text{not C6}))\]

Generalizing from these two examples, we see that there are three different conditions that lead to the state combination (true, false, true); nine that lead to (true, false, false); seven that lead to (false, true, true); twenty one that lead to (false, true, false); and twenty one that lead to (false, false, true). In all there are 65 combinations of input conditions under which E2 can be in the state true. There are 65 for E1, 65 for E3, and 7 for E4. In all there are 202 combinations of the causes that result in one of the effects being true!

Some of these combinations may be the same and yet others may be impossible. Several combinations are impossible in our example. For instance, the cause combination

C1 and C2 and C5 and C6 and (not C8)
that results in the state \((I_4, I_5, I_6) = (\text{true, true, true})\) is not possible. This is so as \(C_2 = x < z\) while \(C_5 = z < x\).

Even after the impossible combinations are weeded out and common combinations combined, the number of cause combinations that remain may be too large. In fact, because of the rules used for “and” nodes in the false state and for “or” nodes in the true state, the number of combinations grows exponentially as we move towards the left end of the graph. In an attempt to curb this explosive growth in the number of cause combinations that get generated, we suggest the following strategies:

1. When considering an “and” ("or") node that is to be in the false (true) state generate only one of the many cause combinations that results in each of the \(2^n - 1\) state combinations. This avoids further explosive growth in the number of generated conditions.

2. When considering an “and” ("or") node that is to be in the false (true) state, consider at most \(n\) different left node state combinations. These should have the property that each of the left nodes is in the false (true) state in at least one of the state combinations and that no state combination is an impossible combination. To the extent possible, state combinations in which exactly one left node is in the false (true) state are to be preferred over those in which more than one left node is in the false (true) state. This cuts down the number of left node state combinations from \(2^n - 1\) to at most \(n\).

To see the difference between these two reduction strategies, consider the cause-effect graph of Figure 51.6. We have already determined that there are 65 cause combinations that result in \(E_2\) being true. If we use reduction strategy (1), then for each of the seven state combinations that result in \(E_2\) being true, we need to generate only one cause combination. The total number of cause combinations to generate is reduced from 65 to 7. If we use strategy (2), then for \(E_2\) we need to pick at most three state combinations for \((I_4, I_5, I_6)\) that are possible. The guidelines recommend the combinations (true, false, false), (false, true, false), and (false, false, true). Let us consider the first one. For the moment, assume that each of these is possible. The sole cause combination that results in \(I_4\) true is:

\[C_1 \text{ and } C_6 \text{ and (not } C_8)\]

We have seen earlier that there are three state combinations for \((C_2, C_6)\) that result in \(I_6\) being false. Under strategy (2), only two of these \(((true, false)\) and \((false, true))\) need be considered. Similarly, there are two state combinations for \((C_1, C_5)\) that need to be considered. In all, therefore, we will obtain four sets of cause conditions that result in \((I_4, I_5, I_6)\) having the state combination (true, false, false). By symmetry, we will obtain four combinations for each of the
remaining two state combinations under consideration for (I4, I5, I6). Hence, the number of generated combinations reduces from 37 to 12.

The four combinations generated for (I4, I5, I6) = (true, false, false) are:

1. (C1 and C6 and (not C8)) and (C1 and (not C5)) and (C2 and (not C6))
2. (C1 and C6 and (not C8)) and (C1 and (not C5)) and ((not C2) and C6)
3. (C1 and C6 and (not C8)) and ((not C1) and C5) and (C2 and (not C6))
4. (C1 and C6 and (not C8)) and ((not C1) and C5) and ((not C2) and C6)

None of these is possible. (1) is not possible as it requires C6 to be both true and false. (3) and (4) are impossible for similar reasons. (2) requires that both C2 and C5 be false. But from the definition of C2 and C5, this implies that C8 be true. This contradicts the requirement in (2) that C8 is false. Since the state combination (true, false, false) is impossible, we can replace it by some other combination that has I4 = true. An examination of the remaining two state combinations being considered for (I4, I5, I6) reveals that neither is possible.

