Detecting the Topological Development in a Complex Moving Region

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Abstract. Exploring the topology of a region object requires the mathematical study of its geometric properties that are preserved under deformations. A moving region whose location and extent change over time can undergo several topological changes such as the splitting of a region or the formation of a hole. The study of this kind of changes is important in many applications, e.g., for the topology control of wireless sensor networks and the processing of animation images in multimedia applications. Since we often lack the ability of capturing the location, extent, and shape changes of a moving region during its lifespan, it is challenging to detect these changes. Further, for a complex moving region containing multiple components, it is difficult to determine which component before a change corresponds to which component after the change. In this article, we propose a model pursuing a three-phase strategy to determine the topological changes of a complex moving region represented by a sequence of snapshots called observations. The first phase partitions the observations into several evaluation units. The second phase uniquely maps each unit before the change to exactly one unit after the change. The third phase interprets the topological changes by integrating all basic topological changes from the evaluation units. We also show the detailed algorithms of this three-phase strategy which turn out to be efficient. Finally, a case study illustrates our concepts.

Categories and Subject Descriptors: H.2.8 [**Information Systems**]: Spatial Databases and GIS

Keywords: moving object, complex region, topological change

1. INTRODUCTION

Investigating the topology of a spatial object requires the mathematical study of its geometric properties that are preserved under deformations such as twisting and stretching. A deformation may or may not change the topology of a spatial object. For example, stretching a circle to an ellipse is a topologically equivalent deformation while splitting a circle into two half circles is considered as a topological change. The study of the topological changes of spatial objects over time, i.e., of time-dependent geometries called *moving objects*, is important in many applications such as geographical information systems (GIS), spatiotemporal databases, the processing of animation images in multimedia applications, and the topology control of wireless sensor networks (WSN). In a forest fire control system, a forest fire can begin at different spots, grow independently in a few days, and finally merge into one fire. This merge leads to a topological change within a *moving region*. The knowledge of this event can help fire fighters prevent the spreading of the fire. In a WSN environment, two mobile networking devices could have been close to each other at the beginning and form a coverage area given by a single connected region. As they move to opposite directions, the coverage area becomes disconnected within a few minutes. The study of this topological change can help researchers perform topology control and improve the performance of the wireless sensor network.

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Several models have been proposed to represent moving objects in computer and database systems. However, the focus has been mainly on trajectories modeled by *moving points*. Another and probably even more important but also more complex category describing the evolution of regions over time as moving regions has rarely attracted much attention. There are some major challenges in detecting topological changes in a moving region. First, we are not able to track the continuous deformation of a moving region at all times due to the shortcomings of the tracking devices. A forest fire is detected through the changing images captured by satellites. However, the satellites can merely collect the needed data every few hours and give us snapshots. But this is insufficient in order to obtain the full picture of all topological changes. Second, for a complex moving region containing multiple components, we cannot tell precisely which component before the change corresponds to which component after the change. Therefore it is difficult to formally determine from two consecutive snapshots, without human intuition and/or background information, whether a spot of fire disappears, or whether it merges with another spot of fire. Third, even if we are able to keep track of all topological changes in a moving region, it is not easy to interpret these changes formally. In recent multimedia technologies such as computer-based animation and MPEG-4 video compression, although images can be captured as frequent as 24 frames per second and content-based coding is used to identify different moving objects, current approaches fail to give a formal interpretation of the topological changes between consecutive frames.

The goal of this article is to provide a solution to the problem of detecting the topological changes in a complex moving region. The detection is based on snapshots capturing the static geometries of a moving region object at different time instances. We name such a snapshot as an *observation*. The snapshot based approach has been widely accepted by researchers in many fields. In computer-based animations, precise images are captured and named as I-frames, and interpolation is performed to fill the gap between two I-frames. Similarly, in our approach, we first give formal definitions of all possible topologies of a static region object. At different time instants we can, e.g., obtain the observations that a moving region object is a *simple region*, a *multi-region without holes*, or a *simple region with holes*. Our objective is to characterize the basic topological changes between two consecutive observations such as the splitting of a region or the formation of a hole. For the detection of the topological changes of a complex moving region, we introduce a three-phase strategy. The first phase partitions the observations before and after a change into *evaluation units*. The second phase uniquely maps a unit before the change to exactly one unit after the change. The third phase interprets the topological changes of the complex moving region by integrating the basic topological changes of all evaluation units. Topological changes that involve more observations can then be evaluated in the same way. Finally, we describe the detailed algorithms for this three-phase strategy and show their efficiency.

The remainder of this article is structured as follows: Section 2 presents available models for topological changes of moving regions. Section 3 formally discusses spatial region objects and their properties, and depicts the concept and the representation of moving region objects. Section 4 introduces our approach to detecting topological changes of a complex moving region. Section 5 presents the three-phase algorithms for evaluating the topological changes in a complex moving region and demonstrates their efficiency. Section 7 draws some conclusions and discusses future work.

2. RELATED WORK

This section summarizes approaches to modeling and implementing complex regions (Section 2.1), topological relationships between complex regions (Section 2.2), moving regions (Section 2.3), as well as topological relationships and topological changes of moving regions (Section 2.4).

2.1 Complex Region Objects

In recent decades, spatial databases [Rigaux et al. 2002; Shekhar and Chawla 2003] have been used to store and query objects in space, such as points, lines, and regions. These *spatial objects* are modeled and represented by means of so-called *spatial data types* [Schneider 1997]. We distinguish two generations of spatial data types. In the first generation, spatial objects are represented by simple structures like single points, single, continuous lines, and simple regions with a connected interior and a connected boundary. However, these structures are insufficient to represent the complex geographic phenomena in the real world. For example, the area affected by a forest fire at a particular time instant may not be a single spot; instead, it might be a complex area which is composed of several spots of fires. Moreover, a region object may have one or more holes inside it, but this aspect has not been captured by the first generation. To represent the geographic phenomenon more properly, the second generation of spatial database research introduces the concept of *complex spatial objects* where complex points, complex lines, and complex regions are defined to represent spatial objects with complex structures [Schneider and Behr 2006]. Especially complex regions are involved in many applications. A complex region is represented by a union of several disjoint connected components called *faces*. Each face may have zero, one, or more holes [Schneider and Behr 2006]. The face representation can show the topological properties of complex regions properly. A similar approach provides a unique hierarchical representation of a region object with multiple components [Worboys and Bofakos 1993]. Complex regions will be the objects for which we explore topological changes.

2.2 Topological Relationships between Region Objects

Another topic related to the contents of this article is the study of topological relationships. *Topological relationships* such as *disjoint*, *meet*, and *overlap* characterize the relative positions between two or more spatial objects. Topological relationships between simple regions with holes, which are a subclass of complex regions, are discussed in [Egenhofer et al. 1994]. Topological relationships between complex regions have been studied formally in [Clementini and Di Felice 1996; Schneider and Behr 2006] and algorithmically in [Schneider 2004; Praing and Schneider 2008; 2009]. For example, the approach in [Praing and Schneider 2009] introduces the concept of *topological feature vectors* to represent topological relationships between complex region objects in a quantitative way. The topological change of a complex region object can be regarded as the result of the change of the topological relationships between its simple components. For example, the fact that two spots of a forest fire grow and merge over time into one large fire area can be seen as a change of the topological relationships between these two spots from *disjoint* to *meet* to *overlap*. There are two main differences between the study of topological relationships and the study in our article. First, we treat the components as an entire complex region. Second, instead of only describing the topological properties of static region objects, we consider time as a third dimension so that the topological changes become time dependent.

2.3 Moving Region Objects

The combination of space and time leads to the category of spatiotemporal objects whose locations, shape, and extent change over time. Such time dependent and continuously evolving spatial objects are called *moving objects* [Güting et al. 2000], and the databases that are able to store and manage them are called *moving objects databases* [Güting and Schneider 2005]. Several approaches have been proposed to model moving objects in databases and GIS. At the conceptual level, a moving object is defined as a function from time to the two-dimensional space. For example, a *moving point* is defined as a function from the data type *time* to the spatial data type *point*. At the implementation level, a moving point is represented as a polyline in the three-dimensional (2D+time) space. Sample points are represented as (x, y, t) tuples, and intermediate locations are approximated through linear interpolation [Sistla et al. 1997; Su et al. 2001]. Similarly, at the conceptual level, a moving region is defined as a function from the data type *time* to the spatial data type *region*. At the implementation level, the approach in [Forlizzi et al. 2000] provides a discrete representation for moving regions. Snapshots of a moving region are captured at time instants, and its entire movement is constructed from a series of snapshots [Tøssebro and Güting 2001]. The limitation of this approach is that it can only compute a simple moving region. A recent approach enables the unique construction of a moving region from two snapshots of a complex region [McKenney and Webb 2010].

2.4 Topological Relationships and Topological Changes of Moving Region Objects

Topological relationships between moving region objects have a time-varying character since they can change over time in parallel to the location, shape, and extent changes of the moving region objects themselves. The approach in [Egenhofer and Al-Taha 1992] discusses the problem of the gradual changes of topological relationships between two objects. This model analyzes the changes of the topological relationships between simple regions that result from their movements. A general concept of time-varying topological predicates, called *spatiotemporal predicates*, has been proposed in [Erwig and Schneider 2002]. A spatiotemporal predicate is an alternating sequence of so-called *basic spatiotemporal predicates* like *Disjoint* or *Inside*, which hold for a time period, and standard topological predicates, which hold for a time instant.

