COP 5536
Advanced Data Structures

Spring 2018

Exam 1 Solutions

Question.1 (12):

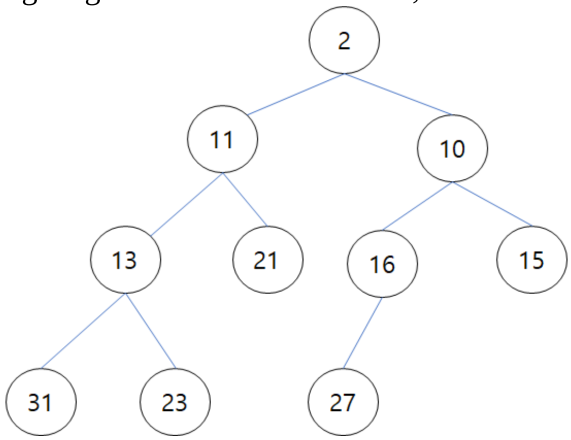
Suppose that a sequence of n operations is performed on a data structure. The k^{th} operation has a cost of $4\sqrt{k}$ whenever *k* is a perfect square (*k=1, 4, 9, etc*), and otherwise it has a cost of 2. Determine the smallest integer amortized cost per operation specifying the method you used.

a. Aggregate method. There are $\lfloor\sqrt{n}\rfloor$ operations that each cost $4\sqrt{k}$, and the other $n - \sqrt{\lfloor n\rfloor}$ operations each cost 2. This yields a total cost of $4 * (1 + 2 + 3 + ... + \sqrt{\lfloor n\rfloor}) + (n - \sqrt{\lfloor n\rfloor}) * 2 = 4* ((\sqrt{\lfloor n\rfloor})* (\sqrt{\lfloor n\rfloor} + 1) / 2) +2* (n - \sqrt{\lfloor n\rfloor}) = 4n$. So the amortized cost is $4n / n = 4$.

b. Accounting method. Charge each operation \$4. When k is not a perfect square, use \$2 to pay for the operation and use the extra \$2 for credit. When k is a perfect square, the preceding $(k - 1) - (\sqrt{k} - 1)^2 = 2\sqrt{k} - 2$ operations have each paid a credit of \$2, which together with the \$4 for the current operation yields exactly enough to pay for its $4\sqrt{k}$ actual cost.

c. Potential method. Define Φ = the number of operations since the most recent perfect square. That is, let $\Phi_K = k - (\sqrt{\lfloor k\rfloor})^2$. When k is not a perfect square, the amortized cost is $c' = c + \Phi_k - \Phi_{k-1} = 2 + 2*((k - (\sqrt{\lfloor k\rfloor})^2) - ((k - 1) - (\sqrt{\lfloor k\rfloor})^2)) = 4$. When k is a perfect square, the amortized cost is $c' = c + \Phi_k - \Phi_{k-1} = 4\sqrt{k} + 2*((k - (\sqrt{k})^2) - ((k - 1) - (\sqrt{k} - 1)^2)) = 4\sqrt{k} + 0 - 2*(2\sqrt{k} - 2) = 4$.

Question 2 (12):
For the following height-biased min leftist tree,



(a) (6) Perfrom Insert(7). Show each step.
(b) (6) DeleteMin from the original tree, showing each step.?

a) Step 1:

```
                2
             /     \
           11       10
          / \    /     \
        13   21 16     15
       /    \    /
     31      23  27
```

Step 2:

```
        10
       /  \
     16    15
    /
  27
```

Step 3:

```
            7
           /
          10
         /  \
       16    15
       /
     27
```

Step 4:

```
               2
             /   \
          11       7
         /  \     /
       13    21  10
      /  \       / \
    31    23   16  15
               /
             27
```

b)

Step 1:

```
         11                                10
        /  \                              /  \
      13    21                          16    15
     /  \                               /
   31    23                           27
```

