The Objects Interaction Graticule for Cardinal Direction Querying in Moving Objects Data Warehouses

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Abstract. Cardinal directions have turned out to be very important qualitative spatial relations due to their numerous applications in spatial wayfinding, GIS, qualitative spatial reasoning and in domains such as cognitive sciences, AI and robotics. They are frequently used as selection criteria in spatial queries. Moving objects data warehouses can help to analyze complex multidimensional data of a spatio-temporal nature and to provide decision support. However, currently there is no available method to query for cardinal directions between spatio-temporal objects in data warehouses. In this paper, we introduce the concept of a *moving objects data warehouse* (*MODW*) for storing and querying multidimensional spatio-temporal data. Further, we also present a novel two-phase approach to model and query for cardinal directions between moving objects by using the MODW framework. First, we apply a tiling strategy that determines the zone belonging to the nine cardinal directions of each spatial object at a particular time and then intersects them. This leads to a *collection of grids* over time called the *Objects Interaction Graticule* (*OIG*). For each grid cell, the information about the spatial objects that intersect it is stored in an Objects Interaction Matrix. In the second phase, an interpretation method is applied to these matrices to determine the cardinal direction between the moving objects. These results are integrated into MDX queries using directional predicates.

1 Introduction

For more than a decade, data warehouses have been at the forefront of information technology applications as a way for organizations to effectively use information for business planning and decision making. The data warehouse contains data that gives information about a particular, decision-making subject instead of about an organization's ongoing operations (*subject-oriented*). Data is gathered into the data warehouse from a variety of sources and then merged into a coherent whole (*integrated*). All the data in a data warehouse can be identified with a particular time period (*time-variant*). Data is periodically added in a data

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warehouse but is hardly ever removed (*non-volatile*). This enables the manager to gain a consistent picture of the business. Thus, the *data warehouse* is a large, subject-oriented, integrated, time-variant and non-volatile collection of data in support of management's decision making process [1, 2]. Online Analytical Processing (OLAP) is the technology that helps perform complex analyses over the information stored in the data warehouse. Data warehouses and OLAP enable organizations to gather overall trends and discover new avenues for growth.

With the emergence of new applications in areas such as geo-spatial, sensor, multimedia and genome research, there is an explosion of complex, spatio-temporal data that needs to be properly managed and analyzed. This data is often complex (with hierarchical, multidimensional nature) and has spatio-temporal characteristics. A good framework to store, query and mine such datasets involves *next-gen* moving objects data warehouses that bring the best tools for data management to support complex, spatio-temporal datasets. The *moving objects data warehouse* (*MODW*) can be defined as a large, subject-oriented, integrated, time-variant, non-volatile collection of analytical, *spatio-temporal* data that is used to support the strategic decision-making process for an enterprise. Moving objects data warehouses help to analyze complex multidimensional geo-spatial data exhibiting temporal variations, and provide enterprise decision support.

Qualitative relations between spatial objects include cardinal direction relations, topological relations and approximate relations. Of these cardinal directions have turned out to be very important due to their application in spatial wayfinding, qualitative spatial reasoning and in domains such as cognitive sciences, robotics, and GIS. In spatial databases and GIS they are frequently used as selection criteria in spatial queries. However, currently there is no available method to model and query for cardinal directions between moving objects (with a spatio-temporal variation).

An early approach to modeling data warehouses with support for several built-in datatypes is presented in [3]. We described a novel system to model cardinal directions between spatial regions in databases using the Object Interaction Matrix (OIM) model in [4]. This model solves the problems found in existing direction relation models like the unequal treatment of the two spatial objects as arguments of a cardinal direction relation, the use of too coarse approximations of the two spatial operand objects in terms of single representative points or MBRs, the lacking property of converseness of the cardinal directions computed, the partial restriction and limited applicability to simple spatial objects only, and the computation of incorrect results in some cases. The basis of the model was a bounded grid called the *objects interaction grid* which helps to capture the information about the spatial objects that intersect each of its cells.

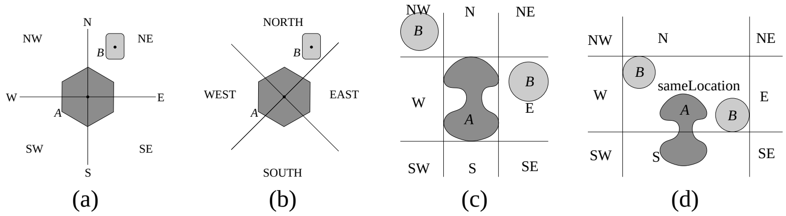


Fig. 1. Projection-based (a) and cone-shaped (b) models, and the Direction-Relation Matrix model with *A* as reference object (c) and with *B* as reference object (d)

Then, we used a matrix to and applied an interpretation method to determine the cardinal direction between spatial objects.