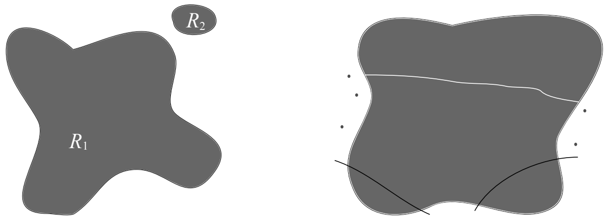
Topological changes characterize the properties and the temporal evolution of an individual moving object. Worboy's group [Worboys and Duckham 2006; Jiang and Worboys 2008; 2009; Jiang et al. 2011] discuss the qualitative changes of an areal object in wireless sensor networks. They propose a set of qualitative changes such as *region_appear*, *region_merge*, and *hole_appear*, and detect these changes with the help of sensor devices. Their research is performed on a specific wireless sensor network environment. They represent an areal object using a graph based method where sub-components are represented as nodes of a graph. Topological changes are detected through the change of node's connectivity in the graph. In our method, instead of using node representation, we partition a snapshot into a set of evaluation units which still keeps the geometric properties of the original moving region. The authors' own previous approach [Liu and Schneider 2011] is able to detect topological changes in a complex moving region that is given as a sequence of snapshots. We define six basic states of a region object and characterize their topological changes as the transitions between these states. We propose a two-phase process which divides every snapshot into so-called *evaluation units* and maps each unit in the first snapshot to exactly one unit in the subsequent snapshot. Thus, the topological changes between different states can be interpreted uniquely. In this article, we will extend the concepts of our previous work with respect to the following aspects. First, we formalize the descriptions of different region data types such as a simple region, a simple region with holes, a multi-region without holes, and a complex region, which are used to represent different static topologies of a moving region object at specific time instances, by giving mathematical definitions of them. Second, in addition to giving a list of basic topological changes, we give formal definitions of them as well. Third, we discuss applications of this model in detail by using a case study.

3. REGIONS AND MOVING REGIONS

In this section, we formally discuss what regions, moving regions, and their properties are. They build the foundation of detecting topological changes of a complex moving region. We first review the data type *region* from spatial databases in Section 3.1. In Section 3.2, we describe the characteristics of moving regions. In Section 3.3, we give the definitions of moving regions and show their properties in a formal way. Finally, in Section 3.4, we introduce the representation of moving regions through the snapshot approach.

3.1 Regions

A moving object describes the temporal evolution of a spatial object over time and is represented by a function from the data type *time* to its corresponding underlying spatial data type such as *point*, *line*, or *region* [Güting et al. 2000]. This means that a moving region object is a mapping from the data type *time* to the spatial data type *region*. The objective of this subsection is to provide a formal definition of different kinds of region objects, which are needed later. For this purpose, we



(a) (b)

Fig. 1. A disconnected region (a) and an irregular region with dangling points and lines which is not regular closed (b).

first introduce some basic concepts like *connectivity*, *closure* and *boundedness*. Our definitions are based on point set theory, point set topology, and our previous research on complex region objects [Schneider and Behr 2006], where regions are embedded into the two-dimensional Euclidean space $\mathbb{R}^2$ and modeled as infinite point sets.

In the simplest case, a region object consists of a single connected component and is called a *simple region*. The point set in the 2D Euclidean space representing a simple region object is connected, closed, and bounded. These three characteristic properties are formally described in the following three definitions. Let X°, X^-, and ∂X denote the *interior*, *exterior*, and *boundary* of a set $X \subseteq \mathbb{R}^2$. Let further $\overline{X}$ denote the *closure* of X with $\overline{X} = \partial X \cup X^\circ$.

***Definition* 3.1 (*Connectivity*)** Two sets $X, Y \subseteq \mathbb{R}^2$ are said to be *separated* if, and only if, $X \cap \overline{Y} = \varnothing = \overline{X} \cap Y$. A set $X \subseteq \mathbb{R}^2$ is *connected* if, and only if, there are no sets $Y, Z \subset X$ such that (i) $Y \neq \varnothing, Z \neq \varnothing$, (ii) $X = Y \cup Z$, and (iii) Y and Z are separated.

Definition 3.1 states that if a region is connected, it is not equal to the union of two nonempty separated sets. A counter-example is shown in Figure 1a, where $R = R_1 \cup R_2$, $R_1 \neq \varnothing$, and $R_2 \neq \varnothing$. However, since R_1 and R_2 are separated, the situation in this figure violates the connectivity property.

***Definition* 3.2 (*Closure*)** Let $X \subseteq \mathbb{R}^2$. X is said to be *regular closed* if, and only if, $X = \overline{X^\circ}$.

Definition 3.2 has the effect that a regular closed region does not have geometric anomalies. The interior operation eliminates dangling points, dangling lines, and boundary parts. The closure operator adds the boundary and eliminates cuts and punctures by supplementing points. An example of a region which is not closed is shown in Figure 1b where we can see dangling points and lines as well as cuts.

***Definition* 3.3 (*Boundedness*)** Let $X \subseteq \mathbb{R}^2$, $p = (x_1, y_1) \in X$, $q = (x_2, y_2) \in X$, and $d(p,q) = \sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2}$. X is said to be *bounded* if holds: $\forall\, p, q \in X\ \exists\, r \in \mathbb{R}^+ : d(p,q) < r$.

Definition 3.3 states that if a point set is said to be bounded, the distance between any two points in it must be bounded. Otherwise, we can find two points for which the distance between them approaches infinity, which is an unrealistic assumption in applications.

Based on Definition 3.1 to Definition 3.3, we are able to formally define the data type *region* as follows.

***Definition* 3.4 (*Region*)** The spatial data type *region* is defined as follows:

$$
\begin{aligned}
region = \{R \subset R^2 \mid & \text{(i)} \quad R \text{ is regular closed} \\
& \text{(ii)} \quad R \text{ is bounded} \\
& \text{(iii)} \quad \text{The number of connected components of } R \text{ is finite}\}
\end{aligned}
$$

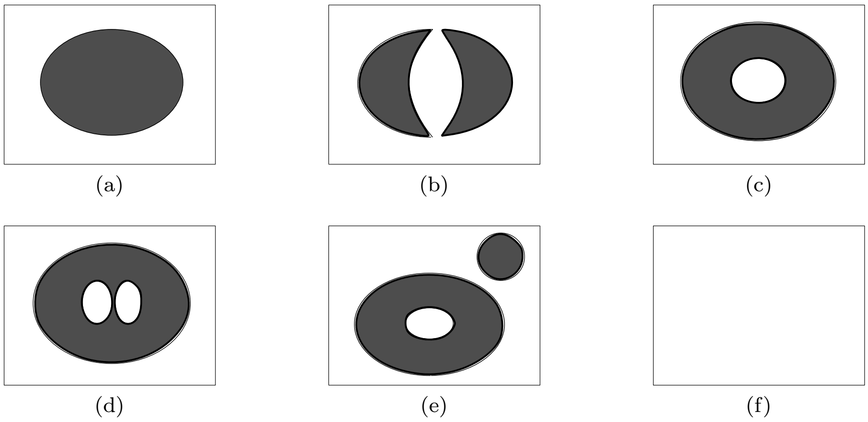


Fig. 2. A simple region (a), a multi-region without holes (b) a simple region with one hole (c), a simple region with multiple holes (d), a complex region (e) and the empty region (f).

In the following, we define six different subtypes of the spatial data type *region* and provide their structural characterization. In the simplest case, a region is composed of a single connected component, and we call it a *simple region.* A simple region is shown in Figure 2a and defined in Definition 3.5.

Definition* 3.5 (*Simple Region) The data type *SR* of *simple regions* is defined as

$$
\begin{aligned}
SR = \{R \,|\, &\text{(i)} \quad R \in \mathit{region}, R \neq \varnothing \\
&\text{(ii)} \quad R \text{ is connected}\}
\end{aligned}
$$

The symbol $\varnothing$ denotes the empty region. There are eight topological relationships between two simple regions [Egenhofer 1989], which are *disjoint*, *meet*, *overlap*, *covers*, *coveredBy*, *equal*, *contains*, and *inside*. They will be used in the definitions of other types of region objects in the rest of this section.

Next, in Definition 3.6, we consider *multi-regions* which are collections of disjoint or at most meeting simple regions without holes. This means that such a region object consists of several components and is thus not connected.

Definition* 3.6 (*Multi-region) The data type *MR* of *multi-regions* is defined as

$$
\begin{aligned}
MR = \{R \,|\, &\text{(i)} \quad R \in \mathit{region} \\
&\text{(ii)} \quad \exists\, n \in \mathbb{N}\ \exists\, R_1, \ldots, R_n \in SR : R = \textstyle\bigoplus_{i=1}^{n} R_i \\
&\text{(iii)} \quad \forall\, 1 \leq i < j \leq n : \mathit{disjoint}(R_i, R_j) \ \vee\ 0\text{-}\mathit{meet}(R_i, R_j)\}
\end{aligned}
$$

The operation $\oplus$ denotes the *geometric union* operation [Schneider and Behr 2006]. The predicate 0-*meet* is a dimension-refined topological predicate [McKenney et al. 2005], i.e., simple regions are only allowed to meet in a finite number of boundary points. An example of a multi-region object is shown in Figure 2b.

Another spatial phenomenon is that a simple region contains one (Definition 3.7) or more (Definition 3.8) holes.

Definition* 3.7 (*Simple Region with One Hole) The data type *SROH* of *simple regions with one hole* is defined as

$$
\begin{aligned}
SROH = \{R \,|\, &\text{(i)} \quad R \in \mathit{region} \\
&\text{(ii)} \quad \exists\, R_0, R_1 \in SR : \mathit{contains}(R_0, R_1) \ \wedge\ R = R_0 \ominus R_1\}
\end{aligned}
$$

The operation $\ominus$ denotes the *geometric difference* operation [Schneider and Behr 2006]. Figure 2c shows an example of a simple region with one hole.

***Definition* 3.8 (*Simple Region with Multiple Holes*)** The data type $SRMH$ of *simple regions with multiple holes* is defined as

$$\begin{array}{rll} SRMH = \{R \mid & \text{(i)} & R \in \mathit{region} \\ & \text{(ii)} & \exists\, n \in \mathbb{N} - \{1\}\ \exists\, R_0, \ldots, R_n \in SR : R = R_0 \ominus \bigoplus_{i=1}^{n} R_i \\ & \text{(iii)} & \forall\, 1 \leq i \leq n : \mathit{contains}(R_0, R_i) \\ & \text{(iv)} & \forall\, 1 \leq i < j \leq n : \mathit{disjoint}(R_i, R_j)\} \end{array}$$

Figure 2d gives an example of a simple region with multiple holes. Finally, we specify the most general region data type *CR* of *complex regions*. It models region objects that consist of at least two components, and at least one component must be a simple region with one or more holes.