Step 2:

```
         15
         /

   21
```

Step 3:

```
     11
    /  \
  13    15
 /  \    /
31  23 21
```

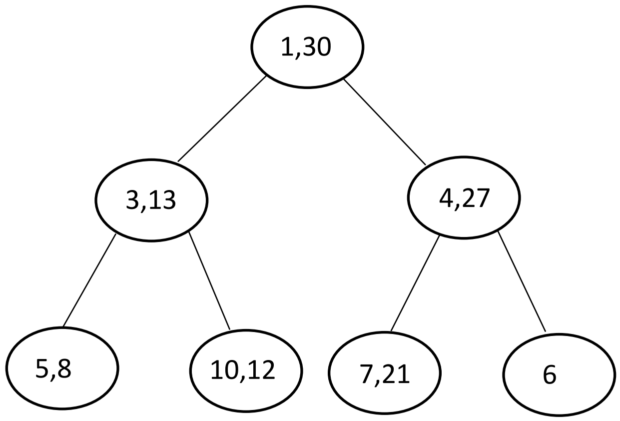
Step 4:

```
     10
    /  \
   16   11
  /    /  \
27   13    15
    /  \  /
   31 23 21
```

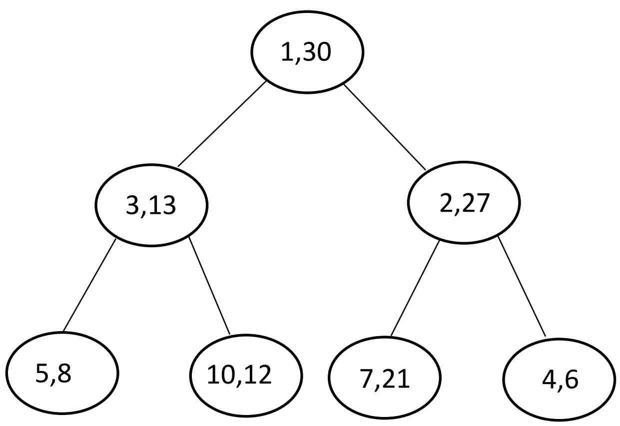
Step 5:

```
       10
     /    \
    11     16
   /  \     /
  13    15 27
 /  \  /
31 23 21
```

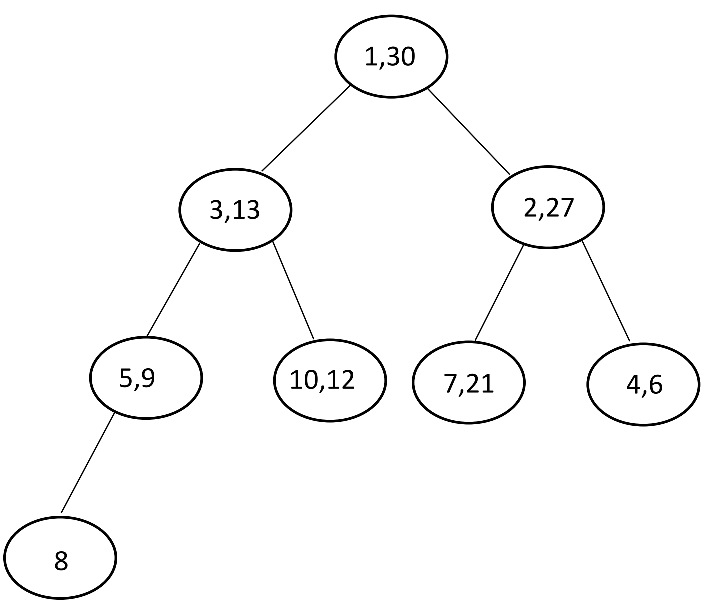
3. (12 points)
Consider the following interval heap.



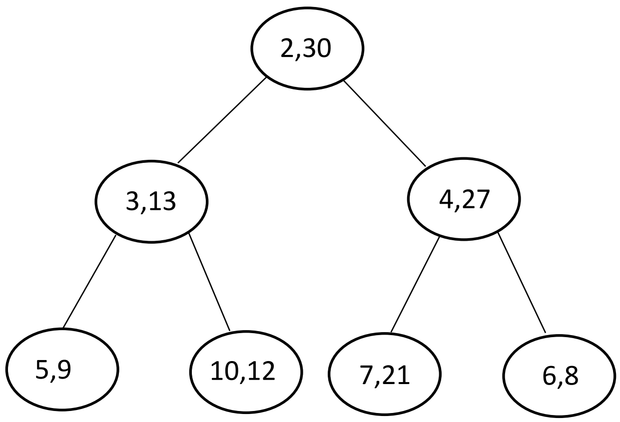
(a) (3) Interval heap after we Insert 2



(3) Interval heap after we Insert 9



(b) (6) Resulting heap after a DeleteMin is performed on <u>resulting heap of (a)</u>



