Modeling Cardinal Directions in the 3D Space with the Objects Interaction Cube Matrix

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ABSTRACT

In GIS and spatial databases, *cardinal directions* are frequently used as selection and join criteria in query languages. However, most cardinal direction models are only able to handle two-dimensional spatial objects. But, e.g., geoscientists and engineers in fields like geography, cartography, soil engineering, and landscape modeling have shown an increasing demand for modeling cardinal directions between objects in the three-dimensional space. Unfortunately, the few available 3D cardinal direction models suffer from several problems like the coarse approximation of the two operand 3D objects in terms of single points and minimum bounding boxes, the lacking property of converseness of the cardinal direction relations computed, and the incomplete coverage of all possible direction relations. All problems mentioned can lead to incorrect results. This paper proposes a new model that solves these problems and in a first stage introduces an *objects interaction cube* and, as its representation, an *objects interaction cube matrix* to capture all possible interactions between any two 3D spatial objects. In a second stage, an interpretation technique is applied to the objects interaction cube matrix to determine the cardinal directions.

Categories and Subject Descriptors

H.2.8 [**Information Systems**]: Spatial databases and GIS

General Terms

Design

Keywords

3D Cardinal Direction, Objects Interaction Cube (Matrix)

1. INTRODUCTION

Cardinal Directions are an important class of qualitative spatial relationships with a long research tradition in spatial databases, GIS, and disciplines like cognitive science, robotics, artificial intelligence, and qualitative spatial reasoning. Cardinal directions represent *absolute* directional relationships like *north* and *southwest* with respect to a given reference system. In spatial databases, they are usually integrated into query languages as selection and join conditions.

So far, most efforts have focused on modeling cardinal directions between objects in the two-dimensional (2D) space while efforts in the three-dimensional space (3D) have been rare. Besides the known 2D cardinal directions, the third dimension introduces new direction components such as *above* and *below* that have to be taken into account in 3D cardinal direction models. The available 3D models suffer from several problems. First, they do not consider the shapes of the 3D objects and either approximate these objects as single 3D points or as minimum bounding cubes. Hence, they loose precision and sometimes lead to incorrect results. Second, in some models the two 3D operand objects are treated in an unequal manner, thus leads to the violation of the principle of converseness. That is, the cardinal direction determined for two 3D spatial objects A and B is not always equal to the inverse of the cardinal relation determined for B and A. Third, some models do not have a complete coverage of all possible relations.

The goal of this paper is to introduce a new two-phase model, called *objects interaction cube matrix* ($OICM$) model that solves these problems and consists of a *representation phase* and an *interpretation phase*. The first phase uses a tiling strategy to determine the 3D zones belonging to the 27 cardinal directions with respect to *each* spatial object and then intersects them. The result leads to a bounded $k \times m \times n$-cube ($k, m, n \in \{1, 2, 3\}$) called *objects interaction cube*. For each cube cell the information about the spatial objects that intersect it is stored in an *objects interaction cube matrix*. In the second phase, an interpretation method is applied to such a matrix to determine the cardinal direction.

The rest of the paper is organized as follows. Section 2 presents the available 3D cardinal direction models and briefly discusses their problems. An overview of our $OICM$ model is given in Section 3. We detail the objects interaction cube and its matrix representation in Section 4. Section 5 describes the interpretation of an objects interaction cube matrix in order to derive the cardinal direction. Finally, Section 6 draws some conclusions and discusses future work.

*This work was partially supported by the National Science Foundation under grant number NSF-IIS-0915914.

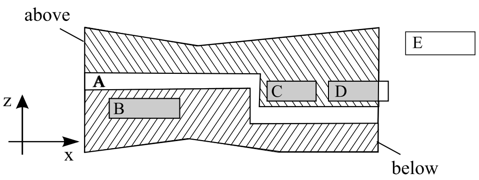

Figure 1: An example of the projection-based directional model.

2. RELATED WORK

Cardinal direction models for 3D spatial objects are rare. Most models developed are *points-based* models so that the extent of objects is ignored. An absolute direction model that considers cardinal directions like *north* and *east* can be found in [5]. It ignores the elevation and projects the space to a 2D coordinate system. In [4], a conceptual framework that depicts positional relationships including both distance and direction relationships is developed. For all the mentioned models, spatial objects are generalized as points, and directions are based on point locations so that the models become rather imprecise.

The *projection-based directional model* [1] extrudes a reference object along the coordinate axes in the direction requested. The extrusion body is tested for the intersection with a target object. The model enables the definition of the semantics of a set of cardinal directions such as *eastOf*, *westOf*, *northOf*, *southOf*, *above* and *below*. Figure 1 shows an example. The two extrusion bodies of the reference object A are identified as *above* and *below*. The target object B intersects the extrusion body *below* so that B is below A. The target objects C and D intersect the extrusion body *above* so that they are above A. The target object E does not intersect any extrusion body so that no statement can be made. Obviously, the expressiveness is a major weakness of this model since the model cannot determine the cardinal direction for all spatial configurations.