In this paper, we present a novel Objects Interaction Graticule system for modeling cardinal directions between moving objects and querying for such relations. We also introduce a moving objects data warehouse framework to help achieve this task. Our method improves upon the OIM model by adding support for moving objects and provides an innovative approach to model cardinal direction relations inside data warehouses. In a first phase, we apply a multi-grid tiling strategy to determine the zone belonging to the the nine cardinal directions of each spatial object at a particular time and then intersects them. This leads to a collection of grids over time called the Objects Interaction Graticule. For each grid cell the information about the spatial objects that intersect it is stored in an Objects Interaction Matrix. In the second phase, an interpretation method is applied to these matrices to determine the cardinal direction between the moving objects. These results are integrated into MDX queries using directional predicates.

In the next section, we provide a survey of existing techniques to model cardinal directions in general, and discuss their applicability to data warehouses and for modeling direction relations between moving objects. In Section 3, we introduce our moving objects data warehouse framework and the Objects Interaction Graticule Model for modeling cardinal direction relations between moving objects. The Tiling Phase of the model (explained in Section 4) helps to generate the OIM matrix; its Interpretation is achieved in Section 5. Section 6 provides direction predicates and MDX queries [5] that illustrate cardinal direction querying using our model. Finally, Section 7 concludes the paper and provides some directions for future research.

2 Related Work

A good survey of existing approaches for modeling cardinal directions between region objects without temporal variation is provided in [4]. The models pro-

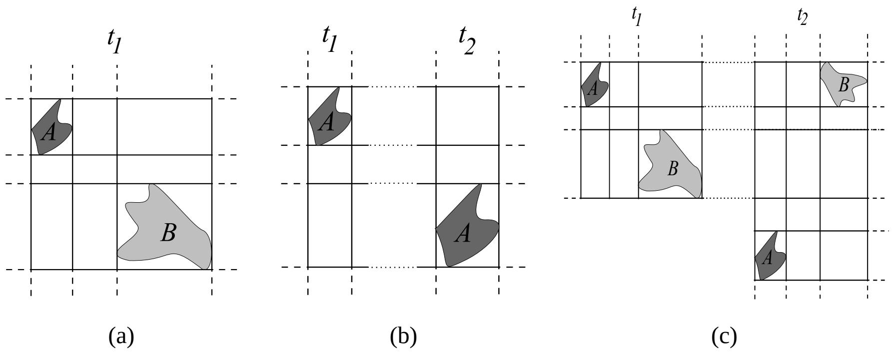


Fig. 2. Possible configurations between two objects: spatial variation *(a)*, spatio-temporal variation in one object *(b)*, spatio-temporal variation in both objects *(c)*.

posed to capture cardinal direction relations between simple spatial objects (like *point*, *line*, and *region* objects) as instances of *spatial data types* [6] can be classified into *tiling-based* models and *minimum bounding rectangle-based* (*MBR-based*) models, some examples of which are shown in Figure 1.

Tiling-based models use partitioning lines that subdivide the plane into tiles. They can be further classified into *projection-based* models and *cone-shaped* models, both of which assign different roles to the two spatial objects involved. The *projection-based* models define direction relations by using partitioning lines parallel to the coordinate axes. The *Direction-Relation Matrix* model [7, 8] helps capture the influence of the objects' shapes as shown in Figure 1c. However, this model only applies to spatial objects with non-temporal variations. It also leads to imprecise results with intertwined objects. We introduced an improved modeling strategy for cardinal directions between region objects in [4]. The *cone-shaped* models define direction relations by using angular zones. The *MBR-based* model [9] approximates objects using minimum bounding rectangles and brings the sides of these MBRs into relation with each other using Allen's interval relations [10].

The data warehouse helps to store and query over large, multidimensional datasets and is hence a good choice for storing and querying spatio-temporal data. We presented a conceptual, user-centric approach to data warehouse design called the *BigCube* model in [3]. Several other models have also been proposed to conceptually model a data warehouse using a cube metaphor as surveyed and extended in [11, 12]. Moving objects, their formal data type characterizations and operations have been introduced in [13, 14]. However there is the lack of a model for *qualitative direction relations between moving objects* in all existing works. This paper provides a clear solution to this problem by first describing the basics of the MODW framework in Section 3 and then introducing the OIG model for gathering direction relations between moving objects in Section 4.

3 Moving Objects Data Warehouses and the Objects Interaction Graticule (OIG) Approach for Modeling Cardinal Directions

The idea behind *moving objects data warehouses* (*MODW*) is to provide a system capable of representing moving entities in data warehouses and be able to ask queries about them. Moving entities could be *moving points* such as people, animals, all kinds of vehicles such as cars, trucks, air planes, ships, etc., where usually only the time-dependent position in space is relevant, not the extent. However, moving entities with an extent, e.g., hurricanes, fires, oil spills, epidemic diseases, etc., could be characterized as *moving regions*. Such entities with a continuous, spatio-temporal variation (in position, extent or shape) are called *moving objects*. With a focus on cardinal direction relations, moving regions are more interesting because of the change in the relationship between their evolving extents over time. In this paper, we focus on simple (single-component, hole-free) moving regions and provide a novel approach to gather direction relations between such objects over time, using a data warehousing framework. The *moving objects data warehouse* is defined by a conceptual cube with moving objects in the data dimensions (containing members) defining the structure of the cube, and its cells containing measure values that quantify real-world facts. The measures and members are instances of moving object data types [13]. The $\mathcal{B}ig\mathcal{C}ube$ [3] is an example of a conceptual, user-centric data warehouse model that can be extended for MODW design. In this paper, our OIG model lies at the conceptual level and the predicates provide the means to implement the model using any logical approach [15]. However, we shall provide MDX queries to help illustrate the versatility of the model in querying for direction relations between moving objects.