If the final interval heap does not contain a swap between 6 and 8. (1 point deducted)

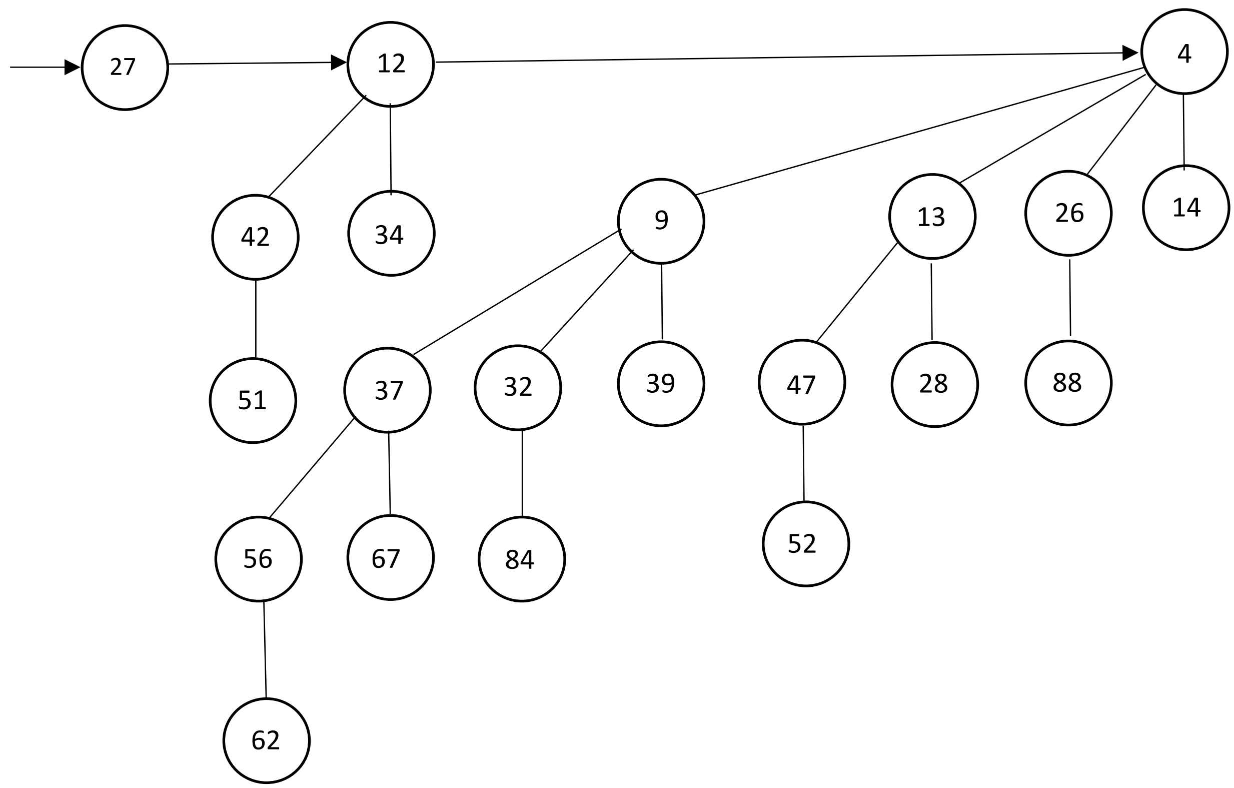
I have given marks in cases where you failed to understand the question or made a copying error but have displayed an understanding of the underlying concepts.

If the resulting heap from (A) is wrong and yet the deletemin is done correctly on the heap you started with you have received partial marks.

4. (14 points)

a) (6) All trees in the binominal heap are binominal trees. Max degree in a binominal tree is O(log n) (2 Marks) (proof by induction involving a statement that says a binomial heap contains 2^k nodes and $B_k = 2B_{k-1}$ – 4 Marks). If you have not provided a proof by induction and just a verbal proof I have given you 2 marks for the proof

b) (8) The resulting Binomial heap when a remove min operation is performed on the given Min Binomial heap is as follows. The order in which you process the tree is not important as pointed out in class. There are 3 possible solutions based on the order you follow.



I have given marks in cases where you made a copying error. If you fail to meld the two subtrees of degree 3 (1 point deducted).

Other possible answers.