The Objects Interaction Matrix for Modeling Cardinal Directions in Spatial Databases

Tao Chen, Markus Schneider*, Ganesh Viswanathan and Wenjie Yuan

Department of Computer & Information Science & Engineering
University of Florida
Gainesville, FL 32611, USA
{tachen, mschneid, gv1, wyuan}@cise.ufl.edu

Abstract. Besides topological relations and approximate relations, *cardinal directions* have turned out to be an important class of qualitative spatial relations. In spatial databases and GIS they are frequently used as selection criteria in spatial queries. But the available models of cardinal relations suffer from a number of problems like the unequal treatment of the two spatial objects as arguments of a cardinal direction relation, the use of too coarse approximations of the two spatial operand objects in terms of single representative points or minimum bounding rectangles, the lacking property of converseness of the cardinal directions computed, the partial restriction and limited applicability to simple spatial objects only, and the computation of incorrect results in some cases. This paper proposes a novel two-phase method that solves these problems and consists of a tiling phase and an interpretation phase. In the first phase, a tiling strategy first determines the zones belonging to the nine cardinal directions of *each* spatial object and then intersects them. The result leads to a bounded grid called *objects interaction grid*. For each grid cell the information about the spatial objects that intersect it is stored in an *objects interaction matrix*. In the second phase, an interpretation method is applied to such a matrix and determines the cardinal direction. These results are integrated into spatial queries using directional predicates.

1 Introduction

Research on *cardinal directions* has had a long tradition in spatial databases, Geographic Information Systems (GIS), and other disciplines like cognitive science, robotics, artificial intelligence, and qualitative spatial reasoning. Cardinal directions are an important qualitative spatial concept and form a special kind of *directional relationships*. They represent *absolute* directional relationships like *north* and *southwest* with respect to a given reference system in contrast to *relative* directional relationships like *front* and *left*; thus, cardinal directions describe an order in space. In spatial databases they are, in particular, relevant as selection and join conditions in spatial queries. Hence, the determination of

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and reasoning with cardinal directions between spatial objects is an important research issue.

In the past, several approaches have been proposed to model cardinal directions. They all suffer from at least one of four main problems. First, some models use quite coarse approximations of the two spatial operand objects of a cardinal direction relation in terms of single representative points or minimum bounding boxes. This can lead to inaccurate results. Second, some models assume that the two spatial objects for which we intend to determine the cardinal direction have different roles. They create a scenario in which a *target object* A is placed relative to a dominating *reference object* B that is considered as the center of reference. This is counterintuitive and does not correspond to our cognitive understanding. Third, some models do not support *inverse cardinal directions*. This means that once the direction $dir(A, B)$ between two objects A and B is computed, the reverse direction $inv(dir(A, B))$ from B to A should be deducible, i.e., $inv(dir(A, B)) = dir(B, A)$. For example, if A is northwest and north of B, then the inverse should directly yield that B is to the southeast and south of A. Fourth, some models only work well if the spatial objects involved in direction computations have a *simple* structure. This is in contrast to the common consensus in the spatial database community that *complex* spatial objects are needed in spatial applications. As a consequence of these problems, some models can yield wrong or counterintuitive results for certain spatial scenarios.

The goal of this paper is to propose and design a computation model for cardinal directions that overcomes the aforementioned problems by taking better into account the shape of spatial operand objects, treating both spatial operand objects as equal partners, ensuring the property of converseness of cardinal directions ($A\ p\ B \Leftrightarrow B\ inv(p)\ A$), accepting complex spatial objects as arguments, and avoiding the wrong results computed by some approaches.

Our solution consists in a novel two-phase method that includes a *tiling phase* followed by an *interpretation phase*. In the first phase, we apply a tiling strategy that first determines the zones belonging to the nine cardinal directions of *each* spatial object and then intersects them. The result leads to a closed grid that we call *objects interaction grid* (*OIG*). For each grid cell we derive the information about the spatial objects that intersect it and store this information in a so-called *objects interaction matrix* (*OIM*). In the second phase, we apply an interpretation method to such a matrix and determine the cardinal direction.

Section 2 discusses related work and summarizes the available approaches to compute cardinal directions. In Section 3, the objects interaction matrix model is introduced in detail. Section 4 compares the OIM model to past approaches. Section 5 defines directional predicates for integrating cardinal directions into spatial queries. Finally, Section 6 draws conclusions and depicts future work.

2 Related Work

Several models have been proposed to capture cardinal direction relations between spatial objects (like *point*, *line*, and *region* objects) as instances of *spatial*

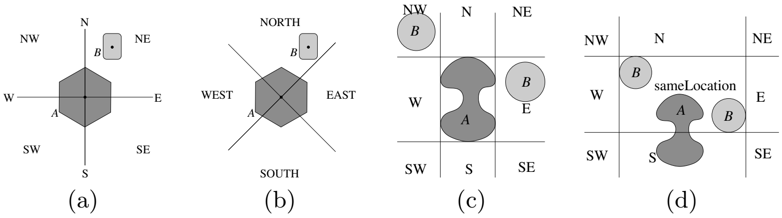


Fig. 1. Projection-based (a) and cone-shaped (b) models, and the Direction-Relation Matrix model with A as reference object (c) and with B as reference object (d)

data types [1]. These models can be classified into *tiling-based* models and *minimum bounding rectangle-based* (*MBR-based*) models.

