Sparse Matrices

sparse … many elements are zero
dense … few elements are zero

Example Of Sparse Matrices

diagonal
tridiagonal
lower triangular (?)

These are structured sparse matrices.
May be mapped into a 1D array so that a mapping function can be used to locate an element.
Unstructured Sparse Matrices

Airline flight matrix.

- airports are numbered 1 through n
- \( \text{flight}(i,j) \) = list of nonstop flights from airport i to airport j
- \( n = 1000 \) (say)
- \( n \times n \) array of list references => 4 million bytes
- total number of flights = 20,000 (say)
- need at most 20,000 list references => at most 80,000 bytes

Unstructured Sparse Matrices

Web page matrix.

- web pages are numbered 1 through n
- \( \text{web}(i,j) \) = number of links from page i to page j

Web analysis.

- authority page … page that has many links to it
- hub page … links to many authority pages
Web Page Matrix

- $n = 2$ billion (and growing by 1 million a day)
- $n \times n$ array of ints $\Rightarrow 16 \times 10^{18}$ bytes ($16 \times 10^9$ GB)
- each page links to 10 (say) other pages on average
- on average there are 10 nonzero entries per row
- space needed for nonzero elements is approximately $20$ billion x 4 bytes $= 80$ billion bytes (80 GB)

Representation Of Unstructured Sparse Matrices

Single linear list in row-major order.

scan the nonzero elements of the sparse matrix in row-major order

each nonzero element is represented by a triple

(row, column, value)

the list of triples may be an array list or a linked list (chain)
### Single Linear List Example

<table>
<thead>
<tr>
<th>row</th>
<th>column</th>
<th>value</th>
</tr>
</thead>
<tbody>
<tr>
<td>0 3 0 4</td>
<td>1 1 2 2 4 4</td>
<td>3 4 5 7 2 6</td>
</tr>
<tr>
<td>0 5 7 0</td>
<td>3 5 3 4 2 3</td>
<td></td>
</tr>
<tr>
<td>0 0 0 0</td>
<td></td>
<td></td>
</tr>
<tr>
<td>0 2 6 0 0</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

### Array Linear List Representation

<table>
<thead>
<tr>
<th>row</th>
<th>column</th>
<th>value</th>
</tr>
</thead>
<tbody>
<tr>
<td>1 1 2 2 4 4</td>
<td>3 5 3 4 2 3</td>
<td>3 4 5 7 2 6</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>element</th>
<th>row</th>
<th>column</th>
<th>value</th>
</tr>
</thead>
<tbody>
<tr>
<td>0 1 2 3 4 5</td>
<td>1 1 2 2 4 4</td>
<td>3 5 3 4 2 3</td>
<td>3 4 5 7 2 6</td>
</tr>
</tbody>
</table>
Chain Representation

Node structure.

Single Chain

row | col
---|---
1  | 1
1  | 2
2  | 2
2  | 4
4  | 4

list = column | value
---|---
3  | 5  | 3  | 4  | 2  | 3 | 3  | 4  | 5  | 7  | 2  | 6

firstNode
### One Linear List Per Row

<p>| | | | | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0</td>
<td>3</td>
<td>0</td>
<td>4</td>
</tr>
<tr>
<td>0</td>
<td>0</td>
<td>5</td>
<td>7</td>
<td>0</td>
</tr>
<tr>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>0</td>
<td>2</td>
<td>6</td>
<td>0</td>
<td>0</td>
</tr>
</tbody>
</table>

row1 = [(3, 3), (5, 4)]
row2 = [(3, 5), (4, 7)]
row3 = []
row4 = [(2, 2), (3, 6)]

### Array Of Row Chains

Node structure.

```
next   col   value
```


Array Of Row Chains

0 0 3 0 4
0 0 5 7 0
0 0 0 0 0
0 2 6 0 0

Orthogonal List Representation

Both row and column lists.

Node structure.
Orthogonal Lists

Variations

May use circular lists instead of chains.
Approximate Memory Requirements

500 x 500 matrix with 1994 nonzero elements

2D array \[500 \times 500 \times 4 = 1\text{million}\] bytes
Single Array List \[3 \times 1994 \times 4 = 23,928\] bytes
One Chain Per Row \[23928 + 500 \times 4 = 25,928\]

Runtime Performance

Matrix Transpose
500 x 500 matrix with 1994 nonzero elements

2D array \[210\text{ ms}\]
Single Array List \[6\text{ ms}\]
One Chain Per Row \[12\text{ ms}\]
Performance

Matrix Addition.
500 x 500 matrices with 1994 and 999 nonzero elements

<table>
<thead>
<tr>
<th>Method</th>
<th>Time</th>
</tr>
</thead>
<tbody>
<tr>
<td>2D array</td>
<td>880 ms</td>
</tr>
<tr>
<td>Single Array List</td>
<td>18 ms</td>
</tr>
<tr>
<td>One Chain Per Row</td>
<td>29 ms</td>
</tr>
</tbody>
</table>