The goal of the OIG model is to enable a data warehouse user to query for cardinal directions between moving region objects. To achieve this goal, we need to take the various possible moving objects' configurations into account and model for direction relations in all of the cases to arrive at the overall direction relation. This is because the direction relation between two moving objects, between two queried time instances, can be arrived at only by considering all the direction relations between them during their *lifetimes*. The possible configurations between moving objects that we need to consider include the following. First, two objects could be at different spatial locations at the same instant of time (dual object, spatial variation) as shown in Figure 2(a). Second, an object could be at two different spatial locations at two different instances of time (single object, spatio-temporal variation) as shown in Figure 2(b). Third, two objects could be at two different spatial locations at two different time instances (dual object, spatio-temporal variation) as shown in Figure 2(c). The dotted lines

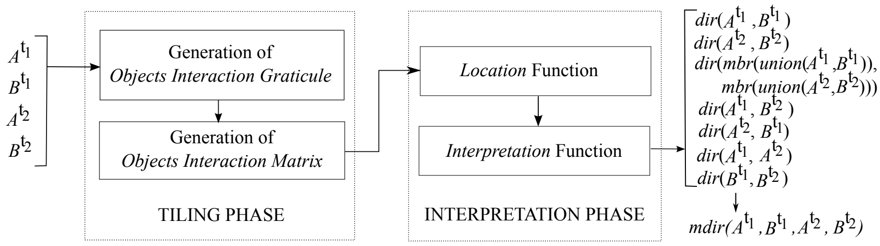


Fig. 3. Overview of the two phases of the Objects Interaction Graticule (OIG) model

between the configuration of objects across time represents the *flux* in the intersection of the coordinate systems used in the space-time continuum. We include this in our model to be able to capture the locations of objects across the time extents. However, the dashed lines indicate the part not bounded by the objects interaction graticule (OIG). The OIG is a closed, bounded region and the dotted lines do not signify any *holes* in the spatio-temporal variation of the moving objects.

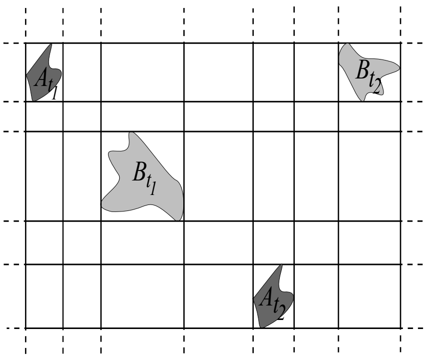
Figure 3 shows the two-phase strategy of our model for calculating the cardinal direction relations between two objects A and B at time instances t_1 and t_2. We assume that A and B (in the general case) are non-empty values of the complex spatial-temporal data type *mregion* [13]. For computing the direction relation between two moving objects' snapshots, we need to consider *all possible direction relations* between the *various combination of objects* in the interacting system. First, we consider the scenario at each snapshot t_1 and t_2, and also the case when $t_1 = t_2$. For these, we default to the OIM approach for directions between objects without temporal variation and gather the direction relations between them. This is given by $dir(A^{t_1}, B^{t_1})$ and $dir(A^{t_2}, B^{t_2})$. The second case arises if $t_1 \neq t_2$. Then five more direction relations can be computed as shown in Figure 3. This includes four combinations for the two objects at t_1 and t_2, given by $dir(A^{t_1}, B^{t_2})$, $dir(A^{t_2}, B^{t_1})$, $dir(A^{t_1}, A^{t_2})$ and $dir(B^{t_1}, B^{t_2})$. Plus, we also relate the entire system (both objects) at each of the different time instances used to determine the query result. This is given by $dir(mbr(union(A^{t_1}, B^{t_1})), mbr(union(A^{t_2}, B^{t_2})))$. Using all these direction relations, we can now compute the moving direction relations between the two regions over time.

Notice that, for clarity, we have used the notation A^t instead of $A(t)$ to refer to the temporal development of the moving region A (A is actually defined by a continuous function $A : time \rightarrow region$). We will use this notation through the rest of the paper. The *tiling phase* in Section 4 details our novel tiling strategy that produces the *objects interaction graticule* and shows how they are represented by *objects interaction matrices*. The *interpretation phase* in Section 5 leverages the objects interaction matrix to derive the directional relationship between two moving region objects.