Tiling-based models define cardinal direction relations by using partitioning lines that subdivide the plane into tiles. They can be further classified into *projection-based* models and *cone-shaped* models, both of which assign different roles to the two spatial objects involved. The first object represents the *target object* that is put into relationship to the second, dominant object called the *reference object*.

The *projection-based* models define direction relations by using partitioning lines parallel to the coordinate axes. The early approach in [2] first generalizes the reference (target) object by a reference (target) point (commonly the centroid of the object). Then it partitions the embedding space according to the reference point into four non-overlapping zones and uses the composition of two basic cardinal directions, that is, *north*, *west*, *south*, and *east*, in each zone to assign one of the four pairwise opposite directions *northwest*, *northeast*, *southeast*, and *southwest* to it (Figure 1a). The direction is then determined by the zone in which the target point falls. A problem of this approach is that the intermediate generalization step completely ignores the shape and extent of the spatial operand objects and thus leads to easy to use but rather inaccurate models. The *Direction-Relation Matrix* model [3, 4] presents a major improvement of this approach by better capturing the influence of the objects' shapes (Figure 1c). In this model, the partitioning lines are given by the infinite extensions of the minimum bounding rectangle segments of the reference object. This leads to a tiling with the nine zones of *north*, *west*, *east*, *south*, *northwest*, *northeast*, *southwest*, *southeast*, as well as a *central zone* named *sameLocation* and given by the minimum bounding rectangle of the reference object. The target object contributes with its exact shape, and a direction-relation matrix stores for each tile whether it is intersected by the target object. Thus this model suffers from the problem of unequal treatment of objects leading to incorrect and counterintuitive determinations of cardinal directions. For example, the Figure 5a shows a map of the two countries China and Mongolia. If China is used as the reference object, Mongolia is located in the minimum bounding rectangle of China, and thus the model only yields *sameLocation* as a result. This is not what we would intuitively expect. A further problem of this model is that it does not enable us to directly imply the inverse relation. For example, Figure 1c and Figure 1d show the same spatial configuration. If A is the reference object (Figure 1c), the model derives that parts of B are northwest and east of A. We would now expect that then A is southeast and west of B. But the model determines *sameLocation* and south as cardinal directions (Figure 1d).

The *cone-shaped* models define direction relations by using angular zones. The early approach in [5] first also generalizes a reference object and a target object by point objects. Two axis-parallel partitioning lines through the reference point are then rotated by 45 degrees and span four zones with the cardinal directions *north*, *west*, *east*, and *south* (Figure 1b). Due to the generalization step, this model can produce incorrect results. The *Cone-Based Directional Relations* concept [6] is an improvement of the early approach and uses the minimum bounding rectangle of the reference object to subdivide the space around it with partitioning lines emanating from the corners of the rectangle with different angles. This model has the problems of an unequal treatment of the operand objects and the lack of inverse cardinal relations.

MBR-based models approximate both spatial operand objects of a directional relationship through minimum bounding rectangles and bring the sides of these rectangles into relation with each other by means of Allen's interval relations [7]. By using these interval relations, the *2D-string* model [8] constructs a direction-specifying 2D string as a pair of 1D strings, each representing the symbolic projection of the spatial objects on the x- and y-axis respectively. The 2D String model may not provide correct inverse direction relations. Another weakness of this model (and its extensions) is the lack of the ability to uniquely define directional relations between spatial objects since they are based on the projection of the objects along both standard axes.

The *Minimum Bounding Rectangle* (*MBR*) model [9, 10] also makes use of the minimum bounding rectangles of both operand objects and applies Allen's 13 interval relations to the rectangle projections on the x- and y-axes respectively. 169 different relations are obtained [11] that are expressive enough to cover all possible directional relation configurations of two rectangles. A weakness of this model is that it can give misleading directional relations when objects are overlapping, intertwined, or horseshoe shaped. A comparison with the Direction-Relation Matrix model reveals that spatial configurations exist whose cardinal direction is better captured by either model.

3 The Objects Interaction Matrix Model

The Objects Interaction Matrix (OIM) model belongs to the tiling-based models, especially to the projection-based models. Figure 2 shows the two-phase strategy of our model for calculating the cardinal direction relations between two objects A and B. We assume that A and B are non-empty values of the complex spatial data type *region* [1]. The *tiling phase* in Section 3.1 details our novel tiling strategy that produces *objects interaction grids*, and shows how they are represented by *objects interaction matrices*. The *interpretation phase* in Section 3.2 leverages

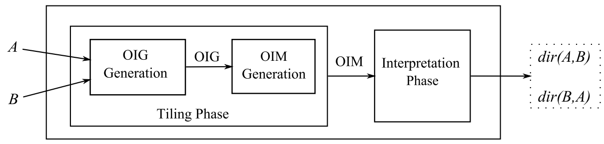


Fig. 2. Overview of the two phases of the Objects Interaction Matrix (OIM) model

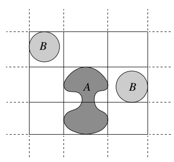
the objects interaction matrix to derive the directional relationship between two spatial region objects.