***Definition* 3.9 (*Complex Region*)** The data type *CR* of *complex regions* is defined as

$$\begin{array}{rll} CR = \{R \mid & \text{(i)} & R \in \mathit{region} \\ & \text{(ii)} & \exists\, n \in \mathbb{N} - \{1\}\ \exists\, R_1, \ldots, R_n \in SR \cup SROH \cup SRMH : R = \bigoplus_{i=1}^{n} R_i \\ & \text{(iii)} & \exists\, 1 \leq i \leq n : R_i \in SROH \cup SRMH \\ & \text{(iv)} & \forall\, 1 \leq i < j \leq n : \mathit{disjoint}(R_i, R_j)\ \vee\ 0\text{-}\mathit{meet}(R_i, R_j)\} \end{array}$$

The topological predicates *disjoint* and 0-*meet* are here also applied to simple regions with holes. According to Definitions 3.5 to 3.9, we obtain five different region data types modeling region objects of different geometric complexity. In addition, we introduce the data type *ER* that includes the empty region object $\varnothing$ as its only value (see Figure 2f). Hence, in total, we obtain six possible region shapes.

For all six region data types we assume the availability of the two functions *comps*, which yields the set of connected components (i.e., the simple region components with or without holes) of a region object, and *holes*, which yields the set of holes (given as simple region components) of a region object.

3.2 What are Moving Regions?

Having discussed the spatial data type *region*, we will next give a formal description of what a moving region is and what properties it has. A moving region describes the temporal evolution and change of a region object with respect to location, shape, and areal extent. Examples of moving regions include continuously moving hurricanes, growing forest fires, and spreading oil spills.

In Section 3.1, we have introduced six different region shapes. We call these six shapes the six *states* of a region object. Since a moving region is a region object evolving over time, the snapshots we take at different time instants can have different states. If the states of two consecutive snapshots are different, we witness a topological change. For example, an erosion area near the coast might divide an area into two separate areas in ten years. This phenomenon illustrates a topological change that is from a simple region to a multi-region without holes. In contrast, the state of a moving region at different time instants may not change. A hurricane moves from Florida to Louisiana in three days. During this period the area of the hurricane also changes, i.e., it grows or shrinks and has a different shape at different time instants. However, it remains one simple region without splitting into two parts or generating holes inside it. This kind of change is called *topology-preserving change* [Jiang and Worboys 2008]. In this article, we mainly focus on the discussion of the internal topological changes of a moving region since they are more important in applications.

In previous work, a moving object has been defined as a function from time to a spatial data type. In particular, a moving region f is defined as a function $f : \mathit{time} \rightarrow \mathit{region}$. However, this definition

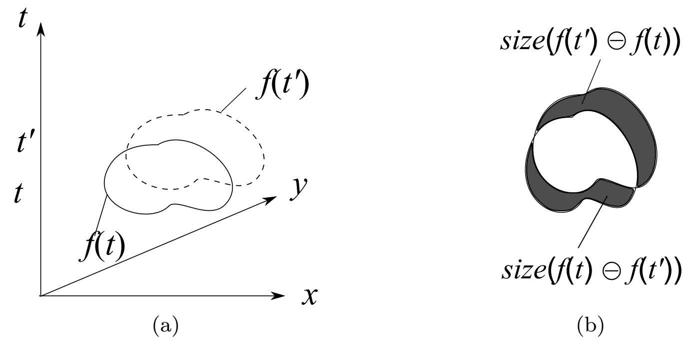


Fig. 3. Snapshots $f(t)$ and $f(t')$ captured from a moving region at instants t_1 and t_2 respectively (a); the dissimilarity area of them in the 2D plane (b).

is too general and cannot capture the properties of a movement in reality. For example, under this definition, a simple region can suddenly split into a complex region. But this situation happens rarely in the real world. Only continuous and smooth changes of shapes exist in reality. Thus, a clear definition is needed which is able to describe the continuous changes of a moving region.

3.3 Definitions of Moving Regions and Their Properties

Moving regions are time dependent. However, not all time-dependent region objects can be considered as moving regions. The most important property a moving region should have is continuity. For example, an instantaneous jump will violate the continuity property. In contrast, the transition of a moving region should be smooth. As moving regions can be defined as functions of time, the continuity of a moving region is similar to the continuity of a function. In our previous work, we have defined the continuity of a moving region by a *slight* change of the area between two instants [Güting et al. 2000]. We briefly review this definition here.

Given $R_1, R_2 \in \mathit{region}$, we introduce a concept called *dissimilarity*, which describes the difference between two region objects in a quantitative way. The dissimilarity function $\psi : \mathit{region} \times \mathit{region} \to \mathbb{R}$ between R_1 and R_2 is defined as

$$\psi(R_1, R_2) = \mathit{area}(R_1 \ominus R_2) + \mathit{area}(R_2 \ominus R_1)$$

where the operator *area* calculates the area of a region object. Figure 3a shows an example of a moving region at two different time instants t and t' with $t < t'$ so that we get two region objects $f(t)$ and $f(t')$ respectively. $f(t)$ is represented by the shape with the solid line, and $f(t')$ is represented by the shape with the dashed line. We observe that $f(t')$ is slightly larger than $f(t)$ and has moved to another location. Their projections to the 2D plane are shown in Figure 3b. We observe that most parts of these two regions overlap with each other. The shaded area shows the difference between two regions and thus forms the dissimilarity. Assume that the region changes smoothly, then if the time difference between t and t' approaches zero, the dissimilarity area should approach zero accordingly. Based on this idea, Definition 3.10 describes the continuity of a moving region.

Definition* 3.10 (*Continuity of Moving Regions) Given a moving region $f : \mathit{time} \to \mathit{region}$,

(i) f is *right-semicontinuous* at t, if and only if, $\lim_{t' \to t^+} \psi(f(t), f(t')) = 0$
(ii) f is *left-semicontinuous* at t, if and only if, $\lim_{t' \to t^-} \psi(f(t'), f(t)) = 0$
(iii) f is *continuous at* t, if and only if, f is right-semicontinuous and left-semicontinuous at t
(iv) f is *continuous in* $[t_1, t_2]$ if, and only if, f is right-semicontinuous at t_1, left-semi-continuous at t_2 and continuous at any $t \in]t_1, t_2[$

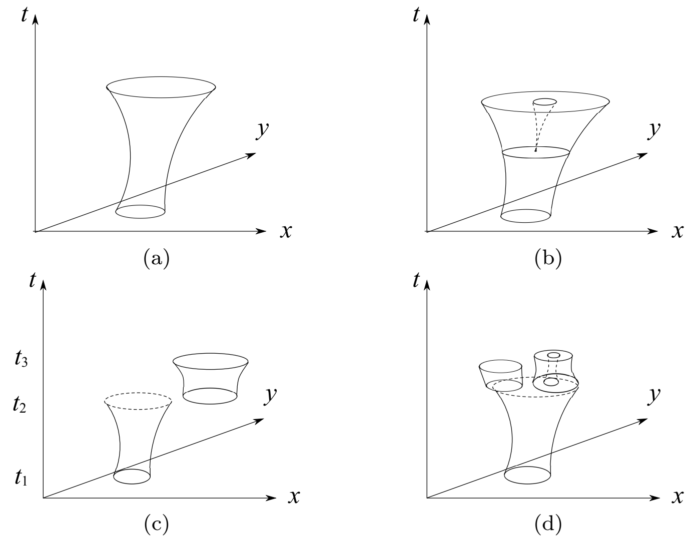


Fig. 4. Moving regions that are continuous during their lifetime (a), (b), instantaneous jump (c), and sudden shape change (d)

Conditions (i) and (ii) distinguish whether time t' approaches time t from the right (future) or from the left (past) of t. The nearer t and t' are to each other, the smaller the dissimilarity value has to be. Condition (iii) is stricter than Conditions (i) and (ii) and unites their behavior. Condition (iv) specifies continuity of a moving region on a time interval in which it is defined.

Figure 4 illustrates examples of moving regions that are continuous and discontinuous respectively. Figure 4a shows a moving region which first shrinks and then grows. The entire changing process is smooth and represents a continuous movement. Figure 4b shows a region that grows and then forms a hole. Although the topology of the region has been changed from a simple region to a simple region with holes, the change is continuous and smooth. Figure 4c shows an example of *discontinuity*. At time t_2, the moving region performs an instantaneous jump to a new location. Thus, the moving region is not left-semicontinuous at t_2 and thus not continuous in $[t_1, t_3]$. In Figure 4d, the moving region shows a significant topology change at time t_2; it changes from a simple region to a complex region, which violates the continuity property.

3.4 Snapshot Representation of Moving Regions

In order to detect the topological development in a moving region, the first important task is to represent the moving region properly. However, this task is challenging. Because a moving region continuously changes its locations and shape, we are not able to track the continuous deformation of that moving region at all times due to the shortcomings of the tracking devices. For example, when detecting a forest fire, we get the report from the sensors at discrete time instances since sensors usually take measurements at discrete times. When studying whether there is a hurricane, we analyze the pictures from the satellites which are captured every few hours. Thus, our idea is to represent a moving object as a sequence of *snapshots*. In this article, we call a snapshot an *observation*.

The snapshot based approach has been widely accepted by researchers in many fields. In computer-based animations, precise images are captured less frequently and named as I-frames, and interpolation is performed to fill the gap between two I-frames. Similarly, in our model, we represent a moving region at different time instances and interpret the transitions in between. At different time instants we can, for example, obtain the observations that a moving region object is a *simple region*, a *multi-region without holes*, or a *simple region with holes*. Our model will be able to characterize the basic topological

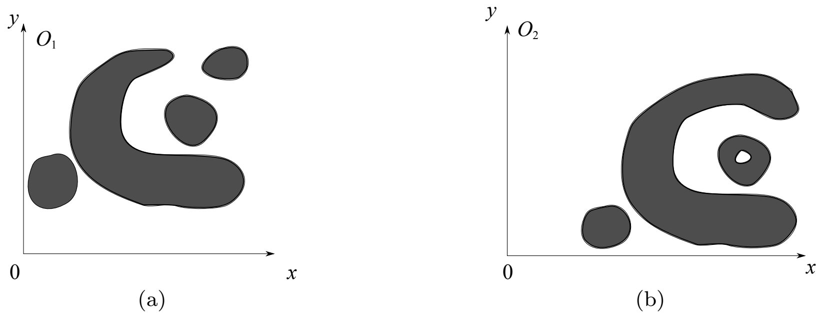


Fig. 5. Two snapshots O_1 and O_2 of a moving region illustrating region merging and hole formation

changes between two consecutive observations such as the splitting of a region or the formation of a hole.

