# CAP5515: Computational Molecular Biology-Homework 2

Due at the **beginning** of the lecture on **04-07-2009**.

**No late** assignment will be accepted.

Consider the modularity function $Q$ which is defined as follows:

**First definition**: The undirected network $G = (V, E)$, with $n$ nodes and $m$ edges, is given by the symmetric matrix $A = (a_{uv})$: $a_{uv} = 1$ if $(u, v) \in E$ and $a_{uv} = 0$ otherwise. Let $deg(u)$ denote the degree of the node $u$. Assume that the network is partitioned into disjoint communities. Define $\delta(u, v) =$ 1if $u, v$ in a same community. Otherwise, $\delta(u, v) = 0$. The modularity $Q$ is defined as

$$Q_1 = \frac{1}{2m}\left(a_{uv} - \frac{deg(u)\ deg(v)}{2m}\right)\delta(u, v) \tag{1}$$

**Second definition**: Assume that the network $G = (V, E)$ is partitioned into communities $\mathcal{C} = \{C_1, C_2, \ldots, C_k\}$. Denote $E(X, Y)$ the number of edges that have one end node in $X$ and other end node in $Y$ with the notice that if $u, v \in X \cap Y$ the edge $(u, v)$ will be counted twice. The modularity is defined as

$$Q_2 = \sum_{C_i \in \mathcal{C}} \left(\frac{E(C_i, C_i)}{2m} - \left(\frac{E(C_i, V)}{2m}\right)^2\right) \tag{2}$$

Do the following questions:

1. (10 pts) Is $Q_1$ equivalent to $Q_2$, i.e. $Q_1(\mathcal{C}) = Q_2(\mathcal{C})$. Justify your answer.
2. (10 pts) We have learnt that $Q_2$ has a resolution limit. How about $Q_1$? Justify your answer.