PLEASE READ THE FOLLOWING INSTRUCTIONS CAREFULLY
1. For all problems, use only the algorithms discussed in class/text.
2. Write your name at the top of every exam sheet.
3. Write your answers directly on the exam question sheet. You may use scrap paper (supplied by your proctor) for work, but these will not be graded.
4. All answers will be graded on correctness, efficiency, clarity, elegance and other normal criteria that determine quality.
5. The points assigned to each question are provided in parentheses.
6. You may use only a pen or a pencil. No calculators allowed.
7. Do not write on the reverse side of the exam sheet.
8. Do not write close to the margins since those areas do not always make it through when faxed.
1. (10 points) Consider the following splay tree:

```
   p
   /  
  m   k
  /  
 c   j
 /  
 d   f
   
 g
 
 h
```

(a) (5) Perform a search for element $h$ under the assumption this is a Top-down splay tree. Show the tree(s) after each step of the splay.

(b) (5) Do question (a) assuming a Bottom-up splay tree.
Name:

Continue work here if necessary.
2. (10 points) You are given a Bloom filter that consists of \( m = 13 \) memory bits and two hash functions \( f_1() \) and \( f_2() \) defined as below:

\[
\begin{align*}
f_1(k) &= k \mod m \\
f_2(k) &= (2 \times k) \mod m
\end{align*}
\]

where \( k \) is a given key. Assume that all \( m \) bits of the Bloom filter are initially set to 0.

(a) (4) Show the Bloom filter bits following the insertion of the key 17.

(b) (4) Into the Bloom filter of (a) (i.e. following the insertion of the key 17) insert 19. Show the resulting Bloom filter bits.

(c) (2) For the filter of (b), give a key value that results in a filter error (i.e. the Bloom filter response is Maybe even though the key is not in the filter).
Name: ____________________________

Continue work here if necessary.
3. (10 points) You are given two strings $S$ and $T$ of length $m$ and $n$, respectively. Describe how to find the Longest Common Substring of $S$ and $T$ using any data structure discussed in the class and provide an example. Your algorithm should run in linear time with respect to $m$ and $n$. 
Name:

Continue work here if necessary.
4. (10) For the min radix-priority search tree (RPST) with range [0,63),

(a) (8) Perform *insert* operations into an initially empty RPST in sequence with the following keys: (9,50), (33,10), (20,1), (60,12), (22,61), (10,37). Show each step. (Note: The elements $x$ and $y$ of a key $(x, y)$ represents the *search* and *priority* key values, respectively.)

(b) (2) Delete key (10,37) from the result RPST of Part (a).
Name:

Continue work here if necessary.
5. (10) Describe the 2-dimensional range tree data structure. Derive the formula for the preprocessing time $P$, the space required $S$, and the query time $Q$. 
Name:

Continue work here if necessary.