PLR Partitions: A Conceptual Model of Maps

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Abstract. The traditional spatial data models model space in terms of points, lines, and regions. These models employ a *disjoint dimension model* in which a spatial object can only consist of a zero, one, or two-dimensional point set. However, such models cannot adequately represent spatial reality. For example, consider a river network that contains both rivers and lakes. Intuitively, this river network is a single object made up of one-dimensional components (the river segments), and two-dimensional components (the lakes). Typically, collection types are used to represent such an object, but they present new problems. In this paper, we propose the *PLR partition* model that is able to model space in the form of a *map geometry* that can contain point, line, and region features within the same object. This model solves the problems associated with the traditional spatial data models.

1 Introduction

Spatially oriented disciplines such as spatial databases, geographical information systems (GIS), digital cartography, CAD, geocomputation, geoinformatics, artificial intelligence, computer vision, image databases, robotics, and cognitive science are fundamentally affected by notions of space and the properties of the models used to define space for these fields. In many cases, the expressiveness and the limitations of the formal spatial model underlying applications in these fields directly impacts the abilities of systems built upon them. Thus, the development of new, more powerful data models has far reaching implications.

Spatial data models attempt to model space using geometric constructs to represent real-world spatial features. For example, the spatial data types defined in most spatial data models consist of points, lines, and regions, which are used to model zero-, one-, and two-dimensional features, respectively. We denote such models as *traditional spatial models*. However, the design of spatial models based on these types reflects fundamental assumptions about space that limit their generality. Specifically, we identify three limitations of models that represent space in this manner. First, traditional spatial models are *disjoint dimension models*, meaning that spatial objects are defined based on their dimensionality (that is, points, lines, and regions). A common problem associated with such models arises with the use of spatial operations. For example, consider the two

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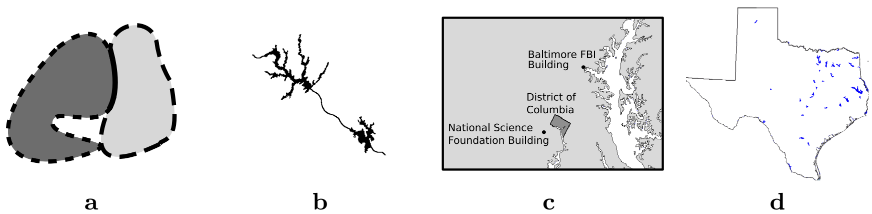

Fig. 1. Scenes that require multiple points, lines, or regions for representation.

regions in Figure 1a. In this case, the intersection of the regions does not result in a region; however, the spatial intersection between regions is typically defined such that a new region is returned. In practice, this situation is handled by defining specific intersection operations that take two regions and return a point, a line, or a region, respectively. The assumption is that the user will indicate what type of object they wish to compute by choosing the appropriate operation. However, the intersection of the regions in this case results in both a point and a line. It is clear that defining multiple intersection operations does not address the fundamental problem. In order to compute this information, the user must compute two different spatial intersection operations. The end result is that no matter which operation is chosen, information about the intersection may be lost. We denote this problem the *dimension reduction problem* for spatial operations.

A second problem that follows from the disjoint dimension model of spatial data types is that the model implicitly imposes the restriction that an object must contain features of only a single dimension. In practice, this constraint causes current spatial data models to have limitations in their abilities to represent geographic reality. For instance, consider Figure 1b, and Figure 1c. These figures each depict an object containing components of multiple dimension: a river system containing lakes and the river feeding them (**b**), and United States Federal Government properties including the District of Columbia, the Baltimore FBI building, and the National Science Foundation building (**c**). Each of these figures contain what can intuitively be modeled as a single object containing features of multiple dimensions. These objects cannot be modeled by a single spatial object using current spatial models. For instance, assuming that the river in **b** is to be represented as a line, then either the disjoint one-dimensional sections must be grouped into an object, or a representative path of the river through the two-dimensional objects (lakes) must be arbitrarily chosen. Similarly in **c**, current spatial models will not allow a single object to represent both the two-dimensional area of the District of Columbia along with the points indicating the federal buildings, even though they are all federal properties. We term this inability to model components of multiple dimensions of a single object the *dimension represention problem* of spatial object models.