Figures 5a and 5b represent two observations of a complex moving region captured at times t_1 and t_2 respectively. We can see that this moving region object moves to the right and down, and there is a merge between the largest region component with the region component at the right upper corner. Also, there is a hole appearing in the region component in the center. However, this interpretation is intuitive and lacks a formal explanation.

For such a complex moving region containing multiple components, we cannot tell precisely which component before the change corresponds to which component after the change. Therefore it is difficult to formally determine from two consecutive snapshots, without human intuition and/or background information, whether a spot of fire disappears, or whether it merges with another spot of fire. In the next two sections, we solve this problem by providing a three-phase strategy which is able to uniquely interpret the topological changes between two consecutive snapshots.

4. MODELING TOPOLOGICAL CHANGES IN MOVING REGIONS

In the following two sections, we present our method for detecting topological changes in a complex moving region through snapshots. We make two reasonable assumptions so that the continuous transition between two consecutive snapshots can be detected. First, the movement of a moving region is considered continuous so that there is no instantaneous jump between two consecutive observations like in Figure 4c. Second, devices such as sensors and satellites update their periodical data with a proper frequency so that there will be no "tremendous" topological changes between two consecutive observations. In this section, we define *basic topological changes*. In the next section, we will introduce how to partition observations into evaluation units, map them between two observations, and interpret topological changes between consecutive observations.

A moving region may have different shapes at different time instants, leading to topological changes. However, because of the continuity property, topological changes cannot happen between every pair of states. For example, a direct change from a simple region to a complex region is impossible. Instead, there must be other intermediate states between them. Therefore, we introduce the *state transition diagram* which shows the validity of transitions between different states of a moving region. The state transition diagram represents all direct topological changes and is shown in Figure 6. An arrow between two states shows that there exists a direct topological change between these two states. If there is no arrow between two states, this means that a direct topological change between them is not valid and does therefore not exist. There can be more than one possible topological change between the same two states. For example, from a simple region to a simple region with holes, two topological changes may happen: either a hole is formed inside the region, or the region touches itself and forms a hole. These two topological changes are named as *hole form* and *region self-touch* respectively.

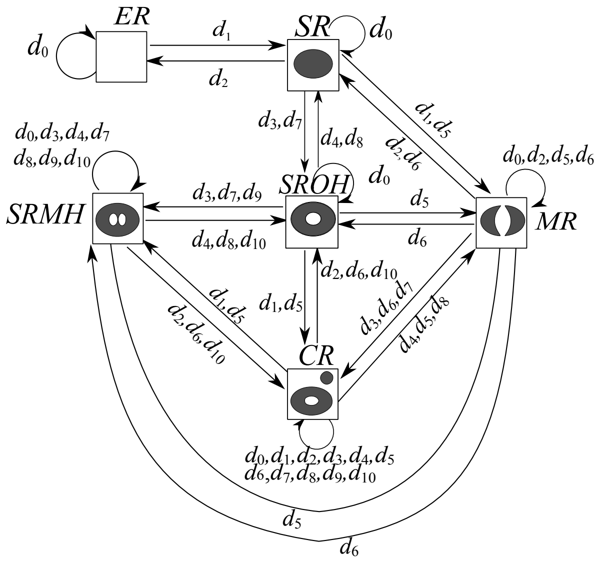


Fig. 6. The state transition diagram representing valid topological changes of a moving region

The six possible snapshot states of a moving region object at a time instant are represented by the six region data types *ER*, *SR*, *SROH*, *SRMH*, *MR*, and *CR* (see Section 3.1) for the empty region, simple regions, simple regions with one hole, simple regions with multiple holes, multi-regions, and complex regions respectively. The six data types are summarized in the set *StateSet*. We have identified the 11 *basic topological change mappings* d_0 to d_{10} (see Table I and Figure 6) that change a moving object m at time instant t_1, i.e., the region object $m(t_1)$, into the moving object m at time instant t_2, i.e., into the region object $m(t_2)$. Formally, a *basic topological change mapping* d_i with $0 \leq i \leq 10$ is a function with the signature $d_i : \alpha \to \beta$ with $\alpha, \beta \in$ *StateSet*. It can be overloaded, i.e., represent different mappings from states (region data types) of *StateSet* to states (region data types) of *StateSet*. This leads to a large number of instances of these mappings shown in Table I and in Figure 6.

The following Definitions 4.1 to 4.11 provide the semantic specifications of the basic topological change mappings. Our observation is that a simple region of the region data type *SR* is the structurally most basic object that can be involved in a topological change either as a component object or as a hole object. The definitions make extensive use of *topological relationships* [Egenhofer and Franzosa 1991; Schneider and Behr 2006] like *disjoint*, *meet*, and *contains*, which characterize the relative position between spatial objects, and *dimension-refined topological relationships* [McKenney et al. 2005] like 0-*meet* and 1-*meet*, which additionally characterize common boundary parts as single, zero-dimensional points or as one-dimensional lines.

The first basic topological change mapping called *topology preserve* (d_0) does *not* change the topological structure of a region object. That is, the region objects of both snapshots are topologically

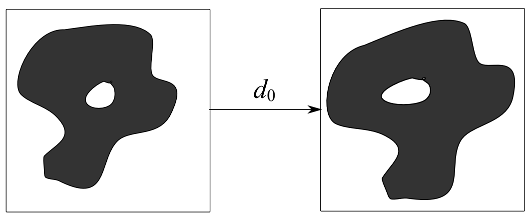


Fig. 7. d_0: Topology preserve

Table I. Basic topological change mappings

d_0 (topology preserve)	:	α	$\longrightarrow$	α
d_1 (region appear)	:	ER	$\longrightarrow$	SR
	:	SR	$\longrightarrow$	MR
	:	$SROH$	$\longrightarrow$	CR
	:	$SRMH$	$\longrightarrow$	CR
	:	CR	$\longrightarrow$	CR
d_2 (region disappear)	:	SR	$\longrightarrow$	ER
	:	MR	$\longrightarrow$	$SR \cup MR$
	:	CR	$\longrightarrow$	$SROH \cup SRMH \cup CR$
d_3 (hole form)	:	SR	$\longrightarrow$	$SROH$
	:	MR	$\longrightarrow$	CR
	:	$SROH$	$\longrightarrow$	$SRMH$
	:	$SRMH$	$\longrightarrow$	$SRMH$
	:	CR	$\longrightarrow$	CR
d_4 (hole fill)	:	$SROH$	$\longrightarrow$	SR
	:	$SRMH$	$\longrightarrow$	$SROH \cup SRMH$
	:	CR	$\longrightarrow$	$MR \cup CR$
d_5 (region split)	:	SR	$\longrightarrow$	MR
	:	MR	$\longrightarrow$	MR
	:	$SROH$	$\longrightarrow$	$MR \cup CR$
	:	$SRMH$	$\longrightarrow$	$MR \cup CR$
	:	CR	$\longrightarrow$	$MR \cup CR$
d_6 (region merge)	:	MR	$\longrightarrow$	$SR \cup MR \cup SROH \cup SRMH \cup CR$
	:	CR	$\longrightarrow$	$SROH \cup SRMH \cup CR$
d_7 (region self-touch)	:	SR	$\longrightarrow$	$SROH$
	:	MR	$\longrightarrow$	CR
	:	$SROH$	$\longrightarrow$	$SRMH$
	:	$SRMH$	$\longrightarrow$	$SRMH$
	:	CR	$\longrightarrow$	CR
d_8 (ring split)	:	$SROH$	$\longrightarrow$	SR
	:	$SRMH$	$\longrightarrow$	$SROH \cup SRMH$
	:	CR	$\longrightarrow$	$MR \cup CR$
d_9 (hole split)	:	$SROH$	$\longrightarrow$	$SRMH$
	:	$SRMH$	$\longrightarrow$	$SRMH$
	:	CR	$\longrightarrow$	CR
d_{10} (hole merge)	:	$SRMH$	$\longrightarrow$	$SROH \cup SRMH$
	:	CR	$\longrightarrow$	$SROH \cup SRMH \cup CR$

equivalent. However, this mapping allows that a region object grows, shrinks, rotates, and changes its shape as long as these changes are performed without splitting or forming holes. An example is shown in Figure 7, and a formal specification is given in Definition 4.1.

***Definition* 4.1 (*Topology Preserve*)** A basic topological change mapping is called *topology preserve* and denoted by $d_0 : \alpha \to \beta$ if $\alpha = \beta$ and d_0 is a homeomorphism.

A homeomorphism is a function that is a bijective mapping between sets such that both the function and its inverse are continuous. Intuitively a homeomorphism is a continuous topological transformation which can be interpreted as an elastic transformation that stretches, twists, or otherwise deforms without cutting. Examples are affine transformations like translation, rotation, or scaling.

The basic topological change mapping called *region appear* (d_1) adds a simple region to a region object of any region data type. For example, an empty region becomes a simple region, and a complex region with n components becomes a complex region with $n+1$ components. An example is shown in Figure 8. It could represent a fire spot arising at a particular location. A formal specification is given in Definition 4.2.