4 The Tiling Phase: Representing Interactions of Objects with the Objects Interaction Graticule and Matrix

In this section, we describe the *tiling phase* of the model. The general idea of our tiling strategy is to superimpose a graticule called *objects interaction graticule* (*OIG*) on a configuration of two moving spatial objects (regions). Such a graticule is determined by four vertical and four horizontal *partitioning lines* of *each* object at available time instances. The four vertical (four horizontal) partitioning lines of an object are given as infinite extensions of the two vertical (two horizontal) segments of the object's minimum bounding rectangle at each of the two time instances. The partitioning lines of both objects create a partition of the Euclidean plane consisting of multiple mutually exclusive, directional *tiles* or *zones*.

$OIG(A^{t_1}, B^{t_1}, A^{t_2}, B^{t_2}) =$



(a)

$$
\begin{aligned}
OIM(A^{t_1}, B^{t_2}) &= \begin{pmatrix} 1 & 0 & 2 \end{pmatrix} \\
OIM(A^{t_2}, B^{t_1}) &= \begin{pmatrix} 2 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix} \\
\begin{array}{r} OIM(mbr(union(A^{t_1}, B^{t_1})), \\ mbr(union(A^{t_2}, B^{t_2}))) \end{array} &= \begin{pmatrix} 1 & 0 & 2 \\ 0 & 0 & 2 \end{pmatrix} \\
OIM(A^{t_1}, A^{t_2}) &= \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix} \\
OIM(B^{t_1}, B^{t_2}) &= \begin{pmatrix} 0 & 0 & 2 \\ 0 & 0 & 0 \\ 2 & 0 & 0 \end{pmatrix}
\end{aligned}
$$

(b)

Fig. 4. The objects interaction graticule $OIG(A, B)$ for the two region objects A and B in Figures 1c and 1d (a) and the derived objects interaction matrices (*OIM*) for OIG components described in Definition 3.

In the most general case, all partitioning lines are different from each other, and we obtain an overlay partition with central, bounded tiles and peripheral, unbounded tiles (indicated by the dashed segments in Figure 4 (a)). The unbounded tiles do not contain any objects and therefore, we exclude them and obtain a graticule space that is a bounded proper subset of $\mathbb{R}^2$, as Definition 1 states.

Definition 1. *Let R= ($A^{t_1}, B^{t_1}, A^{t_2}, B^{t_2}$), $R \in region$ with $A^{t_1} \neq \varnothing \wedge B^{t_1} \neq \varnothing \wedge A^{t_2} \neq \varnothing \wedge B^{t_2} \neq \varnothing$, and let* $min_x^r = \min\{x \,|\, (x,y) \in r\}$, $max_x^r = \max\{x \,|\, (x,y) \in r\}$, $min_y^r = \min\{y \,|\, (x,y) \in r\}$, *and* $max_y^r = \max\{y \,|\, (x,y) \in r\}$ *for* $r \in \{A^{t_1}, B^{t_1}, A^{t_2}, B^{t_2}\}$. *The* objects interaction graticule space (OIGS) *of* $A^{t_1}, B^{t_1}, A^{t_2}$ *and* B^{t_2} *is given as:*

$$\begin{aligned}\mathrm{OIGS}(R) = \{(x,y) \in \mathbb{R}^2 \,|\, &\min(min_x^{A^{t_1}}, min_x^{B^{t_1}}, min_x^{A^{t_2}}, min_x^{B^{t_2}}) \le x \le \\ &\max(max_x^{A^{t_1}}, max_x^{B^{t_1}}, max_x^{A^{t_2}}, max_x^{B^{t_2}}) \wedge \min(min_y^{A^{t_1}}, min_y^{B^{t_1}}, \\ &min_y^{A^{t_2}}, min_y^{B^{t_2}}) \le y \le \max(max_y^{A^{t_1}}, max_y^{B^{t_1}}, max_y^{A^{t_2}}, max_y^{B^{t_2}})\}\end{aligned}$$

Definition 2 determines the bounded graticule formed as a part of the partitioning lines and superimposed on $OIGS(A^{t_1}, B^{t_1}, A^{t_2}, B^{t_2})$.

Definition 2. *Let* seg *be a function that constructs a segment between any two given points* $p, q \in \mathbb{R}^2$, *i.e.,* $seg(p,q) = \{t \,|\, t = (1-\lambda)p + \lambda q, 0 \le \lambda \le 1\}$. *Let* $H_r = \{seg((min_x^r, min_y^r), (max_x^r, min_y^r)), seg((min_x^r, max_y^r), (max_x^r, max_y^r))\}$ *and* $V_r = \{seg((min_x^r, min_y^r), (min_x^r, max_y^r)), seg((max_x^r, min_y^r), (max_x^r, max_y^r))\}$ *for* $r \in \{A^{t_1}, B^{t_1}, A^{t_2}, B^{t_2}\}$. *We call the elements of* $H_{A^{t_1}}$, $H_{B^{t_1}}$, $H_{A^{t_2}}$, $H_{B^{t_2}}$, $V_{A^{t_1}}$, $V_{B^{t_1}}$, $V_{A^{t_2}}$ *and* $V_{B^{t_2}}$ objects interaction graticule segments. *Then, the* objects interaction graticule (OIG) *for A and B is given as:*

$$OIG(A^{t_1}, B^{t_1}, A^{t_2}, B^{t_2}) = H_{A^{t_1}} \cup V_{A^{t_1}} \cup H_{B^{t_1}} \cup V_{B^{t_1}} \cup H_{A^{t_2}} \cup V_{A^{t_2}} \cup H_{B^{t_2}} \cup V_{B^{t_2}}.$$

