1 What is Amdahl's Law?

Amdahl's law is an expression used to find the maximum expected improvement to an overall system when only part of the system is improved. It is often used in parallel computing to predict the theoretical maximum speedup using multiple processors.

(source: https://en.wikipedia.org/wiki/Amdahl%27s_law)

2 What Kinds of Problems Do We Solve with Amdahl's Law?

Recall how we defined performance of a system that has been sped up:

$$\text{Speedup} = \frac{\text{Execution time before improvement}}{\text{Execution time after improvement}}$$

There are three types of problems to be solved using the following Amdahl's Law equation:

$$\text{Speedup} = \frac{1}{(1 - \text{fraction enhanced}) + (\text{fraction enhanced/factor of improvement})}$$

Let Speedup be denoted by "S", fraction enhanced be denoted by "f_E", and factor of improvement be denoted by "f_I". Then we can write the above equation as

$$S = ((1 – f_E) + (f_E / f_I))^{-1} .$$

The three problem types are as follows:

1. Determine S given f_E and f_I
2. Determine f_I given S and f_E
3. Determine f_E given S and f_I

Let us consider an example of each type of problem, as follows.

2.1 Problem Type 1 – Predict System Speedup

If we know f_E and f_I, then we use the Speedup equation (above) to determine S.

Example: Let a program have 40 percent of its code enhanced (so $f_E = 0.4$) to run 2.3 times faster (so $f_I = 2.3$). What is the overall system speedup S?

Step 1: Setup the equation: $S = ((1 – f_E) + (f_E / f_I))^{-1}$

Step 2: Plug in values & solve $S = ((1 – 0.4) + (0.4 / 2.3))^{-1}$

$= (0.6 + 0.174)^{-1} = 1 / 0.774$

$= \mathbf{1.292}$

2.2 Problem Type 2 – Predict Speedup of Fraction Enhanced

If we know f_E and S, then we solve the Speedup equation (above) to determine f_I , as follows:

Example: Let a program have 40 percent of its code enhanced (so $f_E = 0.4$) to yield a system speedup 4.3 times faster (so $S = 4.3$). What is the factor of improvement f_I of the portion enhanced?

Case #1:

Can we do this? In other words, let's determine if by enhancing 40 percent of the system, it is possible to make the system go 4.3 times faster …

Step 1: Assume the limit, where f_I = infinity, so $S = ((1 – f_E) + (f_E / f_I))^{-1} \rightarrow S = 1 / (1 – f_E)$

Step 2: Plug in values & solve $S = ((1 – 0.4))^{-1} = 1 / 0.6 = \mathbf{1.67}$.

Step 3: So S = 1.67 is the **maximum possible speedup**, and we cannot achieve S = 4.3 !!

Case #2:

A different case: Let's determine if by enhancing 40 percent of the system, it is possible to make the system go 1.3 times faster …

Step 1: Assume the limit, where f_I = infinity, so $S = ((1 – f_E) + (f_E / f_I))^{-1} \rightarrow S = 1 / (1 – f_E)$

Step 2: Plug in values & solve $S = ((1 – 0.4))^{-1} = 1 / 0.6 = \mathbf{1.67}$.

Step 3: So S = 1.67 is the **maximum possible speedup**, and we <u>can</u> achieve S = 1.3 !!

Step 4: Solve speedup equation for f_I :

$1/S = (1 – f_E) + (f_E / f_I)$ [invert both sides]

$1/S – (1 – f_E) = f_E / f_I$ [subtract $(1 – f_E)$]

$(1/S – (1 – f_E))^{-1} = f_I / f_E$ [invert both sides]

$f_E \cdot (1/S – (1 – f_E))^{-1} = f_I$ [multiply by f_E]

Step 5: Plug in values & solve:

$f_I = f_E \cdot (1/S – (1 – f_E))^{-1}$

$= 0.4 \cdot (1/1.3 – (1 – 0.4))^{-1}$

$= 0.4 / (0.769 – 0.6) = \mathbf{2.367}$

Step 6: Check your work: $S = ((1 – f_E) + (f_E / f_I))^{-1} = (0.6 + (0.4/2.367))^{-1} = 1.3$ ✓

2.3 Problem Type 3 – Predict Fraction of System to be Enhanced

If we know f_I and S, then we solve the Speedup equation (above) to determine f_E , as follows:

Example: Let a program have a portion f_E of its code enhanced to run 4 times faster (so $f_I = 4$), to yield a system speedup 3.3 times faster (so $S = 3.3$). What is the fraction enhanced (f_E)?

Step 1: *Can this be done*? Assuming f_I = infinity, $S = 3.3 = ((1 – f_E))^{-1}$ so minimum $f_E = 0.697$

Yes, this can be done for maximum f_I, so let's solve the equation to determine <u>actual</u> f_E

Step 2: Solve speedup equation for f_E :

$S = ((1 – f_E) + (f_E / f_I))^{-1}$ [state the equation]

$3.3 = ((1 – f_E) + (f_E / 4))^{-1}$ [plug in values]

$(1 – f_E) + f_E/4= 1/3.3 = 0.303$ [invert both sides]

$1 – 0.75f_E = 0.303$ [regroup]

$0.75f_E = 1 – 0.303 = 0.697$ [commutativity]

$f_E = 0.697 / 0.75 = \mathbf{0.929}$ [divide by 0.75]

Step 3: Check your work: $S = ((1 – f_E) + (f_E / f_I))^{-1} = (0.071 + (0.929/4))^{-1} = 3.3$ ✓