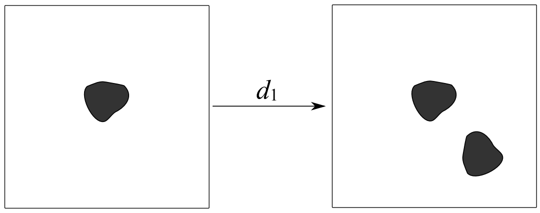


Fig. 8. d_1: Region appear

Definition* 4.2 (*Region Appear) A basic topological change mapping is called *region appear* and denoted by $d_1 : \alpha \to \beta$ with α and β according to Table I if the following holds:

$$\forall\, u \in \alpha\ \exists\, v \in \beta\ \exists\, sr \in SR :\\ d_1(u) = u \oplus sr = v\ \wedge\ u \text{ 1-}\mathit{coveredBy}\ v\ \wedge\ v \text{ 1-}\mathit{covers}\ sr\ \wedge\ (u\ \mathit{disjoint}\ sr\ \vee\ u \text{ 0-}\mathit{meet}\ sr)$$

The interior of the simple region component added must be disjoint from the interior of the original region object.

The basic topological change mapping called *region disappear* (d_2) removes a simple region from a region object of any region data type. Hence, its effect is opposite to the mapping *region appear.* An example is shown in Figure 9, and a formal specification is given in Definition 4.3.

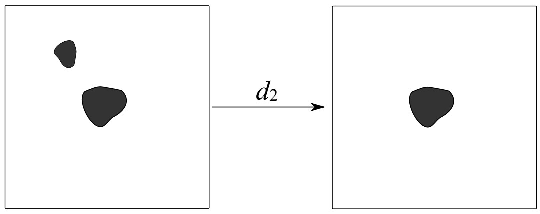


Fig. 9. d_2: Region disappear

Definition* 4.3 (*Region Disappear) A basic topological change mapping is called *region disappear* and denoted by $d_2 : \alpha \to \beta$ with α and β according to Table I if the following holds:

$$\forall\, u \in \alpha\ \exists\, v \in \beta\ \exists\, sr \in SR :\\ d_2(u) = u \ominus sr = v\ \wedge\ u \text{ 1-}\mathit{covers}\ v\ \wedge\ u \text{ 1-}\mathit{covers}\ sr\ \wedge\ (v\ \mathit{disjoint}\ sr\ \vee\ v \text{ 0-}\mathit{meet}\ sr)$$

This topological change mapping is not unique in its codomain. Hence, we can regard β as a union type here. If a simple region disappears from a multi-region u, we either get a simple region v if u has two simple regions as components, or we again get a multi-region v if u has more than two simple region components. The disappearance of a simple region from a complex region can lead to a simple region with one hole or multiple holes if the complex region consists of two components, or to a complex region in all other cases.

The basic topological change mapping called *hole form* (d_3) lets a hole appear in the interior of a simple region, a simple region with one hole, or a simple region with multiple holes, possibly as a component of a complex region. An example is shown in Figure 10. It could represent a forest fire whose central part is extinct while the surrounding area is still on fire. A formal specification is given in Definition 4.4.

Definition* 4.4 (*Hole Form) A basic topological change mapping is called *hole form* and denoted by $d_3 : \alpha \to \beta$ with α and β according to Table I if the following holds:

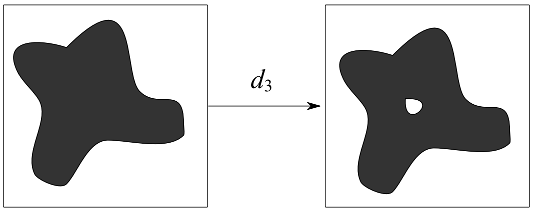


Fig. 10. d_3: Hole form

$$\forall\, u \in \alpha\ \exists\, v \in \beta\ \exists\, sr \in SR:$$
$$d_3(u) = u \ominus sr = v\ \wedge\ u\ 1\text{-}covers\ v\ \wedge\ v\ 1\text{-}meet\ sr\ \wedge\ (u\ 0\text{-}covers\ sr\ \vee\ u\ contains\ sr)$$

The basic topological change mapping called *hole fill* (d_4) removes a hole from a simple region with one or more holes that is possibly a component of a complex region. Hence, its effect is opposite to the mapping *hole form*. An example is given in Figure 11. It could represent an area that is enclosed by fire all around at the beginning and finally becomes a victim of the fire. A formal specification is given in Definition 4.5.

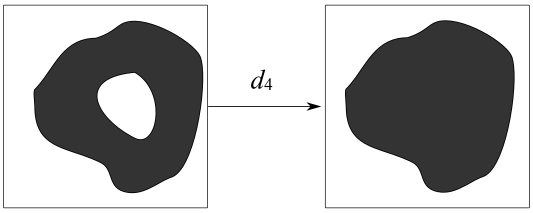


Fig. 11. d_4: Hole fill

***Definition* 4.5 (*Hole Fill*)** A basic topological change mapping is called *hole fill* and denoted by $d_4 : \alpha \to \beta$ with α and β according to Table I if the following holds:

$$\forall\, u \in \alpha\ \exists\, v \in \beta\ \exists\, sr \in SR:$$
$$d_4(u) = u \oplus sr = v\ \wedge\ u\ 1\text{-}coveredBy\ v\ \wedge\ u\ 1\text{-}meet\ sr\ \wedge\ (v\ contains\ sr\ \vee\ v\ 0\text{-}covers\ sr)$$

Note that according to the definition, sr will exactly fill out a complete hole. Since sr is located inside v and at the same time meets u, sr must lie inside the hole of u and fill it out since otherwise sr 1-*coveredBy* v would hold. Again, in two cases, the codomain is not unique. A simple region with two holes is mapped to a simple region with one hole. A simple region with more than two holes remains a simple region with multiple holes after the mapping. A complex region with exactly one component that is a simple region with one hole becomes a multi-region. In all other cases, the mapping produces a complex region again.

The basic topological change mapping called *region split* (d_5) separates a region component (without or with holes) into two region components. The original shape is preserved to a large extent except for the location where the separation occurs. Figure 12 shows an example, and Definition 4.6 gives a formal specification.

***Definition* 4.6 (*Region Split*)** A basic topological change mapping is called *region split* and denoted by $d_5 : \alpha \to \beta$ with α and β according to Table I if the following holds:

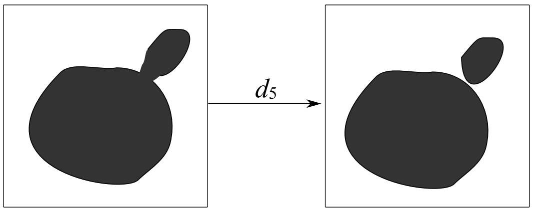


Fig. 12. d_5: Region split

$$\begin{aligned}&\forall\, u \in \alpha\ \exists\, u' \in comps(u)\ \exists\, v \in \beta\ \exists\, v', v'' \in comps(v)\ \exists\, sr \in SR :\\ &\quad d_5(u) = u \ominus sr = v\ \wedge\ u \text{ 1-}covers\ v\ \wedge\ u' \text{ 1-}covers\ v'\ \wedge\ u' \text{ 1-}covers\ v''\ \wedge\ v'\ disjoint\ v''\\ &\qquad \wedge\ v \text{ 1-}meet\ sr\ \wedge\ v' \text{ 1-}meet\ sr\ \wedge\ v'' \text{ 1-}meet\ sr\ \wedge\ u' = v' \oplus sr \oplus v''\end{aligned}$$

The resulting region v is a proper part of u and contains one component more than u. The topological predicate 1-*covers* means that a region object v is contained in another region object u and touches the boundary of u in a one-dimensional line [McKenney et al. 2005].

The basic topological change mapping called *region merge* (d_6) adds a region part to two disjoint components of a region object so that the two components are melted into a single connected component. Hence, its effect is opposite to the mapping *region split*. Figure 13 gives an example. This transition can be found in many applications. For example, two originally disjoint fire spots become merged into a single, larger fire spot as they are moving close to each other. Definition 4.7 provides a formal specification.

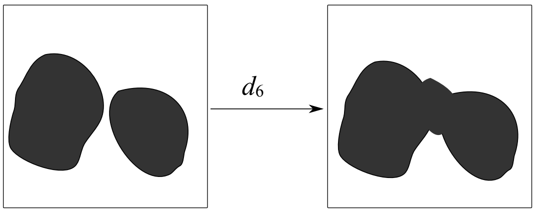


Fig. 13. d_6: Region merge

Definition* 4.7 (*Region Merge) A basic topological change mapping is called *region merge* and denoted by $d_6 : \alpha \rightarrow \beta$ with α and β according to Table I if the following holds:

$$\begin{aligned}&\forall\, u \in \alpha\ \exists\, u', u'' \in comps(u)\ \exists\, v \in \beta\ \exists\, v' \in comps(v)\ \exists\, sr \in SR :\\ &\quad d_6(u) = u \oplus sr = v\ \wedge\ u \text{ 1-}coveredBy\ v\ \wedge\ u' \text{ 1-}coveredBy\ v'\ \wedge\ u'' \text{ 1-}coveredBy\ v'\\ &\qquad \wedge\ u'\ disjoint\ u''\ \wedge\ u \text{ 1-}meet\ sr\ \wedge\ u' \text{ 1-}meet\ sr\ \wedge\ u'' \text{ 1-}meet\ sr\\ &\qquad \wedge\ v \text{ 1-}covers\ sr\ \wedge\ v' \text{ 1-}covers\ sr\ \wedge\ v' = u' \oplus sr \oplus u''\end{aligned}$$

This mapping describes the fusion of two components of a region object (determined by the function *comps*) by a simple region into a single, connected component.

The basic topological change mapping called *region self-touch* (d_7) closes a bay formed by a region component such that a new hole of this component is formed. In order to be able to do this, the region component has to grow. An example is shown in Figure 14. Definition 4.8 gives a formal specification.