3.1 The Tiling Phase: Representing Interactions of Objects with the Objects Interaction Grid and Matrix

In this section, we focus on the *tiling phase* as the first phase of our OIM model. The general idea of our tiling strategy is to superimpose a grid called *objects interaction grid* (*OIG*) on a configuration of two spatial objects (regions). Such a grid is determined by the two vertical and two horizontal *partitioning lines* of *each* object. The two vertical (two horizontal) partitioning lines of an object are given as infinite extensions of the two vertical (two horizontal) segments of the object's minimum bounding rectangle. The four partitioning lines of an object create a partition of the Euclidean plane consisting of nine mutually exclusive, directional *tiles* or *zones* from which one is bounded and eight are unbounded (Figures 1c and 1d). Further, these lines partition an object into non-overlapping components where each component is located in a different tile. This essentially describes the tiling strategy of the Direction-Relation Matrix model (Section 2).

However, our fundamental difference and improvement is that we apply this tiling strategy to *both* spatial operand objects, thus obtain two separate grid partitions (Figures 1c and 1d), and then overlay both partitions (Figure 3a). This leads to an entirely novel cardinal direction model. The overlay achieves a coequal interaction and symmetric treatment of both objects. In the most general case, all partitioning lines are different from each other, and we obtain an overlay partition that shows 9 central, bounded tiles and 16 peripheral, unbounded tiles (indicated by the dashed segments in Figure 3a). The unbounded tiles are irrelevant for our further considerations since they cannot interact with both

$OIG(A, B) =$

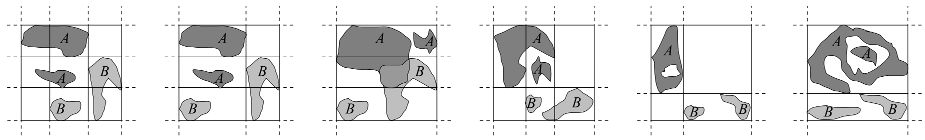


(a)

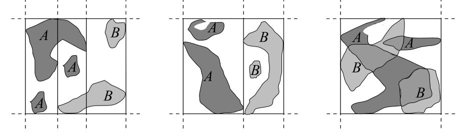
$$OIM(A, B) = \begin{pmatrix} 2 & 0 & 0 \\ 0 & 1 & 2 \\ 0 & 1 & 0 \end{pmatrix}$$

(b)

Fig. 3. The objects interaction grid $OIG(A, B)$ for the two region objects A and B in Figures 1c and 1d (a) and the derived objects interaction matrix $OIM(A, B)$ (b)



(a) 3×3-OIG (b) 3×2-OIG (c) 3×1-OIG (d) 2×3-OIG (e) 2×2 OIG (f) 2×1-OIG



(g) 1×3-OIG (h) 1×2-OIG (i) 1×1 OIG

Fig. 4. Examples of the nine possible sizes of objects interaction grids

objects. Therefore, we exclude them and obtain a grid space that is a bounded proper subset of $\mathbb{R}^2$, as Definition 1 states. This is in contrast to the partitions of all other tiling-based models that are unbounded and equal to $\mathbb{R}^2$.

Definition 1. *Let $A, B \in$ region with $A \neq \varnothing$ and $B \neq \varnothing$, and let* $min_x^r = \min\{x \mid (x, y) \in r\}$, $max_x^r = \max\{x \mid (x, y) \in r\}$, $min_y^r = \min\{y \mid (x, y) \in r\}$, *and* $max_y^r = \max\{y \mid (x, y) \in r\}$ *for* $r \in \{A, B\}$. *Then the* objects interaction grid space (OIGS) *of A and B is given as*

$$\text{OIGS}(A, B) = \{(x, y) \in \mathbb{R}^2 \mid \min(min_x^A, min_x^B) \leq x \leq \max(max_x^A, max_x^B) \wedge \min(min_y^A, min_y^B) \leq y \leq \max(max_y^A, max_y^B)\}$$

Definition 2 determines the bounded grid formed as a part of the partitioning lines and superimposed on $OIGS(A, B)$.

Definition 2. *Let* seg *be a function that constructs a segment between any two given points $p, q \in \mathbb{R}^2$, i.e.,* $seg(p, q) = \{t \mid t = (1 - \lambda)p + \lambda q, 0 \leq \lambda \leq 1\}$. *Let* $H_r = \{seg((min_x^r, min_y^r), (max_x^r, min_y^r)), seg((min_x^r, max_y^r), (max_x^r, max_y^r))\}$ *and* $V_r = \{seg((min_x^r, min_y^r), (min_x^r, max_y^r)), seg((max_x^r, min_y^r), (max_x^r, max_y^r))\}$ *for* $r \in \{A, B\}$. *We call the elements of H_A, H_B, V_A, and V_B* objects interaction grid segments. *Then the* objects interaction grid (OIG) *for A and B is given as*

$$OIG(A, B) = H_A \cup V_A \cup H_B \cup V_B.$$

This definition comprises the description of all grids that can arise. In the most general case, if $H_A \cap H_B = \varnothing$ and $V_A \cap V_B = \varnothing$, we obtain a bounded 3×3-grid. Special cases arise if $H_A \cap H_B \neq \varnothing$ and/or $V_A \cap V_B \neq \varnothing$. Then equal grid segments coincide in the union of all grid segments. As a result, depending on the relative position of two objects to each other, objects interaction grids can be of different size. However, due to the non-empty property of a region object, not all grid segments can coincide. This means that at least two horizontal grid segments and at least two vertical grid segments must be maintained. Figure 4 shows examples for all nine possible sizes of objects interaction grids, and Definition 3 gives a corresponding formal characterization.