So, we need to consider some other state combinations. Since, whenever I5 or I6 is true, I4 is also true, the only possible combinations are (true, true, false) and (true, false, true). There is only one possible cause combination for each. These are:

1. (C1 and C6 and (not C8)) and (C1 and C5) and ((not C2) and C6)
   = C1 and C5 and C6 and (not C2) and (not C8)
   = C1 and C5 and C6 (by definition of the C’s)
2. (C1 and C6 and (not C8)) and (C1 and (not C5)) and (C2 and C6)
   = C1 and C2 and C6 and (not C5) and (not C8)
   = C1 and C2 and C6 (by definition of the C’s)

Similarly, for each of E1 and E3 exactly two cause combinations (which are not impossible) are generated. For E4, the guidelines for strategy (2) suggest the following cause combinations (C7, C8, C9): (true, false, false), (false, true, false), and (false, false, true). Each is possible. So, three cause combinations for E4 are generated. Hence, when strategy (2) is used we are left with 9 cause combinations.

The next step is to determine if these nine cause combinations result in any effects in addition to the one noted for each. To do this, we examine the cause combinations one at a time. The truth value of all causes in the combination is ascertained. For example, the cause combination for (I4, I5, I6) = (true, true, false) has C1, C5, and C6 true and C2 and C8 false. We begin by assigning these values to the cause nodes C1, C2, C5, C6, and C8. These truth values require that C3, C4, C7, and C9 be false. At this time, if there are any causes whose value is undetermined, these are assigned the value ‘?’’. In our example, no cause has an unassigned truth value. So, no ‘?’’s are introduced.
Next, we evaluate the states of the remaining nodes by moving left to right. The state of any node depends only on the states of the nodes on its left. So, its state may be evaluated using the node type information and the truth values of the nodes on its left. The following rules are used to handle the truth value ‘?’:

1. The truth value of an identity or negation node is ‘?’ iff its left node has value ‘?’.
2. The truth value of an ‘or’ node is true iff at least one of its left nodes have value true. It is false iff all its left nodes have value false. Otherwise, its value is ‘?’.
3. An ‘and’ node has value true iff all its left nodes have value true. It has value false iff at least one of its left nodes have value false. Otherwise, it has value ‘?’.

Using these rules, the value of the effect nodes may be determined. For C1, C5, and C6 true and the remaining causes false, we get E1, E3, and E4 false and E2 true.

Let us consider another example. Consider the case E4 = true. As remarked earlier, when strategy (2) is used, only the following state combinations for (C7, C8, C9) are to be considered:

1. (true, false, false)
2. (false, true, false)
3. (false, false, true)

Consider the first of these. This requires C7 to be true and C8 and C9 to be false. When C7 is true, C1 and C3 must be false. The values of C2, C4, C5, and C6 are undetermined and set to ‘?’.

Using these truth values, the values of the effect nodes E1-E4 are, respectively, ‘?’; ‘?’; false, and true.

By repeating this process for each of the twelve cause combinations, we can determine the effects each is expected to produce. This cause-effect relationship may be tabulated in the form of a decision table as in Figure 51.7. A decision table has one row for each cause-effect combination. It contains one column for each cause and each effect. A ‘T’ entry implies that the corresponding cause or effect is true (or present). An ‘F’ entry implies that it is false (or absent). A blank entry signifies a value that is trivially determined from the other values and the cause-effect graph. It might evaluate to T, F, or ‘?’.

For the example of Figure 51.4, there are 9 rows in the decision table. The first row of Figure 51.7, for example, states that when the causes C3, C4, and C5 are true (i.e., are present), the effect E1 is true. From the definitions of the causes C1-C9 and the specified truth values for C3, C4, and C5, we see that the remaining causes must be false. This results in E2, E3, and E4 having the value false. So, all the blanks in first row of the decision table correspond to the value false. For the last row of the table, C7 and C8 are false while C9 and E4 are true. This
implies that C4 and C6 are false. The values of C1, C2, C3, and C5 are ‘?’.
This results in E1 false and E2 and E3 having value ‘?’.