In the OIG of an object, there are two constituent *object interaction coordinate systems* (*OICS*) for each temporal state of the moving objects. These are defined as follows:

$$OICoordS(A^{t_1}, B^{t_1}) = H_{A^{t_1}} \cup V_{A^{t_1}} \cup H_{B^{t_1}} \cup V_{B^{t_1}}, \text{ and}$$
$$OICoordS(A^{t_2}, B^{t_2}) = H_{A^{t_2}} \cup V_{A^{t_2}} \cup H_{B^{t_2}} \cup V_{B^{t_2}}.$$

The definition of OIG comprises the description of all graticules that can arise. In the most general case, if $t_1 = t_2$ and $H_{A^{t_1}} \cap H_{B^{t_1}} = \varnothing$ and $V_{A^{t_1}} \cap V_{B^{t_1}} = \varnothing$, we obtain a bounded 3×3-graticule similar to that for a non-temporal variation in the objects configurations. Special cases arise if $H_{A^{t_1}} \cap H_{B^{t_1}} \neq \varnothing$ and/or $V_{A^{t_1}} \cap V_{B^{t_1}} \neq \varnothing$. Then equal graticule segments coincide in the union of all graticule segments. As a result, depending on the relative position of two objects to each other, the objects interaction graticule can be of different sizes. However, due to the non-empty property of a region object, not all graticule segments can coincide. This means that at least two horizontal graticule segments and at least two vertical graticule segments must be maintained. Definition 3 gives a formal characterization for the OIG.

Definition 3. *An objects interaction graticule* $\mathrm{OIG}(A^{t_1}, B^{t_1}, A^{t_2}, B^{t_2})$ *consists of two objects interaction coordinate systems, at* t_1 *and* t_2, *each of size* $m \times n$, *with*

$m,n \in \{1,2,3\}$, if $|H_A \cap H_B| = 3-m$ and $|V_A \cap V_B| = 3-n$. Further, it also consists of four Objects Interaction Grids for each of the spatio-temporal combinations of the two moving objects and a fifth for the overall system. Together, these are called the objects interaction graticule components.

The objects interaction graticule partitions the objects interaction graticule space into *objects interaction graticule tiles* (*zones*, *cells*). Definition 4 provides their definition for each of the time instances uniquely, using the objects interaction coordinate systems.

Definition 4. *Given $A^{t_1}, B^{t_1}, A^{t_2}, B^{t_2} \in region$ with $A^{t_1} \neq \varnothing \wedge B^{t_1} \neq \varnothing \wedge A^{t_2} \neq \varnothing \wedge B^{t_2} \neq \varnothing$, $\mathrm{OIGS}(A^{t_1}, B^{t_1}, A^{t_2}, B^{t_2})$, and $\mathrm{OIG}(A^{t_1}, B^{t_1}, A^{t_2}, B^{t_2})$, we define $c_H = |H_A \cup H_B| = |H_A| + |H_B| - |H_A \cap H_B|$ and c_V correspondingly at a time instant. Let $H_{AB} = H_A \cup H_B = \{h_1, \ldots, h_{c_H}\}$ such that (i) $\forall\, 1 \leq i \leq c_H : h_i = seg((x_i^1, y_i), (x_i^2, y_i))$ with $x_i^1 < x_i^2$, and (ii) $\forall\, 1 \leq i < j \leq c_H : h_i < h_j$ (we say that $h_i < h_j :\Leftrightarrow y_j < y_i$). Further, let $V_{AB} = V_A \cup V_B = \{v_1, \ldots, v_{c_V}\}$ such that (i) $\forall\, 1 \leq i \leq c_V : v_i = seg((x_i, y_i^1), (x_i, y_i^2))$ with $y_i^1 < y_i^2$, and (ii) $\forall\, 1 \leq i < j \leq c_V : v_i < v_j$ (we say that $v_i < v_j :\Leftrightarrow x_i < x_j$).*

Next, we define four auxiliary predicates that check the position of a point (x,y) with respect to a graticule segment:

$$\begin{aligned}
below((x,y),h_i) &\Leftrightarrow x_i^1 \leq x \leq x_i^2 \wedge y \leq y_i \\
above((x,y),h_i) &\Leftrightarrow x_i^1 \leq x \leq x_i^2 \wedge y \geq y_i \\
right_of((x,y),v_i) &\Leftrightarrow y_i^1 \leq y \leq y_i^2 \wedge x \geq x_i \\
left_of((x,y),v_i) &\Leftrightarrow y_i^1 \leq y \leq y_i^2 \wedge x \leq x_i
\end{aligned}$$

An objects interaction graticule tile *$t_{i,j}$ with $1 \leq i < c_H$ and $1 \leq j < c_V$ is then defined for a particular time instant as*

$$\begin{aligned}
t_{i,j} = \{(x,y) \in \mathrm{OIGS}(A,B) \,|\, & below((x,y),h_i) \wedge above((x,y),h_{i+1}) \wedge \\
& right_of((x,y),v_j) \wedge left_of((x,y),v_{j+1})\}
\end{aligned}$$

The definition indicates that all tiles are bounded and that two adjacent tiles share their common boundary. Let $OIGT(A,B)$ be the set of all tiles $t_{i,j}$ imposed by $OIG(A,B)$ on $OIGS(A,B)$. An $m \times n$-graticule contains $m \cdot n$ bounded tiles.