Definition 3. *An objects interaction grid* OIG(A, B) *is of size* $m \times n$*, with* $m, n \in \{1, 2, 3\}$*, if* $|H_A \cap H_B| = 3 - m$ *and* $|V_A \cap V_B| = 3 - n$.

The objects interaction grid partitions the objects interaction grid space into *objects interaction grid tiles* (*zones*, *cells*). Definition 4 provides their definition.

Definition 4. *Given* $A, B \in$ *region with* $A \neq \varnothing$ *and* $B \neq \varnothing$, OIGS(A, B)*, and* OIG(A, B)*, we define* $c_H = |H_A \cup H_B| = |H_A| + |H_B| - |H_A \cap H_B|$ *and* c_V *correspondingly. Let* $H_{AB} = H_A \cup H_B = \{h_1, \ldots, h_{c_H}\}$ *such that (i)* $\forall\, 1 \leq i \leq c_H : h_i = seg((x_i^1, y_i), (x_i^2, y_i))$ *with* $x_i^1 < x_i^2$*, and (ii)* $\forall\, 1 \leq i < j \leq c_H : h_i < h_j$ *(we say that* $h_i < h_j :\Leftrightarrow y_j < y_i$*). Further, let* $V_{AB} = V_A \cup V_B = \{v_1, \ldots, v_{c_V}\}$ *such that (i)* $\forall\, 1 \leq i \leq c_V : v_i = seg((x_i, y_i^1), (x_i, y_i^2))$ *with* $y_i^1 < y_i^2$*, and (ii)* $\forall\, 1 \leq i < j \leq c_V : v_i < v_j$ *(we say that* $v_i < v_j :\Leftrightarrow x_i < x_j$*).*

Next, we define four auxiliary predicates that check the position of a point (x, y) *with respect to a grid segment:*

$$\begin{aligned}
below((x,y), h_i) &\Leftrightarrow x_i^1 \leq x \leq x_i^2 \;\wedge\; y \leq y_i \\
above((x,y), h_i) &\Leftrightarrow x_i^1 \leq x \leq x_i^2 \;\wedge\; y \geq y_i \\
right_of((x,y), v_i) &\Leftrightarrow y_i^1 \leq y \leq y_i^2 \;\wedge\; x \geq x_i \\
left_of((x,y), v_i) &\Leftrightarrow y_i^1 \leq y \leq y_i^2 \;\wedge\; x \leq x_i
\end{aligned}$$

An objects interaction grid tile $t_{i,j}$ *with* $1 \leq i < c_H$ *and* $1 \leq j < c_V$ *is then defined as*

$$\begin{aligned}
t_{i,j} = \{(x,y) \in \text{OIGS}(A,B) \mid\ & below((x,y), h_i) \;\wedge\; above((x,y), h_{i+1}) \;\wedge \\
& right_of((x,y), v_j) \;\wedge\; left_of((x,y), v_{j+1})\}
\end{aligned}$$

The definition indicates that all tiles are bounded and that two adjacent tiles share their common boundary. Let *OIGT*(A, B) be the set of all tiles $t_{i,j}$ imposed by *OIG*(A, B) on *OIGS*(A, B). An $m \times n$-grid contains $m \cdot n$ bounded tiles.

By applying our tiling strategy, an objects interaction grid can be generated for any two region objects A and B. It provides us with the valuable information which region object intersects which tile. Definition 5 provides us with a definition of the *interaction* of A and B with a tile.

Definition 5. *Given* $A, B \in$ *region with* $A \neq \varnothing$ *and* $B \neq \varnothing$ *and* OIGT(A, B)*, let* ι *be a function that encodes the* interaction *of* A *and* B *with a tile* $t_{i,j}$*, and checks whether no region,* A *only,* B *only, or both regions intersect a tile. We define this function as*

$$\iota(A, B, t_{i,j}) = \begin{cases} 0 \text{ if } & A^\circ \cap t_{i,j}^\circ = \varnothing \;\wedge\; B^\circ \cap t_{i,j}^\circ = \varnothing \\ 1 \text{ if } & A^\circ \cap t_{i,j}^\circ \neq \varnothing \;\wedge\; B^\circ \cap t_{i,j}^\circ = \varnothing \\ 2 \text{ if } & A^\circ \cap t_{i,j}^\circ = \varnothing \;\wedge\; B^\circ \cap t_{i,j}^\circ \neq \varnothing \\ 3 \text{ if } & A^\circ \cap t_{i,j}^\circ \neq \varnothing \;\wedge\; B^\circ \cap t_{i,j}^\circ \neq \varnothing \end{cases}$$

The operator $^\circ$ denotes the point-set topological *interior* operator and yields a region without its boundary. For each grid cell $t_{i,j}$ in the ith row and jth column of an $m \times n$-grid with $1 \leq i \leq m$ and $1 \leq j \leq n$, we store the coded information in an *objects interaction matrix* (*OIM*) in cell $OIM(A,B)_{i,j}$. Since directional relationships have a qualitative and not a quantitative or metric nature, we abstract from the geometry of the *objects interaction grid space* and only keep the information which region intersects which tile. The OIM for $m = n = 3$ is shown below, and Figure 3b gives an example.