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Figure 51.7 Cause-effect decision table for Figure 51.4

Once a decision table has been constructed, it may be reduced using well
known decision table reduction methods. We shall not go into these here. The
effect of the reduction is to eliminate rows of the table that are subsumed by oth-
ers. Following this reduction, a test set is devised so as to include at least one set
of input data for each row of the reduced decision table. Since no row consists
of an impossible set of cause conditions, there is at least one element in the input
domain that satisfies the cause conditions of each of the rows. Because of the
presence of don’t cares in a decision table, it is possible for one input to satisfy
the conditions of more than one row. So, the decision table doesn’t result in a
strict partitioning of the input domain. It does, however, come close to this.

From the table of Figure 51.7, we see that our test problem will include at least 9 test data. We need at least one test data for each of the
following combination of causes:
1. \( y < x, y < z, \) and \( z < x \) (eg.: \( x = 5, y = 3, z = 4 \))
2. \( y < x, z < y, \) and \( z < x \) (eg.: \( x = 5, y = 3, z = 2 \))
3. \( x < y, x < z, \) and \( z < y \) (eg.: \( x = 3, y = 8, z = 4 \))
4. \( x < y, z < x, \) and \( z < y \) (eg.: \( x = 5, y = 7, z = 2 \))
5. \( x < y, x < z, \) and \( y < z \) (eg.: \( x = 1, y = 3, z = 4 \))
6. \( y < x, x < z \), and \( y < z \) (eg.: \( x = 2, y = 1, z = 4 \))
7. \( x = y, x \neq z \), and \( y \neq z \) (eg.: \( x = 2, y = 2, z = 6 \))
8. \( x \neq y, x = z \), and \( y \neq z \) (eg.: \( x = 2, y = 5, z = 2 \))
9. \( x \neq y, x \neq z \), and \( y = z \) (eg.: \( x = 2, y = 6, z = 6 \))

Comparing with the ad hoc I/O partitioning method used in the preceding section, we see that cause-effect graphing has led to 9 test data rather than 4.

### 51.3.3 White Box Methods

White box methods for test generation create test data based on an examination of the code to be generated. The weakest condition one can place on a test set is that it results in each program statement being executed at least once. This is called **statement coverage**. As an example, consider the program:

```plaintext
1 {  
2    max = a;  
3    if (max < b)  
4        max = b;  
5    if (max < c)  
6        max = c;  
7 }
```

This program finds the maximum of the three numbers \( a \), \( b \), and \( c \). Under the statement coverage requirement, the test set should result in each of the seven statements being executed at least once. The single test data \((a = 1, b = 2, c = 3)\) causes each of the five statements to be executed and so results in full statement coverage. This test by itself isn’t enough to give us confidence that the above program contains no errors. This is so as this test data exercises only one of the possible execution paths. The path exercised is \((1, 2, 3, 4, 5, 6, 7)\). The program contains three other execution paths. These are \((1, 2, 3, 5, 6, 7), (1, 2, 3, 4, 5, 7), \) and \((1, 2, 3, 5, 7)\). Exercising these paths is accomplished by using three additional test data: \((3, 1, 6), (4, 6, 1), \) and \((8, 3, 2)\).

The execution paths of a program may be obtained from the **flow graph** of a program. This graph has one vertex for each statement in the program. There is a directed edge from vertex \( i \) to vertex \( j \) if it is possible to execute statement \( j \) immediately after statement \( i \). We shall limit ourselves to flow graphs that have exactly one vertex with no incoming edge and exactly one vertex with no outgoing edge. The single vertex with no incoming edge is called the *entry* vertex and the single vertex with no outgoing edge is called the *exit* vertex. Every execution path begins at the entry vertex and terminates at the exit vertex. The flow graph for the above ‘`max’’ program is shown in Figure 51.8(a). From the flow graph, all execution paths are easily obtained.
Since the flow graph for a program contains as many vertices as program lines, it is usually a very cumbersome graph to deal with. A reduced flow graph is obtained from a flow graph by repeatedly collapsing together pairs of vertices \((i, j)\) with the following properties:

1. There is an edge from \(i\) to \(j\).
2. There is no other edge that enters vertex \(j\).
3. There is no other edge that leaves vertex \(i\).
4. There is no edge from \(j\) to \(i\).
When two vertices $i$ and $j$ are collapsed, the edge from $i$ to $j$ is eliminated. Vertices 1 and 2 of Figure 51.8(a) can be collapsed to get the reduced graph of Figure 51.8(b). This graph can be reduced further to get the graph of Figure 51.8(c). The graph of Figure 51.8(c) cannot be reduced further.