Finally, traditional spatial models do not allow multiple components of an object to be identified separately from each other. For instance, if a region has two faces, both of those faces belong to the region and cannot be identified separately. Thus, two faces in a region cannot meet along a common boundary since they will be indistinguishable from their union. A common consequence of this problem occurs when modelling bodies of water within a state (Figure **1d**). A lake in the interior of a state is often modeled as a hole in the state, even though the lake belongs to the state (the state patrols the lake, builds structures that extend into the lake, etc.). The only choices for representation are to not represent that lake in the region representing the state (which is geographically inaccurate) or to model it as a hole (which does not indicate the state's ownership of the lake). In either case, a tradeoff occurs when modelling geographic or political reality. We term this situation the *feature restriction problem*, because it arises from fundamental restrictions of traditional spatial models.

In this paper, we address the shortcomings of traditional spatial models by introducing a new spatail representation model, called the point-line-region (PLR) partition model, that does not suffer from the dimension reduction problem, the dimension representation problem, or the feature restriction problem. We achieve this by defining a PLR partition as a fundamental unit of spatial representation. A PLR partition is a type of map geometry that can contain multiple features such that each feature can be zero-, one-, or two-dimensional, thus solving the dimension representation problem. We define each feature in a PLR partition to be associated with a *label*. The use of labels allows for PLR partitions to model thematic information of arbitrary complexity, and also provides a solution to the feature restriction problem. For example, two region features in a map geometry can meet along a one-dimensional boundary, but be differentiated based on their labels. Finally, because a PLR partition can model features of multiple dimensions, multiple types of a single spatial operation, such as intersection, need not be defined, thus solving the dimension reduction problem. Furthermore, we design PLR partitions in such a way that the geometries of point, line, and region features in a PLR partition correspond to the definitions of points, lines, and regions used in traditional spatial models. This allows us to leverage the vast amount of knowledge built upon traditional spatial models, such as topological relationships, to be directly applied to the features of map geometries. Thus, our model is conceptually intuitive to people familiar with traditional spatial models, but overcomes the limitations associated with them.

In Section 2 we review relevant related work to map geometries and spatial data models. We then define the type for PLR partitions in Section 3. Finally, in Section 4, we draw some conclusions.

2 Related Work

Research into spatial data types tends to focus on the disjoint dimension models of spatial data. Specifically, early work dealt with simple points, lines, and regions. Increased application requirements and a lack of closure properties of the simple spatial types lead to the development of the complex spatial types: complex points (a single point object consisting of a collection of simple points), complex lines (which can represent networks such as river systems), and complex regions that are made up of multiple faces and holes (that is, a region representing Italy, its islands, and the hole representing the Vatican) [1]. These types are defined based on concepts from point set topology, which allow the identification of the interior, exterior, and boundary of the spatial objects.

The idea of a *map* or *map geometry* as a data type has received significant attention. In [2–5], a map is not defined as a data type itself, but as a collection of spatial regions that satisfy some topological constraints. This approach is similar to the geometry collection concept put forth by the Open Geospatial Consortium. The drawback to this model is that a map is not regarded as a data type, but a collection of other types. Thus, it is unclear in many cases how constraints between participating spatial objects can be enforced and how type closure under spatial operations can be achieved. Other approaches to defining maps have focused on *raster* or *tessellation* approaches [6, 7]. However, such approaches are not general enough for our purposes in the sense that the geometries of maps in these models are restricted to the tessellation scheme in use. In [8], the authors consider a map geometry to be a planar subdivision; however, they do not discuss how a planar subdivision should be modeled except to say that data structures such as winged edge or quad edge structures should be used. In [9] map geometries are represented as planar graphs. However, the plane graph, as defined, is not able to model thematic properties of the map.

The work that comes closest to ours is presented in [10]. The authors of this paper define an abstract, mathematical data model that formally describes the type of *spatial partitions*. A spatial partition is able to model multiple labeled region features together in a single object. However, this model is unable to represent line and point features.

3 The PLR Partition Model

Intuitively, we define a PLR partition as a map geometry containing point, line, and region features such that each feature is associated with a label. For example, a single PLR partition can represent a map of a country that depicts the country's provinces, cities, and roads, complete with their names. PLR partitions are also able to represent the example map geometries shown in Figure 1.

