Overflow Handling

- An overflow occurs when the home bucket for a new pair (key, element) is full.
- We may handle overflows by:
  - Search the hash table in some systematic fashion for a bucket that is not full.
    - Linear probing (linear open addressing).
    - Quadratic probing.
    - Random probing.
  - Eliminate overflows by permitting each bucket to keep a list of all pairs for which it is the home bucket.
    - Array linear list.
    - Chain.

Linear Probing – Get And Put

- divisor = b (number of buckets) = 17.
- Home bucket = key % 17.

0				4				8				12				16
34	0	45				6	23	7			28	12	29	11	30	33

- Put in pairs whose keys are 6, 12, 34, 29, 28, 11, 23, 7, 0, 33, 30, 45

Linear Probing – Remove

0				4				8				12				16
34	0	45				6	23	7			28	12	29	11	30	33

- remove(0)

0				4				8				12				16
34		45				6	23	7			28	12	29	11	30	33

- Search cluster for pair (if any) to fill vacated bucket.

0				4				8				12				16
34	45					6	23	7			28	12	29	11	30	33

Linear Probing – remove(34)

0				4				8				12				16
34	0	45				6	23	7			28	12	29	11	30	33

0				4				8				12				16
	0	45				6	23	7			28	12	29	11	30	33

- Search cluster for pair (if any) to fill vacated bucket.

0				4				8				12				16
0		45				6	23	7			28	12	29	11	30	33

0				4				8				12				16
0	45					6	23	7			28	12	29	11	30	33

Linear Probing – remove(29)

0				4				8				12				16
34	0	45				6	23	7			28	12	29	11	30	33

0				4				8				12				16
34	0	45				6	23	7			28	12		11	30	33

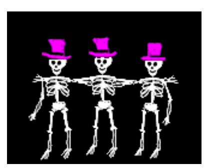
- Search cluster for pair (if any) to fill vacated bucket.

0				4				8				12				16
34	0	45				6	23	7			28	12	11		30	33

0				4				8				12				16
34	0	45				6	23	7			28	12	11	30		33

0				4				8				12				16
34	0					6	23	7			28	12	11	30	45	33

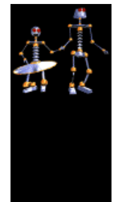
Performance Of Linear Probing



0				4				8				12				16
34	0	45				6	23	7			28	12	29	11	30	33

- Worst-case get/put/remove time is Theta(n), where n is the number of pairs in the table.
- This happens when all pairs are in the same cluster.

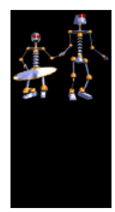
Expected Performance



0				4				8				12				16
34	0	45				6	23	7			28	12	29	11	30	33

- alpha = loading density = (number of pairs)/b.
  - alpha = 12/17.
- S_n = expected number of buckets examined in a successful search when n is large
- U_n = expected number of buckets examined in a unsuccessful search when n is large
- Time to put and remove governed by U_n.

Expected Performance



- $S_n \sim ½(1 + 1/(1 – alpha))$
- $U_n \sim ½(1 + 1/(1 – alpha)^2)$
- Note that 0 <= alpha <= 1.

alpha	S_n	U_n
0.50	1.5	2.5
0.75	2.5	8.5
0.90	5.5	50.5

Alpha <= 0.75 is recommended.

Hash Table Design

- Performance requirements are given, determine maximum permissible loading density.
- We want a successful search to make no more than 10 compares (expected).
  - $S_n \sim ½(1 + 1/(1 – alpha))$
  - alpha <= 18/19
- We want an unsuccessful search to make no more than 13 compares (expected).
  - $U_n \sim ½(1 + 1/(1 – alpha)^2)$
  - alpha <= 4/5
- So alpha <= min{18/19, 4/5} = 4/5.

Hash Table Design

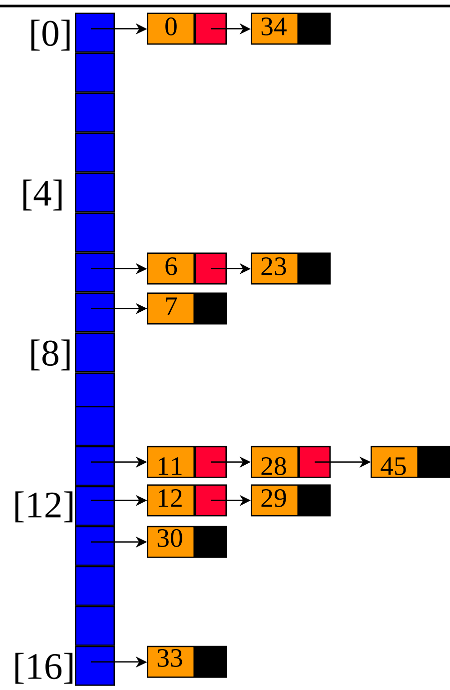
- Dynamic resizing of table.
  - Whenever loading density exceeds threshold (4/5 in our example), rehash into a table of approximately twice the current size.
- Fixed table size.
  - Know maximum number of pairs.
  - No more than 1000 pairs.
  - Loading density <= 4/5 => b >= 5/4*1000 = 1250.
  - Pick b (equal to divisor) to be a prime number or an odd number with no prime divisors smaller than 20.

Linear List Of Synonyms

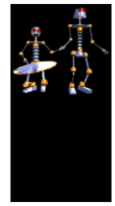
- Each bucket keeps a linear list of all pairs for which it is the home bucket.
- The linear list may or may not be sorted by key.
- The linear list may be an array linear list or a chain.

Sorted Chains

- Put in pairs whose keys are 6, 12, 34, 29, 28, 11, 23, 7, 0, 33, 30, 45
- Home bucket = key % 17.



Expected Performance



- Note that alpha >= 0.
- Expected chain length is alpha.
- S_n ~ 1 + alpha/2.
- U_n <= alpha, when alpha < 1.
- U_n ~ 1 + alpha/2, when alpha >= 1.

java.util.Hashtable

- Unsorted chains.
- Default initial b = divisor = 101
- Default alpha <= 0.75
- When loading density exceeds max permissible density, rehash with newB = 2b+1.