The *three-dimensional cardinal direction* (TCD) model [2] partially considers the shape of objects. Based on the *direction relation matrix* (DRM) model [3] for cardinal relations in the two-dimensional space, the TCD model takes the minimum bounding box (cube) of a reference object, extends its boundary faces to planes, and divides the space into 27 direction tiles, as shown in Figure 2a. The superscripts U, B and M denote the top, middle, and bottom of the reference object, respectively. This model leads to the 27 atomic TCD relations, $NW^U, \ldots, SE^U, NW^M, \ldots, SE^M, NW^B, \ldots, SE^B$ that symbolize the basic cardinal direction relations between the components of the target object and the reference object. The TCD relations form a set of exhaustive and pairwise disjoint relations. Figure 2b shows an example of two volumes A and B. If A is taken as the reference object, then the direction relationship between A and B is $dir(A,B) = N^U : O^U : O^M$. The TCD model presents a major improvement on expressiveness. However, the unequal treatment of the two operand objects leads a *converseness* problem.since the model yields $dir(B,A) = W^M : O^M : E^M : SW^M : S^M : SE^M : W^B : O^B : E^B : SW^B : S^B : SE^B$. But this is unequal to the expected inverse $S^B : O^B : O^M$ of $dir(A,B)$. Further, the model generates incorrect results for some cases. For instance, in Figure 3a, the model yields $dir(A,B) = N^U : O^U : O^M$ as the direction relationships between objects A and B. The direction O^U should not be part of direction relationships since no part of B lies directly above A.

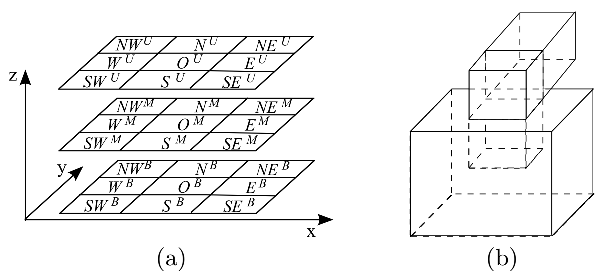

Figure 2: The 27 atomic TCD relations (a). An example where the directional relationships between two objects A and B is $dir(A,B) = N^U : O^U : O^M$ (b).

3. OVERVIEW OF THE OBJECTS INTERACTION CUBE MODEL

The main idea of our novel *Objects Interaction Cube Matrix* ($OICM$) model is to capture and represent the interactions between two general 3D volumes A and B with possibly disconnected components and cavities and to derive the cardinal direction from this information. Our model consists of a representation phase and an interpretation phase. In the *representation phase*, we create an $k \times m \times n$-*Objects Interaction Cube* (OIC) ($k, m, n \in \{1, 2, 3\}$) by intersecting their minimum bounding boxes with each other to capture all possible interactions between two 3D volume objects. We use an *Objects Interaction Cube Matrix* ($OICM$) to keep which object intersects which sub-cube. In the *interpretation phase*, we develop a technique to compute the cardinal direction from the matrix and define the semantic of the result obtained. Each phase will be detailed in the following sections.

4. REPRESENTING INTERACTIONS OF OBJECTS IN 3D SPACE WITH THE OBJECTS INTERACTION CUBE

In this section, we focus on the *representation phase* of our OICM model. Section 4.1 details the strategy of creating an *objects interaction cube* for any two arbitrary complex 3D volume objects. In Section 4.2 we losslessly map such a cube to an *objects interaction cube matrix* ($OICM$).

4.1 The Objects Interaction Cube (OIC)

The general idea is to superimpose a cube called *objects interaction cube* on a configuration of two 3D spatial objects (volumes). Such a cube is constructed from a total of twelve partitioning planes derived from *both* objects. *Partitioning planes* are the infinite extensions of the faces of the axis-aligned minimum bounding box around each object; each partitioning plane is perpendicular to one particular coordinate axis. The six partitioning planes from each object create a partition of the Euclidean space $\mathbb{R}^3$ into 27 mutually exclusive cells from which only the central cell is bounded and the other 26 cells are unbounded. The object itself is located in the bounded, central cell. This essentially describes the tiling strategy of the TCD model.

However, our fundamental improvement is that we employ this tilling strategy to *both* objects, thus obtain two separate space partitions, and then overlay both partitions. The overlay creates a new subdivision of space, and further, also partitions each object into non-overlapping components, where each component of one object lies in a different bounded cell. This overlay achieves coequal interaction and symmetric treatment of the two objects. In the most general case, all partitioning planes are different from each other, and the overlay creates 27 non-overlapping bounded cells and several unbounded cells. Since the components of the objects are always located inside the bounded cells, we can exclude all unbounded cells and only focus on the bounded cells that are relevant to the objects' interactions. Definition 1 formally defines the *objects interaction cube space* that contains all bounded cells.

