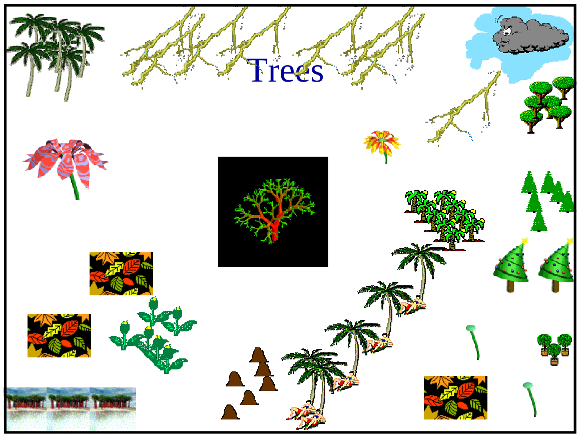
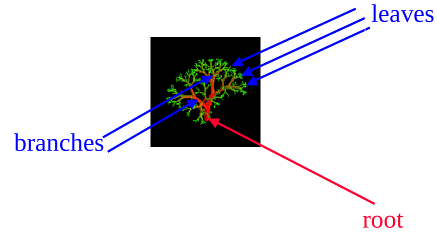
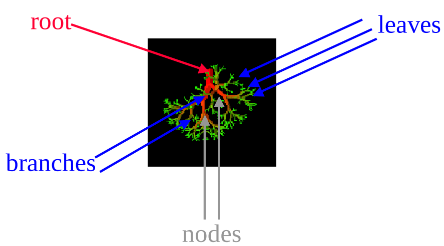
Trees



Nature Lover's View Of A Tree



Computer Scientist's View



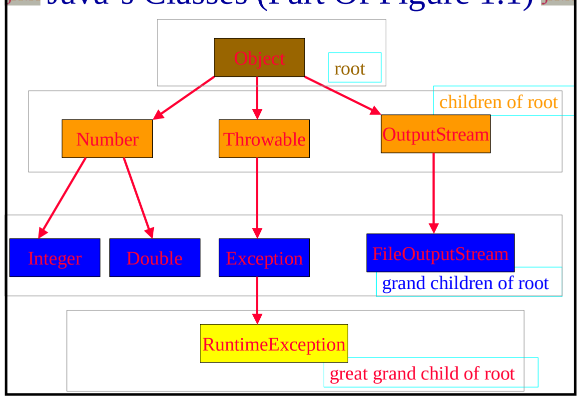
Linear Lists And Trees

- Linear lists are useful for serially ordered data.
  - $(e_0, e_1, e_2, \ldots, e_{n-1})$
  - Days of week.
  - Months in a year.
  - Students in this class.
- Trees are useful for hierarchically ordered data.
  - Employees of a corporation.
    - President, vice presidents, managers, and so on.
  - Java's classes.
    - Object is at the top of the hierarchy.
    - Subclasses of Object are next, and so on.

Hierarchical Data And Trees

- The element at the top of the hierarchy is the root.
- Elements next in the hierarchy are the children of the root.
- Elements next in the hierarchy are the grandchildren of the root, and so on.
- Elements that have no children are leaves.

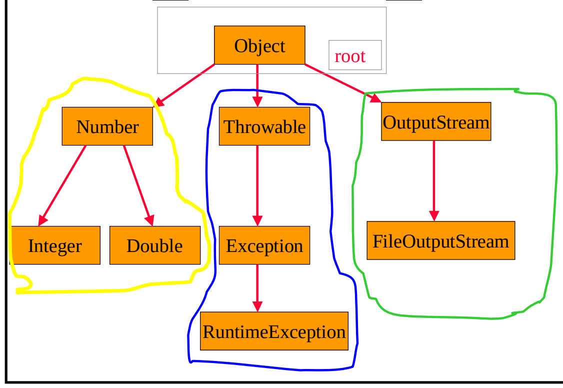
Java's Classes (Part Of Figure 1.1)



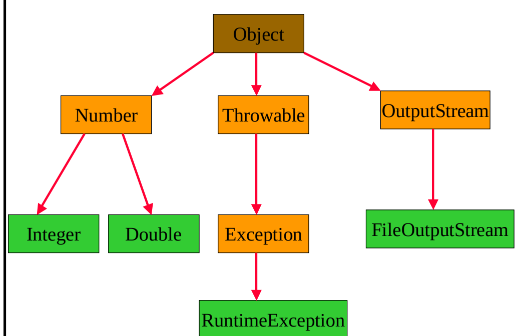
Definition

- A tree t is a finite nonempty set of elements.
- One of these elements is called the root.
- The remaining elements, if any, are partitioned into trees, which are called the subtrees of t.