Definition* 4.8 (*Region Self-touch) A basic topological change mapping is called *region self-touch* and denoted by $d_7 : \alpha \rightarrow \beta$ with α and β according to Table I if the following holds:

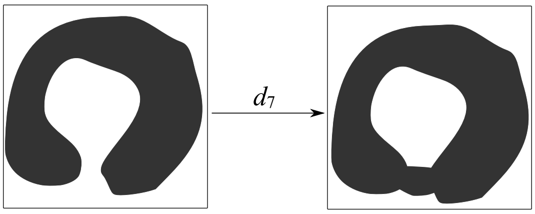


Fig. 14. d_7: Region self-touch

$$\begin{aligned}&\forall\, u \in \alpha\ \exists\, u' \in \mathit{comps}(u)\ \exists\, v \in \beta\ \exists\, v' \in \mathit{comps}(v)\ \exists\, sr \in \mathit{SR} :\\ &\quad d_7(u) = u \oplus sr = v\ \wedge\ u \text{ 1-}\mathit{coveredBy}\ v\ \wedge\ u' \text{ 1-}\mathit{coveredBy}\ v'\ \wedge\ u \text{ 1-}\mathit{meet}\ sr\\ &\quad\quad \wedge\ u' \text{ 1-}\mathit{meet}\ sr\ \wedge\ v \text{ 1-}\mathit{covers}\ sr\ \wedge\ v' \text{ 1-}\mathit{covers}\ sr\ \wedge\ v' = u' \oplus sr\\ &\quad\quad \wedge\ |\mathit{holes}(v)| = |\mathit{holes}(u)| + 1\end{aligned}$$

A self-touch can appear in a single component of an object of any region type. A single component itself can be either a simple region, a simple region with one hole, or a simple region with multiple holes. In all cases, the number of holes is increased by one.

The basic topological change mapping called *ring split* (d_8) breaks a hole of a region component and creates a bay for this component. Its effect is opposite to the mapping *region self-touch*. An illustration is given in Figure 15, and a formal specification is provided in Definition 4.9.

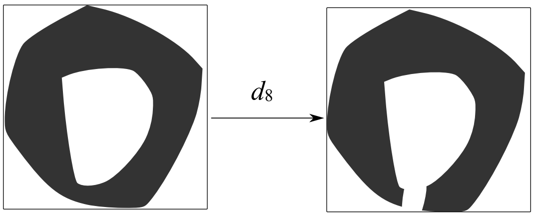


Fig. 15. d_8:Ring split

Definition* 4.9 (*Ring Split) A basic topological change mapping is called *ring split* and denoted by $d_8 : \alpha \to \beta$ with α and β according to Table I if the following holds:

$$\begin{aligned}&\forall\, u \in \alpha\ \exists\, u' \in \mathit{comps}(u)\ \exists\, v \in \beta\ \exists\, v' \in \mathit{comps}(v)\ \exists\, sr \in \mathit{SR} :\\ &\quad d_8(u) = u \ominus sr = v\ \wedge\ u \text{ 1-}\mathit{covers}\ v\ \wedge\ u' \text{ 1-}\mathit{covers}\ v'\ \wedge\ v \text{ 1-}\mathit{meet}\ sr\ \wedge\ v' \text{ 1-}\mathit{meet}\ sr\ \wedge\\ &\quad\quad \wedge\ u \text{ 1-}\mathit{covers}\ sr\ \wedge\ u' \text{ 1-}\mathit{covers}\ sr\ \wedge\ v' = u' \ominus sr\\ &\quad\quad \wedge\ |\mathit{holes}(v)| = |\mathit{holes}(u)| - 1\end{aligned}$$

The basic topological change mapping called *hole split* (d_9) divides a hole of a region component into two holes. Figure 16 gives an example. Definition 4.10 provides a formal specification.

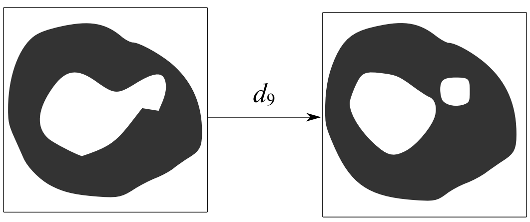


Fig. 16. d_9: Hole split

Definition* 4.10 (*Hole Split) A basic topological change mapping is called *hole split* and denoted by $d_9 : \alpha \rightarrow \beta$ with α and β according to Table I if the following holds:

$$\begin{aligned} &\forall\, u \in \alpha\ \exists\, u' \in comps(u)\ \exists\, v \in \beta\ \exists\, v' \in comps(v)\ \exists\, sr \in SR : \\ &\quad d_9(u) = u \oplus sr = v\ \wedge\ u \text{ 1-}coveredBy\ v\ \wedge\ u' \text{ 1-}coveredBy\ v'\ \wedge\ u \text{ 1-}meet\ sr \\ &\qquad \wedge\ u' \text{ 1-}meet\ sr\ \wedge\ v \text{ 1-}covers\ sr\ \wedge\ v' \text{ 1-}covers\ sr\ \wedge\ v' = u' \oplus sr \\ &\qquad \wedge\ |holes(v)| = |holes(u)| + 1 \end{aligned}$$

The basic topological change mapping called *hole merge* (d_{10}) merges two holes of a region component into one hole so that the number of holes decreases by 1. This is illustrated in Figure 17. A formal specification is given in Definition 4.11.

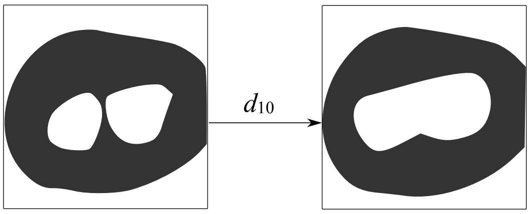


Fig. 17. d_{10}: Hole merge

Definition* 4.11 (*Hole Merge) A basic topological change mapping is called *hole merge* and denoted by $d_{10} : \alpha \rightarrow \beta$ with α and β according to Table I if the following holds:

$$\begin{aligned} &\forall\, u \in \alpha\ \exists\, u' \in comps(u)\ \exists\, v \in \beta\ \exists\, v' \in comps(v)\ \exists\, sr \in SR : \\ &\quad d_{10}(u) = u \ominus sr = v\ \wedge\ u \text{ 1-}covers\ v\ \wedge\ u' \text{ 1-}covers\ v'\ \wedge\ v \text{ 1-}meet\ sr\ \wedge\ v' \text{ 1-}meet\ sr \\ &\qquad \wedge\ u \text{ 1-}covers\ sr\ \wedge\ u' \text{ 1-}covers\ sr\ \wedge\ v' = u' \ominus sr \\ &\qquad \wedge\ |holes(v)| = |holes(u)| - 1 \end{aligned}$$

Basic topological change mappings can be assembled to composite topological change mappings. We define an order relation $\prec$ that specifies the effect of the application of two consecutive basic topological change mappings in Definition 4.12.

Definition* 4.12 (*Observation Order Relation) Let $m : time \rightarrow region$ be a moving region, and let $T = \langle t_1, t_2, t_3 \rangle$ be a sequence of three consecutive *observation times* with $t_1, t_2, t_3 \in dom(f) \subset time$ and the order $t_1 < t_2 < t_3$. Then $m(t_1), m(t_2), m(t_3) \in region$ are three consecutive *observations* of m. Let further $D = \langle D_1, D_2 \rangle$ be a sequence of two basic topological change mappings with $D_1, D_2 \in \{d_0, \ldots, d_{10}\}$ where D_i is supposed to happen at the observation time t_i. We define the order relation $\prec$ between observations in the following sense:

$$D_1(m(t_1)) \prec D_2(m(t_2)) \quad \Leftrightarrow \quad t_1 < t_2 < t_3\ \wedge\ D_1(m(t_1)) = m(t_2)\ \wedge\ D_2(m(t_2)) = m(t_3)$$

Over time, a moving region may have experienced a sequence of n basic topological changes. Hence, we generalize the order relation $\prec$ to n observation times and n observations. We call such a sequence a *topological development*. Definition 4.13 gives the formal definition of a topological development of a moving region with respect to a number of observations.

Definition* 4.13 (*Topological Development) Let $m : time \rightarrow region$ be a moving region, and let $T = \langle t_1, ..., t_n \rangle$ be a sequence of consecutive observation times with $t_1, ..., t_n \in dom(f) \subset time$ for some $n \in \mathbb{N}$ and with the order $t_1 < t_2 < \ldots < t_{n-1} < t_n$. Then $m(t_1), \ldots, m(t_n) \in region$ are n consecutive observations of m. Let further $D = \langle D_1, \ldots, D_{n-1} \rangle$ be a sequence of basic topological change mappings with $D_1, \ldots, D_{n-1} \in \{d_0, \ldots, d_{10}\}$ where D_i is supposed to happen at the observation time t_i. Then we say that $D_1, \ldots, D_{n-1}$ is a *topological development* of m with respect to the observation times in T, written as $dev(m, T)$, if holds:

$$dev(m, T) = D_1(m(t_1)) \prec D_2(m(t_2)) \prec \ldots \prec D_{n-1}(m(t_{n-1})) = m(t_n)$$

Sometimes, we are only interested in showing $D_1, \ldots, D_{n-1}$, i.e., the sequence of basic topological change mappings, for a moving object m. We then allow to write:

$$dev(m, T) = D_1 \prec D_2 \prec \ldots \prec D_{n-1}$$

5. THREE-PHASE EVALUATION OF TOPOLOGICAL CHANGES IN A MOVING REGION

In this section, we introduce a three-phase evaluation method of detecting the topological development of a complex moving region. Section 5.1 introduces the first phase which partitions each observation into a set of evaluation units. Section 5.2 introduces the second phase which maps the evaluation units of one observation to the evaluation units of the other observation. Section 5.3 discusses how to interpret the topological changes from the mapping we obtained in the second phase.

5.1 The Partitioning Phase

The difficulty in detecting the topological changes in a complex region comes from the fact that such a region is composed by several sub-components, and different changes may happen to different sub-components. As we have mentioned in Section 1, one problem is that we do not know which sub-component before a topological change maps which sub-component after the change. Thus, our first step is to partition a complex moving region into a number of sub-components so that later topological changes can be detected easily. The partition is based on the rule that at most one topological change can happen to each sub-component between two observations we consider. For example, assume that the topological changes that we want to detect from a complex region are *region split* and *hole form*. Our partition will result in two sub-components so that we can detect a region split from one of them and a hole form from the other. We call the sub-component which only involves one basic topological change an *evaluation unit*. It is defined in Definition 5.1 and makes use of the geometric intersection operation $\otimes$.