Spatial objects have traditionally been defined by partitioning the plane into point sets that identify the different parts of an object. For example, lines and regions have been defined such that they partition the plane into point sets containing the interior, boundary, and exterior points of the objects. We define PLR partitions in a similar manner by partitioning the plane into point sets. However, we cannot simply define a PLR partition as a set theoretic partition of $\mathbb{R}^2$. In general, a set partition is defined as a complete decomposition of a set S into non-empty subsets $\{S_i \in S | i \in I\}$ called *blocks* such that the union of the blocks is equal to the original set and the blocks are pairwise disjoint. We cannot model PLR partitions as a set partition of the plane (that is, partition $\mathbb{R}^2$ by a function $\pi : \mathbb{R}^2 \rightarrow I$) for two reasons: I does not contain semantically relevant values, and we must impose constraints on blocks in order for the features in a PLR partition to correspond to the traditional spatial types. Therefore, we proceed in three steps: we first define a mapping from points in the plane to blocks identified by semantically relevant labels. We then provide mathematical notations that we use to identify labeled blocks, and we then identify blocks and define the properties of these blocks that must hold in order for such a mapping to be a valid PLR partition. For the remainder of this paper, we use the terms *PLR partition* and *partition* interchangeably.

3.1 Definition of a Spatial Mapping

One of the goals of our PLR partition definition is that it associates labels with spatial features in partitions. Intuitively, this is similar to marking or coloring different portions of a partition. In general, arbitrary identifying values, called *labels*, that model thematic information of any complexity should be able to be assigned to different points, lines, or regions within a partition so that components of partitions can be identified by their label. Thus, the set A of labels used to mark point, line, and region features in a partition determines the type of the PLR partition. We make no assumptions as to the structure or contents of a specific label. For example, a PLR partition showing the countries of Canada, the US, and Mexico such that each country is labeled with its name can have a type $A = \{Canada, US, Mexico\}$. Any area in a PLR partition that is not specifically labeled in a partition is given the $\perp$ label. Thus, in the PLR partition of Canada, the US, and Mexico, the area shown that does not belong to those countries is labeled with $\perp$ and the type of the partition is $A = \{Canada, US, Mexico, \perp\}$. All points with the $\perp$ label are considered to be in the exterior of a partition.

We cannot simply map the plane to the elements of a type A because the boundaries between two features cannot be identified as belonging to one feature or the other. Instead, shared boundaries between two features belong to both features and are thus labeled with the labels of both features. Therefore, a PLR partition of type A is defined by a *spatial mapping* that maps points in the plane to elements of the power set of A. Points in the plane that belong solely to region features are mapped to an element $e \in 2^A$ that correspond to an element in A (that is, $e = \{l\}$ with $l \in A$). For example, the region $R2$ in Figure 2 consists of all points mapped to the label $\{R2\}$. If we consider the exterior of a partition to be an unbounded region, then points and lines always intersect at least one region. For example, line $L1$ in Figure 2 intersects region $R1$ and the exterior of the partition. Therefore, each point that belongs to a line feature also belongs to a region feature, and must carry the labels of both. This is also true for point features. Thus, boundaries between features are mapped to the labels of all features that surround the boundary, and intersecting features take the labels of all participating features. It follows that a PLR partition of type A is defined by a function $\pi : \mathbb{R}^2 \to 2^A$ that maps points to the power set of the set of labels that defines the type of the partition. For example, Figure 2a depicts a scene where each feature is annotated with a name. Figure 2b shows the result of the spatial mapping that corresponds to this scene.

As was mentioned previously, any points in a PLR partition that are not explicitly labeled are considered to be in the *exterior* of the partition and are

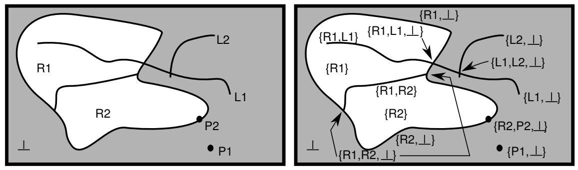

Fig. 2. A scene (a), and its associated spatial mapping (b).

labeled with the *undefined element* or *empty label* $\perp_A \in A$. If no ambiguities arise, we sometimes omit the type subscript and simply use $\perp$. We define a *spatial mapping* as the following:

Definition 1. *A* spatial mapping *of type A is a total function $\pi : \mathbb{R}^2 \rightarrow 2^A$.*

Therefore, a spatial mapping partitions the plane into blocks of identically labeled points. However, blocks defined in this way are not restrictive enough for our purposes. Spatial data types are typically defined not merely as point sets, but are in some sense regular; that is, regions are not allowed to have cuts or punctures, etc. The topological concept of regular open sets as shown in [11] models this well. Therefore, we choose to model regions as two-dimensional, regular open point sets, and lines as one-dimensional, regular open point sets (points are trivial). This implies that a given spatial mapping may not necessarily represent a valid PLR partition. Therefore, we must define the properties a spatial mapping must have if it is supposed to represent a PLR partition. In order to do this, we must first provide a summary of the mathematical notation we use throughout the remainder of the paper.