By applying our tiling strategy, an objects interaction graticule can be generated for any two region objects A and B. It provides us with the valuable information which region object intersects which tile across the temporal variations. With each time event t_1 and t_2, Definition 5 provides us with a definition of the *interaction* of A and B with a tile.

Definition 5. *Given $A, B \in region$ with $A \neq \varnothing$ and $B \neq \varnothing$ and $\mathrm{OIGT}(A,B)$, let ι be a function that encodes the* interaction *of A and B with a tile $t_{i,j}$, and checks* *whether no region, A only, B only, or both regions intersect a tile. We define this function as*

$$\iota(A,B,t_{i,j}) = \begin{cases} 0 \textit{ if} & A^\circ \cap t_{i,j}^\circ = \varnothing \ \wedge \ B^\circ \cap t_{i,j}^\circ = \varnothing \\ 1 \textit{ if} & A^\circ \cap t_{i,j}^\circ \neq \varnothing \ \wedge \ B^\circ \cap t_{i,j}^\circ = \varnothing \\ 2 \textit{ if} & A^\circ \cap t_{i,j}^\circ = \varnothing \ \wedge \ B^\circ \cap t_{i,j}^\circ \neq \varnothing \\ 3 \textit{ if} & A^\circ \cap t_{i,j}^\circ \neq \varnothing \ \wedge \ B^\circ \cap t_{i,j}^\circ \neq \varnothing \end{cases}$$

We use the mbr and union functions for computing the minimum bounding rectangle and the spatial union of two objects, respectively. To support both objects interaction coordinate systems we extend ι to accept $mbr(union(A^{t_1}, B^{t_1}))$ and $mbr(union(A^{t_2}, B^{t_2}))$ as operands. The operator $^\circ$ denotes the point-set topological *interior* operator and yields a region without its boundary. For each graticule cell $t_{i,j}$ in the ith row and jth column of an $m \times n$-graticule with $1 \leq i \leq m$ and $1 \leq j \leq n$, we store the coded information in an *objects interaction matrix* (*OIM*) in cell $OIM(A,B)_{i,j}$.

$$OIM(A,B) = \begin{pmatrix} \iota(A,B,t_{1,1}) & \iota(A,B,t_{1,2}) & \iota(A,B,t_{1,3}) \\ \iota(A,B,t_{2,1}) & \iota(A,B,t_{2,2}) & \iota(A,B,t_{2,3}) \\ \iota(A,B,t_{3,1}) & \iota(A,B,t_{3,2}) & \iota(A,B,t_{3,3}) \end{pmatrix}$$

5 The Interpretation Phase: Assigning Semantics to the Objects Interaction Matrix

The second phase of the OIG model is the *interpretation phase*. This phase takes an objects interaction matrix (OIM)obtained as the result of the tiling phase as input and uses it to generate a set of cardinal directions as output. This is achieved by separately identifying the locations of both objects in the objects interaction matrix and by pairwise interpreting these locations in terms of cardinal directions. The union of all these cardinal directions is the result. This phase is similar to the Interpretation Phase of the OIM model [4].

We use an interpretation function to determine the basic cardinal direction between any two object components on the basis of their (i, j)-locations in the objects interaction matrix. The composite cardinal relation between A and B is then the union of all determined relations.

In a first step, we define a function *loc* (see Definition 6) that acts on one of the region objects A or B and their OIM and determines all locations of components of each object in the matrix for both temporal extents individually. Let $I_{m,n} = \{(i, j) \,|\, 1 \leq i \leq m, 1 \leq j \leq n\}$. We use an index pair $(i, j) \in I_{m,n}$ to represent the location of the element $M_{i,j} \in \{0,1,2,3\}$ and thus the location of an object component from A or B in an $m \times n$ objects interaction matrix.

Definition 6. *Let M be the $m \times n$-objects interaction matrix of two region objects A and B. Then the function loc is defined as:*

$$loc(A,M) = \{(i,j) \,|\, 1 \leq i \leq m, 1 \leq j \leq n, M_{i,j} = 1 \,\vee\, M_{i,j} = 3\}$$
$$loc(B,M) = \{(i,j) \,|\, 1 \leq i \leq m, 1 \leq j \leq n, M_{i,j} = 2 \,\vee\, M_{i,j} = 3\}$$

In a second step, we define an *interpretation function* ψ to determine the cardinal direction between any two object components of A and B on the basis of their locations in the objects interaction matrix. We use a popular model with the nine *basic cardinal directions*: *north* (N), *northwest* (NW), *west* (W), *southwest* (SW), *south* (S), *southeast* (SE), *east* (E), *northeast* (NE), and *origin* (O) to symbolize the possible cardinal directions between *object components*. A different set of basic cardinal directions would lead to a different interpretation function and hence to a different interpretation of index pairs. Definition 7 provides the interpretation function ψ with the signature $\psi : I_{m,n} \times I_{m,n} \rightarrow CD$.