***Definition* 5.1 (*Evaluation Unit*)** Let $O_1, O_2 \in \mathit{region}$ be two observations of a moving region at the time instances t_1 and t_2. Let the set $R_1 = \{c_{1,1}, \ldots, c_{1,n}\}$ be a spatial partition of O_1 such that (i) $n \in \mathbb{N}$, (ii) $c_{1,i} \in \mathit{region}$ for all $1 \leq i \leq n$, (iii) $O_1 = \bigoplus_{i=1}^{n} c_{1,i}$, and (iv) $c_{1,i} \otimes c_{1,j} = \varnothing$ for all $1 \leq i < j \leq n$ holds. Similarly, let $R_2 = \{c_{2,1}, \ldots, c_{2,n}\}$ be a spatial partition of O_2. Each subregion $c_{1,i}$ is called an *evaluation unit* of O_1, and each subregion $c_{2,j}$ is called an *evaluation unit* of O_2 if the following holds:

$$\exists\, \pi : \{1, \ldots, n\} \to \{1, \ldots, n\}, \pi \text{ total and bijective } \forall\, 1 \leq i \leq n\ \exists\, d_{i,\pi(i)} \in \{d_0, d_1, \ldots, d_{10}\} : \\ d_{i,\pi(i)}(c_{1,i}) = c_{2,\pi(i)}$$

We call the process of finding π as well as the evaluation units *partitioning*. As described in Definition 5.1, our goal of partitioning is to find the mapping of all evaluation units between two observations. This partitioning phase contains two steps. In a first step, we perform a preprocessing which adjusts or synchronizes the minimum bounding rectangles of both observations. The purpose of this is to eliminate topology preserving changes such as growing or location changing. In a second step, we partition each observation to evaluation units according to our rules. Given two specific snapshots, the result of the partitioning will be unique.

5.1.1 *Preprocessing: Adjust Minimum Bounding Rectangles.* Assume that we want to detect the topological change of a forest fire. The observations we take at t_1 and t_2 are two complex regions denoted by O_1 and O_2 respectively, as shown in Figure 5. As we have discussed in Section 3.4, from the figure, we can see that the moving region moves to the right and down with respect to the current coordinate system. At the same time, some topological changes, such as *region merge* and *hole appear* happen. Since this complex region changes its location in the coordinate system, it is difficult for us to detect what has happened between these two snapshots. Therefore, our first task is to adjust the coordinate systems before and after the changes, so that these two regions can be mapped to each other later.

Let $x = min_i^x$ denote the leftmost bounding line of O_i $(i \in \{1, 2\})$, $x = max_i^x$ denote the rightmost bounding line of O_i, $y = min_i^y$ denote the lower bounding line of O_i, and $y = max_i^y$ denote the upper bounding line of O_i. The point list $< (min_i^x, min_i^y), (max_i^x, max_i^y) >$ represents the minimum bounding rectangle (MBR) of O_i. The value $w_i = max_i^x - min_i^x$ equals the width of the MBR of O_i, and $h_i = max_i^y - min_i^y$ equals the height of the MBR of O_i. We "shift" the bottom-left point of each rectangle to $(0, 0)$ and adjust their width and height by shrinking or enlarging, so that these two observations will have the *same* MBRs. Let $p_i = (x_i, y_i) \in O_i$ denote a point of O_i before the adjustment, and $p'_i = (x'_i, y'_i)$ denote that point after the adjustment. Then they have to satisfy the following conditions:

(i) $\forall\, p_1 = (x_1, y_1) \in O_1 : p'_1 = (x_1 - min_1^x, y_1 - min_1^y)$
(ii) $\forall\, p_2 = (x_2, y_2) \in O_2 : p'_2 = (\frac{w_1(x_2 - min_2^x)}{w_2}, \frac{h_1(y_2 - min_2^y)}{h_2})$

Condition (i) shifts the entire observation O_1 by making the left bottom point locate at $(0, 0)$. Condition (ii) first shifts the observation O_2 to $(0, 0)$ and then adjusts its size by shrinking or enlarging the length and width with respect to the ratio of O_1. Thus, the result is that these two observations do not only have the same left bottom point but also have the same minimum bounding rectangles. The MBRs of both observations after the adjustment both become $< (0, 0), (max_1^x, max_1^y) >$, as shown in Figures 19a and 19b. These transformations are allowed since shifting, shrinking, and enlarging are topology preserving (topology invariant) operations.

5.1.2 *Partitioning of Complex Regions.* After adjusting the minimum bounding rectangles of these two observations, we perform the second step of the first phase: partitioning. We introduce a partitioning algorithm shown in Figure 18. This algorithm compares two consecutive observations O_1 and O_2 and divides each of them into a set of disjoint subregions represented by R_1 and R_2 respectively. In line 1 we initialize R_1 and R_2 as empty regions. By performing the intersection operation, line 2 finds the common part O of both observations. In line 3, the function *num_ of_ components* determines the number of separated regions in O. From lines 4 to 8, the algorithm adds all components of O to both R_1 and R_2 since they belong to both O_1 and O_2 originally. From lines 9 to 12, the algorithm computes the geometric differences D_1 and D_2 between O_1 and O and between O_2 and O. The difference of an original observation and O is a set of components. The operator *num_ of_ components* determines the number of separated regions in D_1 and in D_2 respectively. From lines 13 to 20, the algorithm adds these components to R_1 and R_2 respectively. R_1 and R_2 represent spatial partitions of O_1 and O_2.

5.2 The Mapping Phase

In this subsection, we describe the *mapping phase* as the second phase of our three-phase evaluation method. Each subregion $c_{1,i}$ of the spatial partition R_1 of O_1 with $1 \leq i \leq |R_1|$ is mapped to exactly one subregion $c_{2,j}$ of the spatial partition R_2 of O_2 by using the following *mapping rules* (MR):

MR1 If $c_{1,i}$ is already mapped, proceed to the next element $c_{1,i+1}$ in R_1. This may happen because $c_{1,i}$ has been mapped to some component of R_2 before according to MR3.

```
method partition (O1, O2)
  R1 ← ∅, R2 ← ∅
  O ← O1 ⊗ O2
  n ← num_of_components(O)
  for i = 1 to n
    R1 ← R1 ∪ O[i]
    R2 ← R2 ∪ O[i]
    i ← i + 1
  endfor
  D1 ← O1 ⊖ O
  m1 ← num_of_components(D1)
  D2 ← O2 ⊖ O
  m2 ← num_of_components(D2)
  for i = 1 to m1
    R1 ← R1 ∪ D1[i]
    i ← i + 1
  endfor
  for j = 1 to m2
    R2 ← R2 ∪ D2[j]
    j ← j + 1
  endfor
end
```

Fig. 18. Thealgorithm*partition* that divides observations into lists of sub-components

MR2 If $c_{1,i}$ is a connected component of R_1, i.e., $c_{1,i}$ is disjoint to any other component, and $c_{2,i}$ is a connected component of R_2, then we map $c_{1,i}$ to $c_{2,i}$, denoted by $c_{1,i} \mapsto c_{2,i}$ (the symbol $\mapsto$ indicates object-wise mapping). This situation happens when $c_{1,i}$ and $c_{2,i}$ are exactly the same in both observations; thus, no topological change happens between them.

MR3 If $c_{1,i}$ is adjacent to other components in R_1 and $c_{2,i}$ is a connected component in R_2, then add all components that can be reached from $c_{1,i}$ in R_1 to the left side of the mapping; add $c_{2,i}$ to the right side of the arrow. If the left side has a component which is also contained in R_2 but has not yet been mapped, append it to the right side. This guarantees that we do not miss any components that are connected to those components we already processed. For example, assume that in R_1 we have $c_{1,1}$ that is adjacent to $c_{1,3}$, and $c_{1,3}$ is adjacent to $c_{1,2}$; in R_2, we have $c_{2,1}$ and $c_{2,3}$ which are disjoint with each other, and we do not have $c_{2,2}$ in R_2, then we have the following mapping: $c_{1,1} \oplus c_{1,3} \oplus c_{1,2} \mapsto c_{2,1} \oplus c_{2,3}$.

MR4 If $c_{1,i}$ is a connected component in R_1 and $c_{2,i}$ is adjacent to other components in R_2, then add all components that can be reached from $c_{2,i}$ in R_2 to the right side of the arrow; add $c_{1,i}$ to the left side. If the right side has a component which is also contained in R_1 but has not yet been mapped, we append it to the left side. This rule is the opposite of rule MR3.

MR5 If there are remaining non-mapped connected components $c_{1,i}$ in R_1 or $c_{2,j}$ in R_2, then they are mapped to empty on the other side. This is trivial. Because the partition process will result different partitions of both observations. Thus, it is possible that a component in one observation cannot be mapped to any other component in the other observation. This rule will help us detect topological changes such as *region appear* or *region disappear*.

As a result of the application of the mapping rules, we obtain a set of object-wise *evaluation unit mappings* of the form $r \mapsto s$ where $r, s \in$ *region* are the *evaluation units* of these mappings.

5.3 The Interpretation Phase

As a result of the partitioning and mapping phases, we obtain the knowledge which evaluation unit from the first observation is mapped to which evaluation unit from the second observation. In the last phase, the *interpretation phase*, we have to determine the topological development between two observations by integrating all topological changes expressed by the evaluation unit mappings.