$$OIM(A,B) = \begin{pmatrix} \iota(A,B,t_{1,1}) & \iota(A,B,t_{1,2}) & \iota(A,B,t_{1,3}) \\ \iota(A,B,t_{2,1}) & \iota(A,B,t_{2,2}) & \iota(A,B,t_{2,3}) \\ \iota(A,B,t_{3,1}) & \iota(A,B,t_{3,2}) & \iota(A,B,t_{3,3}) \end{pmatrix}$$

3.2 The Interpretation Phase: Assigning Semantics to the Objects Interaction Matrix

The second phase of the OIM model is the *interpretation phase*. This phase takes an objects interaction matrix obtained as the result of the tiling phase as input and uses it to generate a set of cardinal directions as output. This is achieved by separately identifying the locations of both objects in the objects interaction matrix and by pairwise interpreting these locations in terms of cardinal directions. The union of all these cardinal directions is the result.

In a first step, we define a function *loc* (see Definition 6) that acts on one of the region objects A or B and their common objects interaction matrix and determines all locations of components of each object in the matrix. Let $I_{m,n} = \{(i,j) \,|\, 1 \leq i \leq m, 1 \leq j \leq n\}$. We use an index pair $(i,j) \in I_{m,n}$ to represent the location of the element $M_{i,j} \in \{0,1,2,3\}$ and thus the location of an object component from A or B in an $m \times n$ objects interaction matrix.

Definition 6. *Let M be the $m \times n$-objects interaction matrix of two region objects A and B. Then the function loc is defined as:*

$$loc(A,M) = \{(i,j) \,|\, 1 \leq i \leq m, 1 \leq j \leq n, M_{i,j} = 1 \;\vee\; M_{i,j} = 3\}$$
$$loc(B,M) = \{(i,j) \,|\, 1 \leq i \leq m, 1 \leq j \leq n, M_{i,j} = 2 \;\vee\; M_{i,j} = 3\}$$

For example, in Figure 3b, object A occupies the locations (2,2) and (3,2), and object B occupies the locations (1,1) and (2,3) in the objects interaction matrix $OIM(A,B)$. Therefore, we obtain $loc(A, OIM(A,B)) = \{(2,2),(3,2)\}$ and $loc(B, OIM(A,B)) = \{(1,1),(2,3)\}$.

In a second step, we define an *interpretation function* ψ to determine the cardinal direction between any two object components of A and B on the basis of their locations in the objects interaction matrix. We use a popular model with the nine cardinal directions *north* (N), *northwest* (NW), *west* (W), *southwest* (SW), *south* (S), *southeast* (SE), *east* (E), *northeast* (NE), and *origin* (O) to symbolize the possible cardinal directions between *object components*. In summary, we obtain the set $CD = \{N, NW, W, SW, S, SE, E, NE, O\}$ of *basic cardinal directions*. A different set of basic cardinal directions would lead to a different interpretation function and hence to a different interpretation of index pairs. Definition 7 provides the interpretation function ψ with the signature $\psi : I_{m,n} \times I_{m,n} \to CD$.

Table 1. Interpretation table for the interpretation function ψ

(i,j) \ (i',j')	(1,1)	(1,2)	(1,3)	(2,1)	(2,2)	(2,3)	(3,1)	(3,2)	(3,3)
(1,1)	O	W	W	N	NW	NW	N	NW	NW
(1,2)	E	O	W	NE	N	NW	NE	N	NW
(1,3)	E	E	O	NE	NE	N	NE	NE	N
(2,1)	S	SW	SW	O	W	W	N	NW	NW
(2,2)	SE	S	SW	E	O	W	NE	N	NW
(2,3)	SE	SE	S	E	E	O	NE	NE	N
(3,1)	S	SW	SW	S	SW	SW	O	W	W
(3,2)	SE	S	SW	SE	S	SW	E	O	W
(3,3)	SE	SE	S	SE	SE	S	E	E	O

Definition 7. *Given* $(i,j),(i',j') \in I_{m,n}$*, the* interpretation function ψ *on the basis of the set* $CD = \{\mathrm{N}, \mathrm{NW}, \mathrm{W}, \mathrm{SW}, \mathrm{S}, \mathrm{SE}, \mathrm{E}, \mathrm{NE}, \mathrm{O}\}$ *of* basic cardinal directions *is defined as*

$$\psi((i,j),(i',j')) = \begin{cases} \mathrm{N} & \textit{if } \; i < i' \;\wedge\; j = j' \\ \mathrm{NW} & \textit{if } \; i < i' \;\wedge\; j < j' \\ \mathrm{W} & \textit{if } \; i = i' \;\wedge\; j < j' \\ \mathrm{SW} & \textit{if } \; i > i' \;\wedge\; j < j' \\ \mathrm{S} & \textit{if } \; i > i' \;\wedge\; j = j' \\ \mathrm{SE} & \textit{if } \; i > i' \;\wedge\; j > j' \\ \mathrm{E} & \textit{if } \; i = i' \;\wedge\; j > j' \\ \mathrm{NE} & \textit{if } \; i < i' \;\wedge\; j > j' \\ \mathrm{O} & \textit{if } \; i = i' \;\wedge\; j = j' \end{cases}$$

For example, in Figure 3b, we obtain that $\psi((3,2),(1,1)) = SE$ and $\psi((2,2),(2,3)) = W$ where holds that $(2,2),(3,2) \in loc(A, OIM(A,B))$ and $(1,1),(2,3) \in loc(B, OIM(A,B))$. Table 1 called *interpretation table* shows the possible results of the interpretation function for all index pairs.