Definition 7. *Given $(i,j),(i',j') \in I_{m,n}$, the* interpretation function ψ *on the basis of the set $CD = \{\mathrm{N}, \mathrm{NW}, \mathrm{W}, \mathrm{SW}, \mathrm{S}, \mathrm{SE}, \mathrm{E}, \mathrm{NE}, \mathrm{O}\}$ of* basic cardinal directions *is defined as*

$$\psi((i,j),(i',j')) = \begin{cases} \mathrm{N} & \textit{if } i < i' \wedge j = j' \\ \mathrm{NW} & \textit{if } i < i' \wedge j < j' \\ \mathrm{W} & \textit{if } i = i' \wedge j < j' \\ \mathrm{SW} & \textit{if } i > i' \wedge j < j' \\ \mathrm{S} & \textit{if } i > i' \wedge j = j' \\ \mathrm{SE} & \textit{if } i > i' \wedge j > j' \\ \mathrm{E} & \textit{if } i = i' \wedge j > j' \\ \mathrm{NE} & \textit{if } i < i' \wedge j > j' \\ \mathrm{O} & \textit{if } i = i' \wedge j = j' \end{cases}$$

The main difference compared to the OIM approach however is in the following third and final step. We temporally *lift* the *dir* cardinal direction relation function to include objects over their temporal extents. Here, we specify the *cardinal direction function* named *mdir* (moving-direction) which determines the *composite moving cardinal direction* for two moving region objects A and B. This function has the signature $mdir : region_{t_1} \times region_{t_2} \rightarrow 2^{CD}$ and yields a set of basic cardinal directions as its result. In order to compute the function *dir*, we first generalize the signature of our interpretation function ψ to $\psi : 2^{I_{m,n}} \times 2^{I_{m,n}} \rightarrow 2^{CD}$ such that for any two sets $X, Y \subseteq I_{m,n}$ holds: $\psi(X,Y) = \{\psi((i,j),(i',j')) \,|\, (i,j) \in X, (i',j') \in Y\}$. We are now able to specify the cardinal direction function *mdir* in Definition 8.

Definition 8. *Let* $A, B \in region$ *and* $dir(A,B) = \psi(loc(A, OIM(A,B)),$
$loc(B, OIM(A,B)))$. *Then the* cardinal direction function mdir *is defined as*

$$\begin{aligned} mdir(A^{t_1}, B^{t_1}, A^{t_2}, B^{t_2}) = \; & dir(A^{t_1}, B^{t_1}) \cup dir(A^{t_2}, B^{t_2}) \cup dir((A^{t_1}, B^{t_1}), (A^{t_2}, B^{t_2})) \cup \\ & dir(A^{t_1}, B^{t_2}) \cup dir(A^{t_2}, B^{t_1}) \cup dir(A^{t_1}, A^{t_2}) \cup dir(B^{t_1}, B^{t_2}) \end{aligned}$$

We apply this definition to our example in Figure 4. With $loc(A^{t_1}, OIM(At_1, Bt_1))$ $= \{(1,1)\}$ and $loc(B^{t_1}, OIM(A^{t_1}, B^{t_1})) = \{(3,3)\}$, and so on, we obtain

$$\begin{aligned} mdir(A^{t_1}, B^{t_1}, A^{t_2}, B^{t_2}) &= \{\psi((1,1),(3,3)), \psi((3,1),(1,3)), \psi(\{(1,1)\}, \\ &\quad \{(1,3),(2,3)\}), \psi((1,1),(1,3)), \psi((3,1),(1,2)), \\ &\quad \psi((1,1),(3,1)), \psi((3,1),(1,3))\} \\ &= \{NW, SW, W, SE\} \end{aligned}$$

Finally we can say regarding Figure 4 that "Object A is *partly northwest*, *partly southwest*, *partly west*, and *partly southeast* of object B over the period from time t_1 to t_2". Each of the individual directions between the moving objects for the three possible configurations described in Section 3 can also be provided by using the results from each application of *dir*, that is used to finally arrive at the moving direction relations (given by *mdir*).

6 Directional Predicates for OLAP Querying in Moving Object Data Warehouses

Based on the OIG model and the interpretation mechanism described in the previous sections, we can identify the cardinal directions between any given two moving region objects. To integrate the cardinal directions into moving object data warehouses as selection and join conditions in spatial queries, binary *directional predicates* need to be formally defined. For example, a query like "Find all hurricanes that affect states which lie strictly to the north of Florida" requires a directional predicate like *strict_north* as a selection condition of a spatial join.