3.2 Notations

A treatment of regular open point sets can be found in [11]. Let (X, T) be a topological space[1] with topology $T \subseteq 2^X$, and let $S \subseteq X$. The *interior* of S, denoted by S°, is defined as the union of all open sets that are contained in S. The *closure* of S, denoted by $\overline{S}$ is defined as the intersection of all closed sets that contain S. The *exterior* of S is given by $S^- := (X - S)^\circ$, and the *boundary* or *frontier* of S is defined as $\partial S := \overline{S} \cap \overline{X - S}$. An open set is *regular* if $A = \overline{A}^\circ$. The type of regular open sets is closed under intersection. In this paper, we deal with topological space $\mathbb{R}^2$.

The application of a function $f : A \rightarrow B$ to a set of values $S \subseteq A$ is defined as $f(S) := \{f(x) | x \in S\} \subseteq B$. For example, given a spatial mapping π of type A, the function $\pi(\{(1, 2), (2, 3)\})$ would return the set of labels of the given points. In some cases we know that $f(S)$ returns a singleton set, in which case we write $f[S]$ to to denote the single element, that is $f(S) = \{y\} \rightarrow f[S] = y$. For doubly

[1] In a topological space, the following three axioms hold [12]: (i) $U, V \in T \rightarrow U \cap V \in T$, (ii) $S \subseteq T \rightarrow \bigcup_{U \in S} U \in T$, and (iii) $X \in T, \emptyset \in T$. The elements of T are called *open sets*, their complements in X are called *closed sets*, and the elements of X are called *points*.

nested singleton sets, we use $f[[S]]$ similarly. The inverse function $f^{-1} : B \to 2^A$ of f is defined $f^{-1}(y) := \{x \in S | f(x) = y\}$. Given a spatial mapping π of type A and label $l \in A$, the inverse function $\pi^{-1}(l)$ returns the block of points with the label l. It is important to note that f^{-1} is a total function and that f^{-1} applied to a set yields a set of sets.

3.3 Defining PLR Partitions

In this section, we show how to identify each feature in a spatial mapping. We then provide constraints on these features in order to define the type of PLR partitions. However, identifying features in a spatial mapping is somewhat non-intuitive. Consider region $R1$ in Figure 2b and the portion of line $L1$ that intersects it. The points that make up $R1$ are associated with one of two blocks, the block labeled $\{R1\}$ and the block labeled $\{R1, L1\}$. To find the point set consisting of all points that define the region $R1$, we cannot simply use the function $\pi^{-1}(\{R1\})$, since it returns a point set containing a cut (the block labeled $\{R1, L1\}$ will not be returned). We make the observation that although points in $R1$ have one of two different labels, $\{R1\}$ is a subset of both labels. Therefore, we say that the label $\{R1\}$ is the *discriminant label* of the region, since it can be used to distinguish all points that make up that region. We now need a method to identify and retrieve blocks from a spatial mapping based on discriminant labels.

In order to identify the different features of a partition based on their discriminant labels, we take advantage of the property of regular open sets in the plane that every point that is a member of a regular open set is contained in a *neighborhood* of points that are also members of that set. We define the neighborhood of a given point $p = (p_x, p_y)$ in two dimensions as the set of points forming a circle around p with an infinitesimally small radius r. In order to retrieve the neighborhood of a point p, we define the neighborhood operation which returns the set of points contained in the neighborhood of p:

$$N := \mathbb{R}^2 \to 2^{\mathbb{R}^2}$$
$$N(p) = \{(x, y) | (x - p_x)^2 + (y - p_y)^2 = (r)^2\}$$

We can now determine if a point in a partition belongs to a point, line, or region feature based on the properties of that point's neighborhood. For example, a point p that belongs solely to a region (as opposed to the intersection of a region and a line) will have the same label as every point in its neighborhood. In other words, the label of p will be equivalent to the union of the labels of all points in $N(p)$. The predicate *isBasicRegion* takes a point and returns a value of *true* if it belongs solely to a region feature, and *false* otherwise. For instance, all points in the interior of region $R2$ in Figure 2b are basic region points.