In a third and final step, we specify the *cardinal direction function* named *dir* which determines the *composite cardinal direction* for two region objects A and B. This function has the signature $dir : region \times region \rightarrow 2^{CD}$ and yields a set of basic cardinal directions as its result. In order to compute the function *dir*, we first generalize the signature of our interpretation function ψ to $\psi : 2^{I_{m,n}} \times 2^{I_{m,n}} \rightarrow 2^{CD}$ such that for any two sets $X, Y \subseteq I_{m,n}$ holds: $\psi(X,Y) = \{\psi((i,j),(i',j')) \mid (i,j) \in X, (i',j') \in Y\}$. We are now able to specify the cardinal direction function *dir* in Definition 8.

Definition 8. *Let* $A, B \in region$*. Then the* cardinal direction function dir *is defined as*

$$dir(A,B) = \psi(loc(A, OIM(A,B)), loc(B, OIM(A,B)))$$

We apply this definition to our example in Figure 3. With $loc(A, OIM(A, B)) = \{(2, 2), (3, 2)\}$ and $loc(B, OIM(A, B)) = \{(1, 1), (2, 3)\}$ we obtain

$$\begin{aligned} dir(A, B) &= \psi(\{(2, 2), (3, 2)\}, \{(1, 1), (2, 3)\}) \\ &= \{\psi((2, 2), (1, 1)), \psi((2, 2), (2, 3)), \psi((3, 2), (1, 1)), \psi((3, 2), (2, 3))\} \\ &= \{SE, W, SW\} \end{aligned}$$

Similarly, we obtain the inverse cardinal direction as:

$$\begin{aligned} dir(B, A) &= \psi(\{(1, 1), (2, 3)\}, \{(2, 2), (3, 2)\}) \\ &= \{\psi((1, 1), (2, 2)), \psi((1, 1), (3, 2)), \psi((2, 3), (2, 2)), \psi((2, 3), (3, 2))\} \\ &= \{NW, E, NE\} \end{aligned}$$

Syntactically function *dir* yields a set of basic cardinal directions. The question is what the exact meaning of such a set is. We give the intended semantics of the function result in Lemma 1.

Lemma 1. *Let* $A, B \in region$*. Then* $dir(A, B) = \{d_1, \ldots, d_k\}$ *if the following conditions hold:*

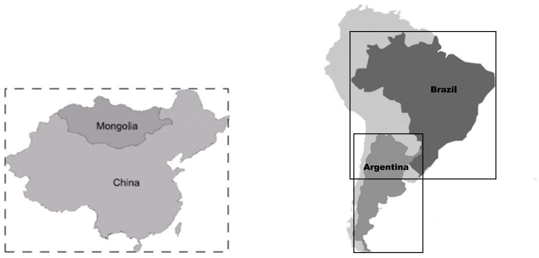
(i) $1 \leq k \leq 9$
(ii) $\forall\, 1 \leq i \leq k : d_i \in CD$
(iii) $\exists\, r_{11}, \ldots, r_{1k}, r_{21}, \ldots, r_{2k} \in region :$
  (a) $\forall\, 1 \leq i \leq k : r_{1i} \subseteq A, r_{2i} \subseteq B$
  (b) $dir(r_{11}, r_{21}) = d_1 \ \wedge\ \ldots\ \wedge\ dir(r_{1k}, r_{2k}) = d_k$

Several r_{1i} from A as well as several r_{2i} from B might be equal. Thus at most k parts from A and at most k parts from B are needed to produce the k basic cardinal directions of the result. There can be further parts from A and B but their cardinal direction is not a new contribution to the result.

Finally we can say regarding Figure 3 that "Object A is *partly southeast*, *partly west*, and *partly southwest* of object B" and that "Object B is *partly northwest*, *partly east*, and *partly northeast* of object A", which is consistent.

4 Comparison to Past Approaches

We now review the problems raised in the Introduction and show how our OIM model overcomes them. The first problem is the *coarse approximation problem* that leads to imprecise results. Models that capture directions between region objects have evolved from reducing these objects to points, to the use of minimum bounding rectangles to approximate their extent, and ultimately to the final goal of considering their shapes. From this perspective, the *Directional-Relation Matrix* (DRM) model is superior to the MBR model due to the fact that it captures the shape of the target object. However, it only represents an intermediate step between the MBR model and the final goal because only the shape of one object is considered and the shape of the other object does not contribute at all. The OIM model that we propose in this paper is the first model that considers the shapes of both region objects.