Two hypothetical flow graphs are shown in Figures 51.9(a), 51.10(a), and 51.11(a), respectively. The corresponding reduced graphs are shown in Figures 51.9(b), 51.10(b), and 51.11(b).

From the reduced flow graph of Figure 51.9(b) we see that the corresponding program has three execution paths: \((1-8, 9-15, 28), (1-8, 16, 17-20, 28),\) and \((1-8, 16, 21-27, 28)\). All the execution paths of this program can be tested using three test data; one for each of these execution paths. From the reduced flow graph Figure 51.11(b), we see that there are many execution paths. These are given by the expression \((3-4, (5, 6)^j, 5, 7-8)\) for \(j \geq 0\). \((5, 6)^j\) is an abbreviation for \(5, 6, 5, 6, ..., 5, 6\) where the number of \(5\)'s (and hence of \(6\)'s) is \(j\).

When the number of iterations of a loop is a function of the input size the number of execution paths is infinite. Even when we restrict ourselves to programs whose loops are iterated at most 10 or 1000 times, the number of execution paths may be very large. Further, even programs that contain no loops at all may have a very large number of execution paths. As an example, consider the flow graph of Figure 51.12. There are no loops in the program it represents. However, the number of execution paths is exponential in the number of vertices.

While it would be nice to be able to use a test set that exercises each execution path in a program at least once, the size of such a test set will often be infinite or impractically large. Hence, it is necessary to lower our sights and develop criteria for an adequate test set whose coverage is somewhere between the two extremes of total statement coverage and total execution path coverage.

One possibility is to require that each edge of the flow graph be traversed by the test set. This is called edge coverage. Note that if a reduced flow graph has at least one edge, then traversing all edges of the reduced flow graph results in traversing all edges of the full flow graph. If the reduced flow graph has no edges, then the empty test set provides a total edge coverage for the reduced flow graph but not for the full graph. Note that the full graph has at least one edge as it contains at least the two vertices: entry and exit. Hence, the edge coverage criterion may be applied to the reduced graph except in the case that this graph has no edges.

For the graph of Figure 51.8(c), two sets of test data suffice to obtain edge coverage. The test data \(a = 6, b = 2, c = 3\) causes the edges \((1-3, 5)\) and \((5, 7)\) to be traversed. The data \(a = 1, b = 2, c = 3\) causes the edges \((1-3, 4), (4, 5), (5, 6),\) and \((6, 7)\) to be traversed. Only two of the possible four execution paths are traversed using this test set.

For the graph of Figure 51.9(b), edge coverage requires all execution paths to be traversed. The edges of Figure 51.11(b) can be covered using a single execution path. In fact, every path of the form \((3-4, (5, 6)^j, 5, 7-8)\) for \(1 \leq j \leq n\)
causes each edge to be traversed. Hence, edge coverage requires only one of the infinitely many execution paths to be tested.

An edge coverage of the graph of Figure 51.10(b) cannot be accomplished using a single test. Two tests will suffice. We need to traverse (for example) the paths: (4-5, 6, 7a, 6, 8-9) and (4-5, 6, 7a, 7b, 8-9). Generating the test data for
The edge coverage criterion requires that all decisions in a program evaluate to both true and false (for two way decisions) during the test.

Decision coverage is a criterion that is closely related to edge coverage. In this criterion, we require that the test set cause each decision to take on all its possible values (true and false in case of a two way decision). For single entry single exit graphs, edge coverage implies decision coverage. However, decision coverage may not imply edge coverage. This is, in fact, the case when the flow graph has no decisions. Decision coverage results in an empty test set whereas
edge coverage results in a test set with one test data. Note, however, that decision coverage together with statement coverage is equivalent to edge coverage.