$$isBasicRegion := \mathbb{R}^2 \to \mathbb{B}$$
$$isBasicRegion(p) = (\pi(p) = \bigcup_{q \in N(p)} \pi(q))$$

In order to determine if a point p belongs to a region in general (and not a boundary between regions) regardless of how many line and point features also include that point, we must determine that all points in the neighborhood of p that happen to be identified as basic region points have the same label (that is, the set containing the labels of all basic region points in $N(p)$ has a cardinality equal to 1). If p is on a boundary between regions, then its neighborhood will contain basic region points from each differently labeled region. For a spatial mapping π of type A, we also define the *discriminant spatial mapping* for regions $\pi_r : \mathbb{R}^2 \to 2^A$ that maps a point to its discriminant region label. For instance, a point p that lies on the intersection of region $R1$ and line $L1$ in Figure 2a is a region point but not a basic region point, and $\pi_r(p) = \{R1\}$.

$$\begin{aligned}
&\mathit{isRegion} := \mathbb{R}^2 \to \mathbb{B}\\
&\mathit{isRegion}(p) = |\{\pi(q) | q \in N(p) \ \wedge \ \mathit{isBasicRegion}(q) \ \wedge \ \pi(q) \subseteq \pi(p)\}| = 1\\
&\pi_r : \mathbb{R}^2 \to 2^A\\
&\pi_r(p) = \{\pi[q] | q \in N(p) \ \wedge \ \mathit{isBasicRegion}(q) \ \wedge \ \mathit{isRegion}(p)\}
\end{aligned}$$

A region boundary point is characterized as belonging to multiple regions; therefore, a point p lies on a region boundary if at least two points in its neighborhood are basic region points from different regions (that is, the set of basic region labels belonging to points in $N(p)$ must contain at least two elements). We define the predicate *isRegionBound* to test if a point lies on a region boundary, and the discriminant spatial mapping for region boundaries below. For example, point p labeled $\{R2, P2, \perp\}$ in Figure 2 is a region boundary because $N(p)$ contains basic region points from both $R2$ and the exterior region $\perp$, and $\pi_{rb}(p) = \{R2, \perp\}$.

$$\begin{aligned}
&\mathit{isRegionBound} := \mathbb{R}^2 \to \mathbb{B}\\
&\mathit{isRegionBound}(p) = |\{\pi(q) | q \in N(p) \ \wedge \ \mathit{isBasicRegion}(q) \ \wedge \ \pi(q) \subseteq \pi(p)\}| > 1\\
&\pi_{rb} : \mathbb{R}^2 \to 2^A\\
&\pi_{rb}(p) = \{\pi[q] | q \in N(p) \ \wedge \ \mathit{isBasicRegion}(q) \ \wedge \ \mathit{isRegionBound}(p)\}
\end{aligned}$$

Point features are unique in that the neighborhood of a point feature will not contain the discriminant label of the point. Therefore, to determine if point p is a point feature, we compute the difference of each label in $N(p)$ with the label of p. The resulting label will either be the discriminant label of p, if it is a point feature, or the empty set if p is not a point feature:

$$\begin{aligned}
&\mathit{isPoint} := \mathbb{R}^2 \to \mathbb{B}\\
&\mathit{isPoint}(p) = |\pi(p) - \textstyle\bigcup_{q \in N(p)} \pi(q)| > 0\\
&\pi_p : \mathbb{R}^2 \to 2^A\\
&\pi_p(p) = \pi(p) - \textstyle\bigcup_{q \in N(p)} \pi(q)
\end{aligned}$$

To identify line features in a spatial mapping, we must first disregard the discrimant labels for point and region features. We define the label stripping function S that takes a point and returns the label of that point with any point, region, and region boundary discrimant labels removed.

$$\begin{aligned}
&S := \mathbb{R}^2 \to 2^A\\
&S(p) = \pi(p) - \pi_r(p) - \pi_{rb}(p) - \pi_p(p)
\end{aligned}$$

Lines present a problem due to the fact that the interior of two lines can intersect along a line or at a point. Thus, we must treat lines differently than regions or points. We can determine that a point p is part of the interior of a line if there are at least two points, q and s, in its neighborhood such that the stripped labels for those points are identical. However, if two line interiors intersect at a point, the discriminant spatial mapping must return two discriminant labels for that point, one for each line. Furthermore, the inverse of the discriminant spatial mapping must return the point in question if given either of the discriminant labels for the intersecting lines. An example of such a scenario is the intersection of lines $L1$ and $L2$ in Figure 2b.