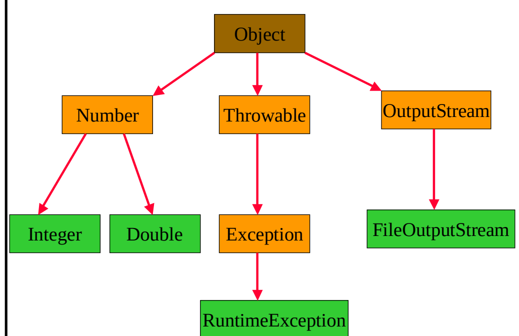
Subtrees



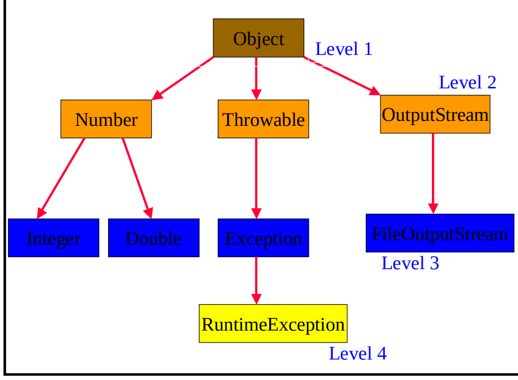
Leaves



Parent, Grandparent, Siblings, Ancestors, Descendants



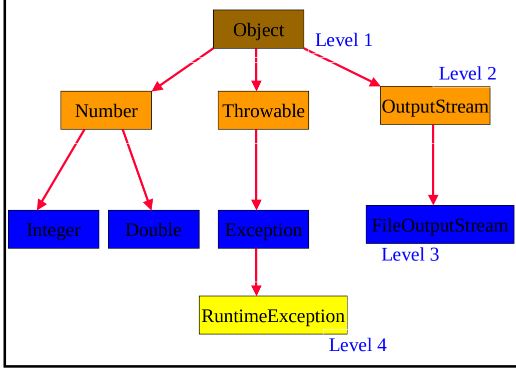
Levels



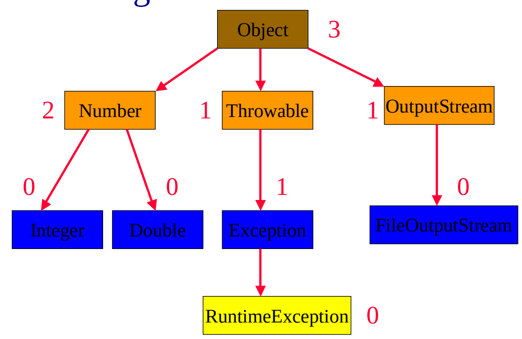
Caution

- Some texts start level numbers at 0 rather than at 1.
- Root is at level 0.
- Its children are at level 1.
- The grand children of the root are at level 2.
- And so on.
- We shall number levels with the root at level 1.

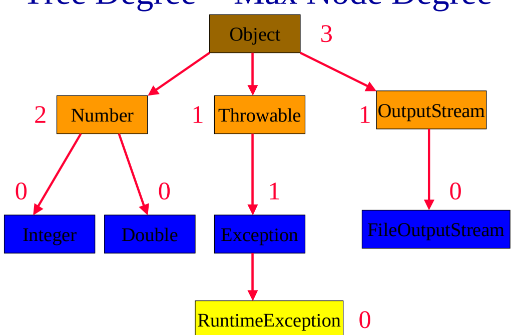
height = depth = number of levels



Node Degree = Number Of Children



Tree Degree = Max Node Degree



Degree of tree = 3.

Binary Tree

- Finite (possibly empty) collection of elements.
- A nonempty binary tree has a root element.
- The remaining elements (if any) are partitioned into two binary trees.
- These are called the left and right subtrees of the binary tree.

Differences Between A Tree & A Binary Tree

- No node in a binary tree may have a degree more than 2, whereas there is no limit on the degree of a node in a tree.
- A binary tree may be empty; a tree cannot be empty.

Differences Between A Tree & A Binary Tree

- The subtrees of a binary tree are ordered; those of a tree are not ordered.

- Are different when viewed as binary trees.
- Are the same when viewed as trees.

Arithmetic Expressions

- (a + b) * (c + d) + e – f/g*h + 3.25
- Expressions comprise three kinds of entities.
  - Operators (+, -, /, *).
  - Operands (a, b, c, d, e, f, g, h, 3.25, (a + b), (c + d), etc.).
  - Delimiters ((,)).