The test criterion can be further strengthened to require that each condition of each decision take on all possible values. This is called *condition coverage*. A *condition* is formally defined to be a Boolean expression that contains no Boolean operator (i.e., and, or, not). Some examples are:

1. \(x > y\)
2. \(x + y < y*z\)
3. \(c\) (where \(c\) is of type Boolean)

---

**Figure 51.11** Yet another flow graph

**Figure 51.12** Loopless graph with many execution paths
Consider the statement:

\[ \text{if } ((C1 \& \& C2) || (C3 \& \& C4)) \text{ S1; else S2;} \]

where \( C1, C2, C3, \) and \( C4 \) are conditions and S1 and S2 are statements. Under the edge or decision coverage criteria, we need to use one test set that causes \((C1 \& \& C2) || (C3 \& \& C4)\) to be true and another that results in this decision being false. Condition coverage requires us to use a test set that causes each of the four conditions to evaluate to true at least once and to false at least once. This requires at least two data sets. Some of the possible pairs of truth value combinations for \( C1-C4 \) that satisfy the requirements of condition coverage are:

1. \((\text{true, true, true, true}), (\text{false, false, false, false})\)  
2. \((\text{true, false, true, false}), (\text{false, true, false, true})\)  
3. \((\text{false, false, true, true}), (\text{true, true, false, false})\)

Each of the above requires two data sets. Note that both data sets for (2) result in the decision for the \textit{if} being false while both data sets for (3) result in this decision being true. So, a test set that results in condition coverage need not result in decision coverage.

Note also that there may be no input data that corresponds to any of the above three pairs of truth value combinations. For example, there may be no data that results in \( C1-C4 \) being true simultaneously. Hence, condition coverage may require the use of more than two data sets. Two truth value combinations for \( C1-C4 \) that result in condition coverage and require three data sets are:

1. \((\text{true, true, true, true}), (\text{true, true, false, false}), (\text{false, false, false, true})\)  
2. \((\text{true, false, false, true}), (\text{true, true, false, true}), (\text{false, true, true, false})\)

Condition coverage can be further strengthened to require that all combinations of condition values be tested for. In the above \textit{if} example, this will require the use of 16 test data; one for each truth combination of \( C1-C4 \). Several of these combinations may not be possible. So, the number of test data will generally be smaller.

Of the test coverage criteria we have discussed so far, execution path coverage is generally the most demanding. A test set that results in total execution path coverage also results in statement and decision coverage. It may, however, not result in condition coverage. Total execution path coverage often requires an infinite number of test data or at least a prohibitively large number of test data. Hence, total path coverage is often impossible in practice.
It is often possible to use a test set that satisfies all three of the following:

1. statement coverage
2. decision coverage
3. all combinations condition coverage

The all combinations condition coverage requirement may result in too many test data. The number of test data can be reduced by changing this requirement to “condition coverage”. In programming languages such as C++, decisions are evaluated in short circuit mode. In this mode, the evaluation of a decision terminates as soon as its value has been determined. For example, consider the statement:

\[ \text{if } ((C1 \&\& C2) || (C3 \&\& C4)) \text{ S1; else S2;} \]

If \((C1 \&\& C2)\) evaluates to true, then \((C3 \&\& C4)\) is not evaluated as regardless of its value, the decision is true. When this mode of decision evaluation is being used, it is necessary to strengthen condition coverage to \textit{condition coverage and evaluation}. This requires that each condition actually be evaluated during the testing and that it take on the value true on at least one such evaluation and the value false on another. This is necessary as the evaluation of a condition may have side effects that impact the run time behavior of a program. For instance, a condition may involve a function invocation (say \(f(x, y, z)\)) that results in the values of \(x, y,\) and \(z\) being changed. To study the effects of this, the condition must be forced to evaluate.

In many cases, condition coverage and evaluation may be impossible. For example, suppose that the above \textit{if} statement is evaluated as in the flow chart of Figure 51.13. There may be no input data that causes \(C2\) to actually evaluate to false. This is so as \(C2\) may be false only when \(C1\) is. But, when \(C1\) is false, \(C2\) is not evaluated.