The *mdir* function, which produces the final moving cardinal directions between two complex region objects A and B across temporal variation, yields a subset of the set $CD = \{N, NW, W, SW, S, SE, E, NE, O\}$ of *basic cardinal directions*. As a result, a total number of $2^9 = 512$ cardinal directions can be identified. Therefore, at most 512 directional predicates can be defined to provide an *exclusive* and *complete* coverage of all possible directional relationships. We can assume that users will not be interested in such a large, overwhelming collection of detailed predicates since they will find it difficult to distinguish, remember and handle them. Instead we provide a mechanism for the user to define and maintain several levels of predicates for querying. As a first step, in Definition 9, we propose nine *existential directional predicates* that ensure the existence of a particular basic cardinal direction between parts of two region objects A and B.

Definition 9. *Let* $R = (A^{t_1}, B^{t_1}, A^{t_2}, B^{t_2}), R \in region$*. Then the existential directional predicate for north is defined as:*

$$exists\_north(R) \equiv (N \in mdir(R))$$

Eight further existential direction predicates for S, E, W, O, NE, SE, NW, and SW are also defined correspondingly. Later, by using $\neg$, $\vee$ and $\wedge$ operators, the user will be able to define any set of composite *derived directional predicates* from this set for their own applications.

We shall provide two examples for these. The first set of predicates is designed to handle *similarly oriented directional predicates* between two regions. *Similarly oriented* means that several cardinal directions facing the same general orientation belong to the same group. Definition 10 shows an example of *northern* by using the existential predicates.

Definition 10. *Let* $R = (A^{t_1}, B^{t_1}, A^{t_2}, B^{t_2}), R \in region$*. Then* northern *is defined as:*

$$northern(R) = exists\_north(R) \vee exists\_northwest(R) \vee exists\_northeast(R)$$

The other similarly oriented directional predicates *southern*, *eastern*, and *western* are defined in a similar way.

The second set of predicates is designed to handle *strict directional predicates* between two region objects. *Strict* means that two region objects are in exactly one basic cardinal direction to each other. Definition 11 shows an example of *strict_north* by using the existential predicates.

Definition 11. *Let* $R = (A^{t_1}, B^{t_1}, A^{t_2}, B^{t_2}), R \in region$*. Then* strict_north *is defined as:*

$$\begin{aligned} strict\_north(R) = {} & exists\_north(R) \wedge \neg exists\_south(R) \wedge \neg exists\_west(R) \wedge \\ & \neg exists\_east(R) \wedge \neg exists\_northwest(R) \wedge \neg exists\_northeast(R) \wedge \\ & \neg exists\_southwest(R) \wedge \neg exists\_southeast(R) \wedge \neg exists\_origin(R) \end{aligned}$$

The other strict directional predicates *strict_south*, *strict_east*, *strict_west*, *strict_origin*, *strict_northeast*, *strict_northwest*, *strict_southeast*, *strict_southwest*, *strict_northern*, *strict_southern*, *strict_eastern*, and *strict_western* are defined in a similar way.

We can now employ these predicates in MDX queries in the moving objects data warehouse. For example, assuming we are given a sample *WeatherEvents* cube (analogous to a spreadsheet table) with hurricane names (ordered in categories according to their intensity) from several years and containing geographic information, we can pose the following query:

Determine the names of hurricanes, ordered in categories according to their intensity, which had a path moving towards the east from their point of origin, and affected states strictly in the northern part of Florida, during the period from 2005 to 2009.
The corresponding MDX query is as follows:

```
SELECT { [Date].[2005] : [Date].[2009] } ON ROWS,
{ NON EMPTY Filter( {[Measures].[Hurricanes].[Category].MEMBERS},
   exists_east( [Measures].[Hurricanes].CurrentMember,
  [Measures].[Hurricanes])) } ON COLUMNS,
{ [Geography].[Country].[State]} ON PAGES,
FROM Cube_WeatherEvents
WHERE ( strict_northern( [Geography].[Country].[State].MEMBERS,
        [Geography].[Country].[USA].[FL] ))
```

A sample result of this query is shown below.

		2005	2006	2007	2008	2009
Georgia	Cat-2	Kevin	Bronco		Tracy	Nobel
	Cat-3	Cindy	Alberto	Barry	Fay	Ida
	Cat-4	Katrina	Alberto			Ida
North Carolina	Cat-2	Cindy	Alberto		Hought	Jives
	Cat-3	Katrina	Ernesto	Sabley	Hanna	Vorice

7 Conclusion and Future Work

In this paper, we introduce the concept of a *moving objects data warehouse* (MODW) for storing and querying multidimensional, spatial-temporal data. We also present a novel approach called the *Objects Interaction Graticule* (*OIG*) model to determine the cardinal directions between moving, simple (single-component, hole-free) region objects. We also show how directional predicates can be derived from the cardinal directions and use them in MDX queries.

In the future, we plan to extend our approach to include complex moving points, lines and other mixed combinations of moving object data types. Further work includes an efficient implementation of the moving objects data warehouse, and the design of spatial reasoning techniques for direction relations using the objects interaction graticule model.

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