Minimum Number Of Nodes

- Minimum number of nodes in a binary tree whose height is $h$.
- At least one node at each of first $h$ levels.
Maximum Number Of Nodes

- All possible nodes at first \( h \) levels are present.

![Tree Diagram]

Maximum number of nodes

\[
= 1 + 2 + 4 + 8 + \ldots + 2^{h-1} \\
= 2^h - 1
\]

Number Of Nodes & Height

- Let \( n \) be the number of nodes in a binary tree whose height is \( h \).
- \( h \leq n \leq 2^h - 1 \)
- \( \log_2(n+1) \leq h \leq n \)
Full Binary Tree

- A full binary tree of a given height $h$ has $2^h - 1$ nodes.

Numbering Nodes In A Full Binary Tree

- Number the nodes 1 through $2^h - 1$.
- Number by levels from top to bottom.
- Within a level number from left to right.
Node Number Properties

- Parent of node $i$ is node $i/2$, unless $i = 1$.
- Node 1 is the root and has no parent.

Node Number Properties

- Left child of node $i$ is node $2i$, unless $2i > n$, where $n$ is the number of nodes.
- If $2i > n$, node $i$ has no left child.
Node Number Properties

- Right child of node \( i \) is node \( 2i+1 \), unless \( 2i+1 > n \), where \( n \) is the number of nodes.
- If \( 2i+1 > n \), node \( i \) has no right child.

Complete Binary Tree With \( n \) Nodes

- Start with a full binary tree that has at least \( n \) nodes.
- Number the nodes as described earlier.
- The binary tree defined by the nodes numbered 1 through \( n \) is the unique \( n \) node complete binary tree.
Example

- Complete binary tree with 10 nodes.

Binary Tree Representation

- Array representation.
- Linked representation.
Array Representation

- Number the nodes using the numbering scheme for a full binary tree. The node that is numbered $i$ is stored in $\text{tree}[i]$.

```
0 5 10
a b c d e f g h i j
```

Right-Skewed Binary Tree

- An $n$ node binary tree needs an array whose length is between $n+1$ and $2^n$.

```
0 5 10 15
a - b - - c - - - - - - - - d
```
Linked Representation

- Each binary tree node is represented as an object whose data type is `BinaryTreeNode`.
- The space required by an $n$ node binary tree is $n \times (\text{space required by one node})$.

The Class `BinaryTreeNode`

```java
package dataStructures;

public class BinaryTreeNode {

    Object element;
    BinaryTreeNode leftChild; // left subtree
    BinaryTreeNode rightChild; // right subtree

    // constructors and any other methods
    // come here

}```
Some Binary Tree Operations

- Determine the height.
- Determine the number of nodes.
- Make a clone.
- Determine if two binary trees are clones.
- Display the binary tree.
- Evaluate the arithmetic expression represented by a binary tree.
- Obtain the infix form of an expression.
- Obtain the prefix form of an expression.
- Obtain the postfix form of an expression.
Binary Tree Traversal

• Many binary tree operations are done by performing a traversal of the binary tree.
• In a traversal, each element of the binary tree is visited exactly once.
• During the visit of an element, all action (make a clone, display, evaluate the operator, etc.) with respect to this element is taken.

Binary Tree Traversal Methods

• Preorder
• Inorder
• Postorder
• Level order