\textit{51.3.4 Summary}

When generating test data, one must keep in mind that the objective of testing is to expose the presence of errors. Generating a minimal test set that serves this purpose is a challenge. All too often, programmers rely on randomly generated test data. This is about the worst way to generate test data. We have discussed two functionally different approaches to the generation of test data: black box and white box. Both approaches aim at generating test data with maximum likelihood of exposing errors. To get a good test set, it is essential to use both the black and white box methods.
Test data obtained from black box methods can expose errors resulting from missing code segments, boundary conditions, numerical errors, interesting combinations of input conditions, etc. Test data obtained from white box methods may not detect such errors.

Test data obtained from white box methods will expose errors resulting from uninitialized variables, misspelled variable names, incorrect conditions (for example $x < y$ instead of $y < x$), unreachable code segments, etc. These errors may not be detected by the black box data.

The black and white box methods provide guidelines for a minimum test set. In practice, it may be possible to use more than a minimum test set. In this case, it is desirable to use additional tests in some uniform way. We would like to exercise all code segments uniformly. To accomplish this, one may use automatic program instrumentation aids. These aids provide statistics on the frequency of execution of each statement or flow graph edge. They can also record the number of times each decision took on a certain value, the number of times each condition had a certain evaluation, etc. Based on this information, one can devise additional tests to exercise those parts of the program that have not been sufficiently exercised. The interested reader is referred to the paper on program...
instrumentation by J. Huang that is cited in the references section of this chapter.

Program instrumentation is also useful in obtaining a test set that satisfies certain criteria. After a few test data have been used, the results from the program instrumentation can be used to determine what needs to be done to meet the test criteria. Additional test data to meet this end can now be obtained.

EXERCISES

5. List all the conditions under which (I4, I5, and I6) take on each of the following state combinations. See Figure 51.4. For each set of conditions determine whether it is an impossible condition.
   a. (true, false, true)
   b. (true, false, false)
   c. (false, true, true)
   d. (false, true, false)
   e. (false, false, true)

6. For the cause-effect graph of Figure 51.14, obtain all cause combinations that result in each of the effects E1, E2, and E3. Present these in a decision table in which each different cause combination is represented by a single row. For each set of cause combinations, all effects should be marked T, F, or “?”.  

![Figure 51.14](image)

7. Do the previous exercise for the cause-effect graph of Figure 51.15.
8. You are to test a program that inputs the lengths of the three sides of a triangle and classifies the triangle into one of the categories:
   a. equilateral
   b. isosceles
   c. scalene
   Obtain a cause-effect graph for this problem. Next, obtain the complete decision table that contains the combination of causes that are to be tested for. Only possible combinations are to be included and the suggestions to reduce growth in the number of rows are to be used.

9. Give an example of a program in which execution path coverage does not result in condition coverage.

51.4 DEBUGGING

Testing exposes the presence of errors in a program. Once a test run produces a result different from the one expected, we know that something is wrong with the program. The process of determining and correcting the cause of this discrepancy between the behavior as predicted by the specifications and that observed is called debugging. A thorough study of debugging methods is beyond the scope of this book. We simply provide some suggestions for debugging and refer the interested reader to the book by G. Myers that is cited in the
references section.

1. Use incremental testing so that the cause of the error is localized.

2. Try to determine the cause of an error by logical reasoning. If this fails, then you may wish to perform a program trace to determine when the program started performing incorrectly. This becomes infeasible when the program executes many instructions with that test data. The program trace is too long to be examined manually. When this happens, testing must be refined to isolate the part of the code that is suspect and a trace of this part obtained.

3. Do not attempt to correct errors by creating exceptions. Soon, the number of exceptions will be very large. Errors should be corrected by first determining their cause and then redesigning your solution as necessary.

4. When correcting an error, be certain that your correction does not result in errors where there were none before. Run your corrected program on the test data on which it worked alright before to be sure that it still works correctly on this data.

51.5 REFERENCES AND SELECTED READINGS

The text: *The art of software testing*, by G. Myers, John Wiley, New York, 1979 contains many do's and don'ts of program testing and debugging. It also contains chapters on walk-throughs, test design, and test generation tools. The terminology "big bang" is borrowed from this book.


A system that generates test data and several references to other such systems can be found in the paper: A system to generate test data and symbolically execute programs, by L. Clarke, *IEEE Trans. On Software Engineering*, Sept. 1976, pp. 215-222.