(a) (b)

Fig. 5. Determining the cardinal direction

Table 2. Cardinal directions between Argentina (A) and Brazil (B) in Figure 5b from different models

Model α	$dir_\alpha(A, B)$	$dir_\alpha(B, A)$
MBR	$\{weak\_bounded\_south\}$	$\{weak\_north\}$
DRM	$\{sL^\dagger, S\}$	$\{sL^\dagger, NW, N, NE, E\}$
2D-String	$\{S\}$	$\{NW, N, NE\}$
OIM	$\{S, W, SW, O, SE\}$	$\{N, E, NE, O, NW\}$

† *sL* means *sameLocation*

Unlike the MBR model and the 2D string model, where the region objects play the same role, the DRM model suffers from the *unequal treatment problem*. A target object is tested for intersection against the tiles created around a reference object. As a result, components of the target object inside different tiles contribute to the final direction relations while the reference object contributes as a whole object. This unequal treatment causes imprecision. Let $dir_{DRM}(A, B)$ be a function that determines the cardinal directions for two simple regions A and B in the DRM model where A is the target object and B is the reference object. Then, in Figure 5b, the DRM model determines the cardinal direction between Argentina (A) and Brazil (B) as $dir_{DRM}(A, B) = \{sameLocation, S\}$. This is not precise because for the major part of Brazil, Argentina lies to the *southwest*, and it also lies to the *west* of some part of Brazil. In our OIM model, objects are treated equally, and thus both contribute to the final cardinal direction. Our model yields the result $dir(A, B) = \{SE, S, SW, W, O\}$, which captures the cardinal directions precisely.

The *converse problem* is a common problem shared by most models. It means that models generate inconsistent results when swapping their operand objects. Table 2 shows the different cardinal directions between Argentina and Brazil in Figure 5b as they are obtained by different cardinal direction models. The results show that the MBR model, the DRM model, and the 2D-String model do not maintain the converseness property, i.e., $dir_\alpha(A, B) \neq inv(dir_\alpha(B, A))$ for $\alpha \in \{MBR, DRM, 2D\text{-}String\}$. Only the OIM model supports the inverse operation, i.e., $dir(A, B) = inv(dir(B, A))$ holds. Therefore, by applying the OIM model, we obtain consistent results corresponding to human intuition.

Further, the MBR model, the DRM model, and the 2D string model have originally been designed for simple regions only. Since all these models are based on the moving bounding rectangle approximation of at least one object, an extension to complex regions and their minimum bounding rectangles is feasible without difficulty. However, this procedure usually generates rather poor results. For example, in Figure 1d, if we take the minimum bounding rectangle of the entire object B, then object A is to the *weak_bounded_south* of object B according to the MBR model, and object A is to the *sameLocation* and *south* of object B according to the DRM model. Both results are imprecise since the western direction of A to one component of B is not captured. Although variants exist for the models to handle complex objects more precisely, considerable efforts are required. Our model natively supports complex objects and is able to yield much more precise results. For the same example in Figure 3a, our model generates the result $dir(A, B) = \{SE, W, SW\}$, which describes object A to be partly *southeast*, partly *west*, and partly *southwest* of object B.

As a summary, we show in Table 3 the evaluation of the four major models based on the four criteria of shape capturing, equal treatment of operand objects, support for the inverse operation, and support for complex objects.

5 Defining Directional Predicates within Databases

Based on the OIM model and the interpretation mechanism described in the previous sections, we can identify the cardinal directions between any given two complex region objects. To integrate the cardinal directions into spatial databases as selection and join conditions in spatial queries, binary *directional predicates* need to be formally defined. For example, a query like "Find all states that are strictly north of Florida" requires a directional predicate like *strict_north* as a selection condition of a spatial join. Assuming a relation *states* with attributes *sname* of type string and *loc* of type *region*, we can express the last query in an SQL-like style as follows:

```
SELECT s1.sname FROM states s1, states s2
WHERE  s2.sname='Florida' and strict_north(s1.loc,s2.loc);
```

The *dir* function, which produces the final cardinal directions between two complex region objects A and B, yields a subset of the set $CD = \{N, NW, W, SW,$

Table 3. Comparison of the OIM model with other cardinal direction models

Models	Shape Capturing	Equal Treatment	Inverse Operation	Complex Objects
MBR	no	yes	no	ps†
DRM	partially	no	no	ps†
2D-String	no	yes	no	ps†
OIM	yes	yes	yes	yes

† "ps" means "poorly supported"

$S, SE, E, NE, O\}$ of *basic cardinal directions*. As a result, a total number of $2^9 = 512$ cardinal directions can be identified. Therefore, at most 512 directional predicates can be defined to provide an *exclusive* and *complete* coverage of all possible directional relationships. We can assume that users will not be interested in such a large, overwhelming collection of detailed predicates since they will find it difficult to distinguish, remember and handle them. Instead, a reduced and manageable set is preferred. Such a set should be user-defined and/or application specific. It should be application specific since different applications may have different criteria for the distinction of directional relationships. For example, one application could require a clear distinction between the cardinal direction *north* and *northwest*, whereas another application could perhaps accept no distinction between the two and regard them both as *northern*. Thus it should also offer user the flexibility of defining their own set of predicates.

As a first step, in Definition 9, we propose nine *existential directional predicates* that ensure the existence of a particular basic cardinal direction between parts of two region objects A and B.

Definition 9. *Let $A, B \in region$. Then the existential directional predicates are defined as*

$$\begin{array}{lcl}
exists_north(A,B) & \equiv & (N \in dir(A,B)) \\
exists_south(A,B) & \equiv & (S \in dir(A,B)) \\
exists_east(A,B) & \equiv & (E \in dir(A,B)) \\
exists_west(A,B) & \equiv & (W \in dir(A,B)) \\
exists_origin(A,B) & \equiv & (O \in dir(A,B)) \\
exists_northeast(A,B) & \equiv & (NE \in dir(A,B)) \\
exists_southeast(A,B) & \equiv & (SE \in dir(A,B)) \\
exists_northwest(A,B) & \equiv & (NW \in dir(A,B)) \\
exists_southwest(A,B) & \equiv & (SW \in dir(A,B))
\end{array}$$

For example, $exists\_north(A, B)$ returns *true* if a part of A is located to the north of B; this does not exclude the existence of other cardinal directions. Later, by using this set of existential directional predicates together with $\neg$, $\vee$ and $\wedge$ operators, the user will be able to define any set of composite directional predicates for their own applications.