Operator Degree

- Number of operands that the operator requires.
- Binary operator requires two operands.
  - a + b
  - c / d
  - e - f
- Unary operator requires one operand.
  - + g
  - - h

Infix Form

- Normal way to write an expression.
- Binary operators come in between their left and right operands.
  - a * b
  - a + b * c
  - a * b / c
  - (a + b) * (c + d) + e – f/g*h + 3.25

Operator Priorities

- How do you figure out the operands of an operator?
  - a + b * c
  - a * b + c / d
- This is done by assigning operator priorities.
  - priority(*) = priority(/) > priority(+) = priority(-)
- When an operand lies between two operators, the operand associates with the operator that has higher priority.

Tie Breaker

- When an operand lies between two operators that have the same priority, the operand associates with the operator on the left.
  - a + b - c
  - a * b / c / d

Delimiters

- Subexpression within delimiters is treated as a single operand, independent from the remainder of the expression.
  - (a + b) * (c – d) / (e – f)

Infix Expression Is Hard To Parse

- Need operator priorities, tie breaker, and delimiters.
- This makes computer evaluation more difficult than is necessary.
- Postfix and prefix expression forms do not rely on operator priorities, a tie breaker, or delimiters.
- So it is easier for a computer to evaluate expressions that are in these forms.

Postfix Form

- The postfix form of a variable or constant is the same as its infix form.
  - a, b, 3.25
- The relative order of operands is the same in infix and postfix forms.
- Operators come immediately after the postfix form of their operands.
  - Infix = a + b
  - Postfix = ab+

Postfix Examples

- Infix = a + b * c
  - Postfix = a b c * +
- Infix = a * b + c
  - Postfix = a b * c +
- Infix = (a + b) * (c – d) / (e + f)
  - Postfix = a b + c d - * e f + /

Unary Operators

- Replace with new symbols.
  - + a => a @
  - + a + b => a @ b +
  - - a => a ?
  - - a-b => a ? b -

Postfix Evaluation

- Scan postfix expression from left to right pushing operands on to a stack.
- When an operator is encountered, pop as many operands as this operator needs; evaluate the operator; push the result on to the stack.
- This works because, in postfix, operators come immediately after their operands.

Postfix Evaluation

- (a + b) * (c – d) / (e + f)
- a b + c d - * e f + /
- a b + c d - * e f + /
- a b + c d - * e f + /
- a b + c d - * e f + /

b
a

stack

Postfix Evaluation

- (a + b) * (c – d) / (e + f)
- a b + c d - * e f + /
- a b + c d - * e f + /
- a b + c d - * e f + /
- a b + c d - * e f + /
- a b + c d - * e f + /
- a b + c d - * e f + /
- a b + c d - * e f + /

d
c
(a + b)

stack

Postfix Evaluation

- (a + b) * (c – d) / (e + f)
- a b + c d - * e f + /
- a b + c d - * e f + /

(c – d)
(a + b)

stack

Postfix Evaluation

- (a + b) * (c – d) / (e + f)
- a b + c d - * e f + /
- a b + c d - * e f + /
- a b + c d - * e f + /
- a b + c d - * e f + /
- a b + c d - * e f + /

f
e
(a + b)*(c – d)

stack

Postfix Evaluation

- (a + b) * (c – d) / (e + f)
- a b + c d - * e f + /
- a b + c d - * e f + /
- a b + c d - * e f + /
- a b + c d - * e f + /
- a b + c d - * e f + /
- a b + c d - * e f + /

(e + f)
(a + b)*(c – d)

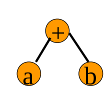
stack

Prefix Form

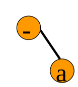
- The prefix form of a variable or constant is the same as its infix form.
  - a, b, 3.25
- The relative order of operands is the same in infix and prefix forms.
- Operators come immediately before the prefix form of their operands.
  - Infix = a + b
  - Postfix = ab+
  - Prefix = +ab

Binary Tree Form

- a + b

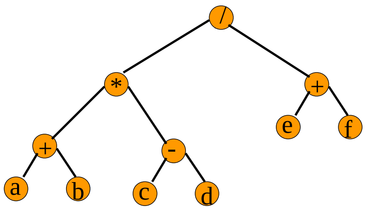


- - a



Binary Tree Form

- (a + b) * (c – d) / (e + f)



Merits Of Binary Tree Form

- Left and right operands are easy to visualize.
- Code optimization algorithms work with the binary tree form of an expression.
- Simple recursive evaluation of expression.