$$
\begin{aligned}
&\mathit{isLine} := \mathbb{R}^2 \to \mathbb{B}\\
&\mathit{isLine}(p) = \exists q, s \in N(p) | q \neq s \ \wedge \ S(q) = S(s) \ \wedge \ S(q) \neq \varnothing \ \wedge \ \pi(q) \subseteq \pi(p)\\
&\pi_l : \mathbb{R}^2 \to \{2^A\}\\
&\pi_l(p) = \{S(q) | q, s \in N(p) \ \wedge \ \mathit{isLine}(p) \ \wedge \ S(q) = S(s) \ \wedge \ q \neq s \ \wedge \ S(q) \neq \varnothing\}\\
&\pi_l^{-1} := 2^A \to 2^{\mathbb{R}^2}\\
&\pi_l^{-1}(Y) = \{x \in \mathbb{R}^2 | Y \in \pi_l(x)\}
\end{aligned}
$$

A point p is on the boundary of a line if the line extends in only one direction from p; therefore, if there exists a point q in the neighborhood of p such that $S(q) \subseteq S(p)$ and no other point in p's neighborhood contains the same stripped label as q, then point p is a boundary of a line.

$$
\begin{aligned}
&\mathit{isLineBoundary} := \mathbb{R}^2 \to \mathbb{B}\\
&\mathit{isLineBoundary}(p) = \exists q \in N(p) | \pi(q) \subseteq \pi(p) \ \wedge \ S(q) \neq \varnothing\\
&\qquad\qquad\qquad\qquad\qquad\qquad \wedge \ (\nexists s \in N(p) | q \neq s \ \wedge \ S(q) = S(s))\\
&\pi_{lb} : \mathbb{R}^2 \to 2^A\\
&\pi_{lb}(p) = \{S[q] | (\nexists s \in N(p) | q \neq s \ \wedge \ S(q) = S(s)) \ \wedge \ \mathit{isLineBoundary}(p)\\
&\qquad\qquad \wedge \ S(q) \neq \varnothing\}
\end{aligned}
$$

Given a spatial mapping, we can now identify its features:

Definition 2. Let π be a spatial mapping of type A

(i) $\rho(\pi) := \pi_r^{-1}(2^A)$ (*regions*)
(ii) $\omega_\rho(\pi) := \pi_{rb}^{-1}(2^A)$ (*region borders*)
(iii) $\lambda(\pi) := \pi_l^{-1}(2^A)$ (*lines*)
(iv) $\omega_\lambda(\pi) := \pi_{lb}^{-1}(2^A)$ (*line borders*)
(v) $\varphi(\pi) := \pi_p^{-1}(2^A)$ (*points*)

Finally, a spatial mapping is a PLR partition if its lines and regions are regular, open point sets, and the borders between features are labeled with the union of the labels of all features sharing the border.

Definition 3. A *PLR partition* is a spatial mapping π of type A with:

(i) $\forall r \in \rho(\pi) : r = \overline{r}^\circ$
(ii) $\forall l \in \lambda(\pi) : l = \overline{l}^\circ$
(iii) $\forall b \in \omega_\rho : \pi_{rb}[b] = \{\pi_r[[r]] | r \in \rho(\pi) \ \wedge \ b \subseteq \partial r\}$
(iv) $\forall b \in \omega_\lambda : \pi_{lb}[b] = \{\pi_l[[l]] | l \in \lambda(\pi) \ \wedge \ b \subseteq \partial l\}$

4 Conclusions and Future Work

In this paper, we have defined the PLR partition model which is able to overcome the dimension reduction, dimension representation, and feature restriction problems associated with traditional spatial data models. PLR partitions are able to represent map geometries containing point, line, and region features along with the thematic information associated with each feature. Furthermore, the individual features within a PLR partition are defined as open point sets so that they directly correspond to the traditional spatial types of complex points, lines, and regions. Thus, existing concepts defined on the traditional spatial types can be directly applied to the individual features in a PLR partition.

Future work includes formally defining operations over PLR partitions and investigating an implementation model for use in spatial systems. We also plan to explore the use of PLR partitions as a mechanism to support spatial query processing in a spatial database system. Furthermore, it is unclear if traditional spatial querying mechanisms are adequate to query PLR partitions. Therefore, we plan to investigate SQL extensions that can support PLR partition queries.

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