The following Lemma 2 shows that by using the existential directional predicates and the logical operators $\neg$, $\vee$ and $\wedge$, we can obtain a complete coverage and distinction of all possible 512 basic and composite cardinal directions from the OIM model based on the CD set.

Lemma 2. *Let the list $\langle d_1, d_2, d_3, d_4, d_5, d_6, d_7, d_8, d_9\rangle$ denote the cardinal direction list $\langle N, S, E, W, O, NE, SE, NW, SW\rangle$ and let the list $\langle p_1, p_2, p_3, p_4, p_5, p_6, p_7, p_8, p_9\rangle$ denote the basic directional predicates list $\langle exists\_north, exists\_south, exists\_east, exists\_west, exists\_origin, exists\_northeast, exists\_southeast, exists\_northwest, exists\_southwest\rangle$. Let $A, B \in region$ and $1 \leq i, j \leq 9$. Then for any basic or composite cardinal direction provided by $dir(A, B)$, the following logical expression returns true:*

$$\bigwedge_{d_i \in dir(A,B)} p_i \ \wedge \bigwedge_{d_j \notin dir(A,B)} \neg p_j$$

The existential predicates provide an interface for the user to define their own *derived directional predicates*. We give two examples.

The first set is designed to handle *strict directional predicates* between two region objects. *Strict* means that two region objects are in exactly one basic cardinal direction to each other. Definition 10 shows an example of *strict_north* by using the existential predicates.

Definition 10. *Let* $A, B \in$ *region. Then* strict_north *is defined as:*

$$\begin{aligned} strict\_north(A,B) \ = \ & exists\_north(A,B) \wedge \neg exists\_south(A,B) \\ & \wedge \neg exists\_west(A,B) \wedge \neg exists\_east(A,B) \\ & \wedge \neg exists\_northwest(A,B) \wedge \neg exists\_northeast(A,B) \\ & \wedge \neg exists\_southwest(A,B) \wedge \neg exists\_southeast(A,B) \\ & \wedge \neg exists\_origin(A,B) \end{aligned}$$

The other strict directional predicates *strict_south*, *strict_east*, *strict_west*, *strict_origin*, *strict_northeast*, *strict_northwest*, *strict_southeast*, *strict_southwest* are defined in a similar way.

The second set of predicates is designed to handle *similarly oriented directional predicates* between two regions. *Similarly oriented* means that several cardinal directions facing the same general orientation belong to the same group. Definition 11 shows an example of *northern* by using the existential predicates.

Definition 11. *Let* $A, B \in$ *region. Then* northern *is defined as:*

$$\begin{aligned} northern(A,B) \ = \ & (exists\_north(A,B) \vee exists\_northwest(A,B) \\ & \vee exists\_northeast(A,B)) \\ & \wedge \neg exists\_east(A,B) \wedge \neg exists\_west(A,B) \\ & \wedge \neg exists\_south(A,B) \wedge \neg exists\_southwest(A,B) \\ & \wedge \neg exists\_southeast(A,B) \wedge \neg exists\_origin(A,B) \end{aligned}$$

The other similarly oriented directional predicates *southern*, *eastern*, and *western* are defined in a similar way. From Definition 11, we can see that due to the disjunction of three existential directional predicates each similarly oriented directional predicate represents multiple directional relationships between two objects. For example, if A is in the northern part of B, then $dir(A,B) \in \{\{N\}, \{NW\}, \{NE\}, \{N,NW\}, \{N,NE\}, \{NW,NE\}, \{N,NW,NE\}\}$.

We can now employ predicates like *strict_north*, *northern* and *exists_north* in queries. For example, assuming we are given the two relations:

```
states(sname:string, area:region)
national_parks(pname:string, area:region)
```

We can pose the following three queries: *Determine the national park names where the national park is located* (a) *strictly to the north of Florida,* (b) *to the northern of Florida, and* (c) *partially to the north of Florida.* The corresponding SQL queries are as follows:

(a)
```
SELECT P.pname FROM national_park P, states S
WHERE  S.sname='Florida' and strict_north(P.area, S.area)
```

(b)
```
SELECT P.pname FROM national_park P, states S
WHERE  S.sname='Florida' and northern(P.area, S.area)
```

(c)
```
SELECT P.pname FROM national_park P, states S
WHERE  S.sname='Florida' and exists_north(P.area, S.area)
```

6 Conclusions and Future Work

In this paper, we have laid the foundation of a novel concept, called *Objects Interaction Matrix* (*OIM*), for determining cardinal directions between region objects. We have shown how different kinds of directional predicates can be derived from the cardinal directions and how these predicates can be employed in spatial queries. In the future, we plan to extend our approach to handle two complex point objects, two complex line objects, and all mixed combinations of spatial data types. Other research issues refer to the efficient implementation and the design of spatial reasoning techniques based on the OIM model.

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