Balanced Binary Search Trees

- height is $O(\log n)$, where $n$ is the number of elements in the tree
- AVL (Adelson-Velsky and Landis) trees
- red-black trees
- get, put, and remove take $O(\log n)$ time

Balanced Binary Search Trees

- Indexed AVL trees
- Indexed red-black trees
- Indexed operations also take $O(\log n)$ time
Balanced Search Trees

- weight balanced binary search trees
- 2-3 & 2-3-4 trees
- AA trees
- B-trees
- BBST
- etc.

AVL Tree

- binary tree
- for every node $x$, define its balance factor
  \[
  \text{balance factor of } x = \text{height of left subtree of } x - \text{height of right subtree of } x
  \]
- balance factor of every node $x$ is -1, 0, or 1
This is an AVL tree.

**Balance Factors**

**Height**

The height of an AVL tree that has \( n \) nodes is at most \( 1.44 \log_2 (n+2) \).

The height of every \( n \) node binary tree is at least \( \log_2 (n+1) \).
RR imbalance => new node is in right subtree of right subtree of blue node (node with $bf = -2$)

RR rotation.
AVL Rotations

- RR
- LL
- RL
- LR

Red Black Trees

**Colored Nodes Definition**
- Binary search tree.
- Each node is colored red or black.
- Root and all external nodes are black.
- No root-to-external-node path has two consecutive red nodes.
- All root-to-external-node paths have the same number of black nodes
Red Black Trees

Colored Edges Definition

- Binary search tree.
- Child pointers are colored red or black.
- Pointer to an external node is black.
- No root to external node path has two consecutive red pointers.
- Every root to external node path has the same number of black pointers.
Red Black Tree

- The height of a red black tree that has $n$ (internal) nodes is between $\log_2(n+1)$ and $2\log_2(n+1)$.
- java.util.TreeMap => red black tree
Graphs

• $G = (V, E)$
• $V$ is the vertex set.
• Vertices are also called nodes and points.
• $E$ is the edge set.
• Each edge connects two different vertices.
• Edges are also called arcs and lines.
• Directed edge has an orientation $(u, v)$.

$u \rightarrow v$

Graphs

• Undirected edge has no orientation $(u, v)$.
$u \quad v$

• Undirected graph $\Rightarrow$ no oriented edge.
• Directed graph $\Rightarrow$ every edge has an orientation.
Undirected Graph

Directed Graph (Digraph)
Applications—Communication Network

- Vertex = city, edge = communication link.

Driving Distance/Time Map

- Vertex = city, edge weight = driving distance/time.
Street Map

- Some streets are one way.

Complete Undirected Graph

Has all possible edges.
Number Of Edges—Undirected Graph

- Each edge is of the form \((u,v)\), \(u \neq v\).
- Number of such pairs in an \(n\) vertex graph is \(n(n-1)\).
- Since edge \((u,v)\) is the same as edge \((v,u)\), the number of edges in a complete undirected graph is \(n(n-1)/2\).
- Number of edges in an undirected graph is \(\leq n(n-1)/2\).

Number Of Edges—Directed Graph

- Each edge is of the form \((u,v)\), \(u \neq v\).
- Number of such pairs in an \(n\) vertex graph is \(n(n-1)\).
- Since edge \((u,v)\) is not the same as edge \((v,u)\), the number of edges in a complete directed graph is \(n(n-1)\).
- Number of edges in a directed graph is \(\leq n(n-1)\).
Vertex Degree

Number of edges incident to vertex.

degree(2) = 2, degree(5) = 3, degree(3) = 1

Sum Of Vertex Degrees

Sum of degrees = 2e (e is number of edges)
In-Degree Of A Vertex

in-degree is number of incoming edges

indegree(2) = 1, indegree(8) = 0

Out-Degree Of A Vertex

out-degree is number of outbound edges

outdegree(2) = 1, outdegree(8) = 2
Sum Of In- And Out-Degrees

each edge contributes 1 to the in-degree of some vertex and 1 to the out-degree of some other vertex

sum of in-degrees = sum of out-degrees = \( e \), where \( e \) is the number of edges in the digraph