Assume that we have two observations O_{i-1} and O_i of a moving region object m at the time instances t_{i-1} and t_i as well as a number of evaluation unit mappings for O_{i-1} and O_i. For each evaluation unit mapping of the form $r \mapsto s$ with $r, s \in$ *region*, we can detect exactly one basic topological change mapping $d \in \{d_0, \ldots, d_{10}\}$ that describes the topological change in a manner such that $d(r) = s$ holds. Let the basic topological changes be $D_{i-1,1}, D_{i-1,2}, \ldots, D_{i-1,k}$ for $k \in \mathbb{N}$ and $D_{i-1,1}, D_{i-1,2}, \ldots, D_{i-1,k} \in \{d_0, \ldots, d_{10}\}$. Then the topological development between O_{i-1} and O_i is

$$dev(m, \{t_{i-1}, t_i\}) = D_{i-1,1} \prec D_{i-1,2} \prec \ldots \prec D_{i-1,k}$$

The order of $D_{i,1}, D_{i,2}, \ldots, D_{i,k}$ is not strict since they are detected together and can be seen as happening at the same time or in any order. Similarly, assume that later we obtain the third observation O_{i+1} at time t_{i+1}, partition it, and perform the mapping between O_i and O_{i+1}. Again we detect the topological development between O_i and O_{i+1}, denoted by $dev(m, \{t_i, t_{i+1}\})$. Then we obtain:

$$dev(m, \{t_{i-1}, t_i, t_{i+1}\}) = dev(m, \{t_{i-1}, t\}) \prec dev(m, \{t_i, t_{i+1}\})$$

Assume that we have n observations $O_1, O_2, \ldots, O_n$ of a complex moving region m with the observation times $T = \{t_1, \ldots, t_n\}$ and the order $t_1 < t_2 < \ldots < t_n$. Based on the topological development of each pair of consecutive observations, we get the topological development of m with respect to T as

$$\begin{aligned} dev(m, T) \;=\; & dev(m, \{t_1, t_2\}) \prec \ldots \prec dev(m, \{t_{i-1}, t_i\}) \prec dev(m, \{t_i, t_{i+1}\}) \prec \ldots \\ & \prec dev(m, \{t_{n-1}, t_n\}) \end{aligned}$$

6. CASE STUDY: DETECT TOPOLOGICAL CHANGES OF A COMPLEX MOVING REGION

To illustrate the three-phase strategy, we perform a case study in this section. We take the diagram in Figure 5 as an example. It shows two observations O_1 and O_2 of a complex moving region at two time instants. From the diagrams we notice that this complex region moves to the right and down during the interval $[t_1, t_2]$. In addition, we make a few findings of the changes between these two diagrams: (i) the left bottom component in O_1 shrinks compared to O_2, (ii) O_1 has four separated components while O_2 has only three separated components, indicating that either a *region merge* or a *region disappear* happens, and (iii) the component in the middle of O_1 forms a hole, which is not the case in O_2.

However, these are only intuitive considerations. Now we explain what happens exactly in a formal way using our method. Our first step is to adjust the minimum bounding rectangles of both observations. This eliminates the fact of a location change or other topology preserving changes and thus enables us to focus on the topological changes. The results of the adjustment are shown in Figures 19a and 19b where two observations have the same MBRs. Then we perform the second step. We apply the intersection operation on O_1 and O_2 which returns their overlapping part, denoted by O, as shown in Figure 19c. We then determine the difference between both observations. Thus, we apply the geometric difference operation between O_1 and O as well as between O_2 and O. The difference between O_1 and O is shown in Figure 19d. The left bottom part $c_{1,5}$ is adjacent to $c_{1,1}$ in R_1, and $c_{1,1}$ corresponds to $c_{2,1}$ in R_2. The component $c_{1,6}$ in the middle right of the figure is the part that disappears in O_2 but forms a hole inside c_3. We obtain Figure 19e which is a new partition of O_1, and we name it as R_1. Similarly, we get the new partition of O_2, which is R_2, shown in Figure 19f. We observe that Figure 19e is identical to Figure 19a, and Figure 19f is identical to Figure 19b with respect to geometry.

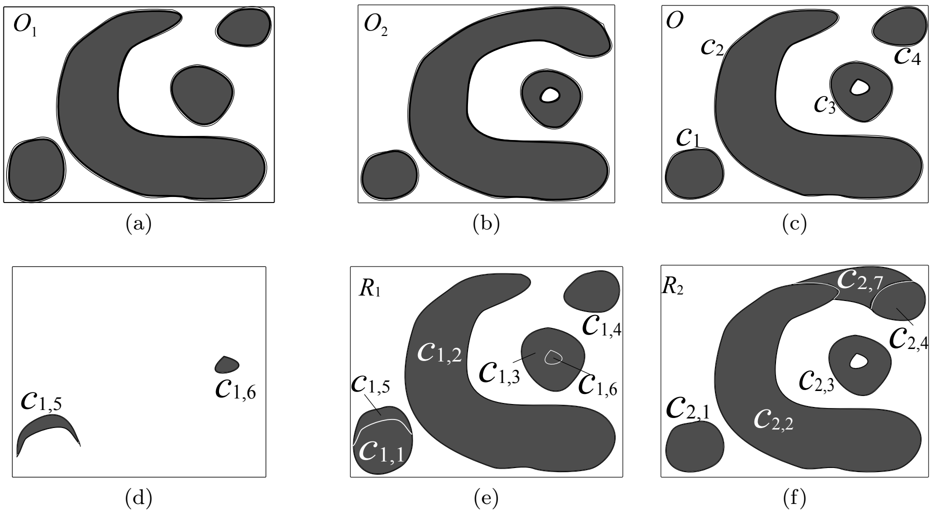


Fig. 19. Adjust MBR of O_1 (a) and O_2 (b), intersection O of the two snapshots O_1 and O_2 (c), geometric difference between O_1 and O (d); spatial partitions R_1 of O_1 (e) and R_2 of O_2 (f)

After performing the mapping phase, we obtain the following evaluation unit mappings between two observations,

$$c_{1,1} \oplus c_{1,5} \mapsto c_{2,1}$$
$$c_{1,2} \oplus c_{1,4} \mapsto c_{2,2} \oplus c_{2,7} \oplus c_{2,4}$$
$$c_{1,3} \oplus c_{1,6} \mapsto c_{2,3}$$

For each of the above evaluation unit mappings, we find a unique basic topological change mapping $d \in \{d_0, \ldots, d_{10}\}$:

$$d_0(c_{1,1} \oplus c_{1,5}) = c_{2,1}$$
$$d_6(c_{1,2} \oplus c_{1,4}) = c_{2,2} \oplus c_{2,7} \oplus c_{2,4}$$
$$d_3(c_{1,3} \oplus c_{1,6}) = c_{2,3}$$

Since d_0 is the topology preserving change, we do not need to report this change. Since d_6 and d_3 both happen in the interval $[t_1, t_2]$ for O_1 and O_2, we do not take care of their ordering. Thus the topological change of the moving region according to the first two snapshots is described as $d_6 \prec d_3$ or $d_3 \prec d_6$, which can be explained as *region merge* followed by *hole form*, or vice versa.

Assume that we have further observations O_3, O_4, O_5, and O_6 of the complex moving region at later time instants t_3, t_4, t_5, and t_6, as shown in Figure 20. We apply the same strategy to each consecutive pair of observations. Then we obtain a topological development for each such pair. From O_2 to O_3 we detect d_8 (*ring split*), from O_3 to O_4 we detect d_5 (*region split*), from O_4 to O_5 we detect $d_2 \prec d_5$ (*region disappear* followed by *region split*), and from O_5 to O_6 we detect d_1 (*region appear*).

In the end we assemble all topological developments together by the $\prec$ operator. For a moving object m with the set $T = \{t_1, t_2, t_3, t_4, t_4, t_5, t_6\}$ of observations times and $O_i = m(t_i)$ for $1 \leq i \leq 6$ we obtain the composite topological development

$$dev(m, T) = d_3(d_6(m(t_1))) \prec d_8(m(t_2)) \prec d_5(m(t_3)) \prec d_5(d_2(m(t_4))) \prec d_1(m(t_5)) = m(t_6)$$

or, if we prefer the short version,

$$dev(m, T) = [d_6 \prec d_3] \prec d_8 \prec d_5 \prec [d_2 \prec d_5] \prec d_1$$

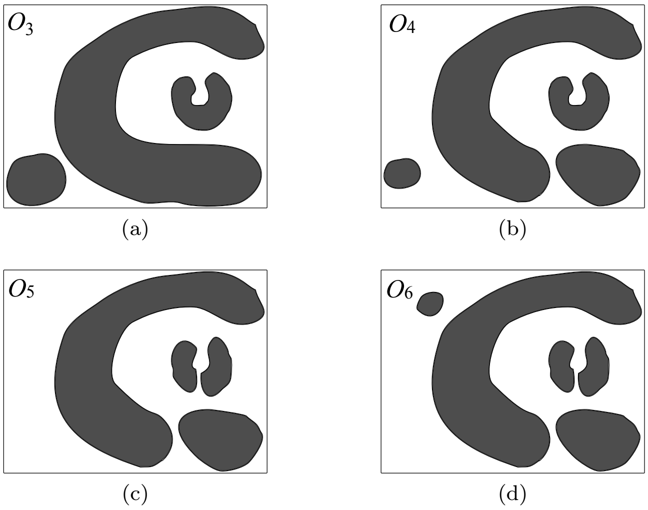


Fig. 20. Later observations of the complex moving region: O_3 (a), O_4 (b), O_5 (c), and O_6 (d)

The brackets [and] indicate that all basic topological change mappings listed between them belong to the same time interval of two consecutive observations. They can be omitted if this is not of interest.

7. CONCLUSIONS AND FUTURE WORK

In this article, we address the problem of identifying the topological development of a complex moving region object based on a list of observations or snapshots. We propose a solution to this problem by means of an algorithm. The change of topology of a moving region can be widely seen in many applications and thus studying this change is of great importance. In the spatiotemporal database context, a moving region is represented as a function from the data type *time* to the data type *region*. Thus, we first study the non-moving spatial data type *region*, discuss all possible topological shapes of a region object, and give formal definitions of them. Since it is difficult to track and store continuous movements, we capture complex moving regions at different time instants as snapshots. The snapshots are called observations, and the topological development of a moving region can be detected from a sequence of observations. Because a complex moving region is composed of several separated components, a major problem of finding topological changes from snapshots is that we do not know which component in the snapshot before a change maps to which component in the snapshot after the change. In our method, we provide a three-phase evaluation strategy which first partitions each observation into a set of components, then performs the mapping process between components, and then interprets the topological changes between two consecutive observations. We compose all topological changes between any two consecutive observations to express the topological development of the moving object. In the future we will implement complex moving regions and our evaluation algorithm in the context of databases and perform queries to detect the topological development of moving regions.

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