Definition 1. Let $A, B \in volume$ with $A \neq \varnothing$ and $B \neq \varnothing$, and let $min_x^v = \min\{x \,|\, (x,y,z) \in v\}$, $max_x^v = \max\{x \,|\, (x,y,z) \in v\}$, $min_y^v = \min\{y \,|\, (x,y,z) \in v\}$, $max_y^v = \max\{y \,|\, (x,y,z) \in v\}$, $min_z^v = \min\{z \,|\, (x,y,z) \in v\}$, and $max_z^v = \max\{z \,|\, (x,y,z) \in v\}$ for $v \in \{A, B\}$. Then the *objects interaction cube space* (*OICS*) of A and B is given as

$$\begin{aligned}
OICS(A,B) = \{(x,y,z) \in \mathbb{R}^3 \,|\, & \\
\min(min_x^A, min_x^B) \leq x \leq \max(max_x^A, max_x^B) \wedge & \\
\min(min_y^A, min_y^B) \leq y \leq \max(max_y^A, max_y^B) \wedge & \\
\min(min_z^A, min_z^B) \leq z \leq \max(max_z^A, max_z^B)\} &
\end{aligned}$$

Definition 2 defines *partitioning faces* as part of the *partitioning planes* and superimposes them on the objects interaction cube space to obtain the *objects interaction cube*.

Definition 2. Let $f^v_{min\_x}$, $f^v_{max\_x}$, $f^v_{min\_y}$, $f^v_{max\_y}$, $f^v_{min\_z}$, and $f^v_{max\_z}$ denote the six *partitioning faces* of v for $v \in \{A, B\}$, then

$$\begin{aligned}
f^v_{min_x} &= \{(x,y,z) \in OICS(A,B) \,|\, x = min_x^v\} \\
f^v_{max_x} &= \{(x,y,z) \in OICS(A,B) \,|\, x = max_x^v\} \\
f^v_{min_y} &= \{(x,y,z) \in OICS(A,B) \,|\, y = min_y^v\} \\
f^v_{max_y} &= \{(x,y,z) \in OICS(A,B) \,|\, y = max_y^v\} \\
f^v_{min_z} &= \{(x,y,z) \in OICS(A,B) \,|\, z = min_z^v\} \\
f^v_{max_z} &= \{(x,y,z) \in OICS(A,B) \,|\, z = max_z^v\}
\end{aligned}$$

Next, let F_x, F_y and F_z denote the three sets that contain the partitioning faces perpendicular to the x-axis, the y-axis and the z-axis respectively. We obtain

$$\begin{aligned}
F_x &= \{f^A_{min_x}, f^A_{max_x}, f^B_{min_x}, f^B_{max_x}\} \\
F_y &= \{f^A_{min_y}, f^A_{max_y}, f^B_{min_y}, f^B_{max_y}\} \\
F_z &= \{f^A_{min_z}, f^A_{max_z}, f^B_{min_z}, f^B_{max_z}\}
\end{aligned}$$

Finally, we get the ("cell walls" of the) *objects interaction cube* (*OIC*) for A and B (see Figures 3(a) and 3(b)) as

$$OIC(A,B) = F_x \cup F_y \cup F_z$$

This definition comprises all cases for OICs. In general, we obtain a $3 \times 3 \times 3$ OIC if $|F_x| = |F_y| = |F_z| = 4$, meaning that there are a total of 12 different partitioning faces that forms 27 bounded cells. Special cases arise if at least one of the three sets F_x, F_y, and F_z contains less than four partitioning faces; this means that at least two partitioning faces from the two objects coincide. This results in different sizes of cubes. Definition 3 formally describes the situation.

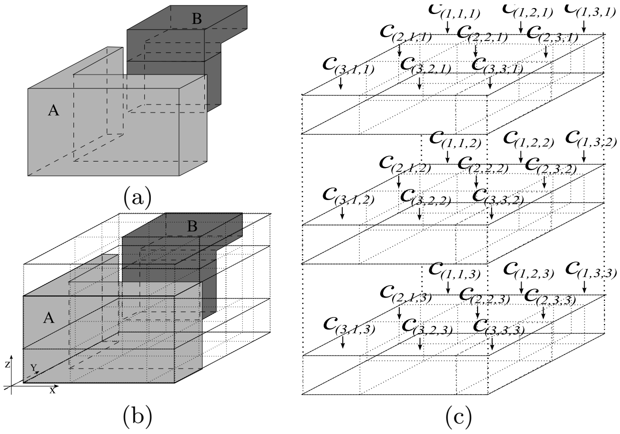

Figure 3: Examples of two volumes A and B (a), their OIC representation (b), and the labeling of the OIC cubic cells (c).

Definition 3. An objects interaction cube $OIC(A, B)$ is of size $k \times m \times n$, with $k, m, n \in \{1, 2, 3\}$, if $|F_x| = k + 1$, $|F_y| = m + 1$, and $|F_z| = n + 1$.

The objects interaction cube space is partitioned into several *cubic cells* by the partitioning faces (see Definition 4).

Definition 4. Since each partitioning face is perpendicular to a specific coordinate axis, we first define an ordering of the partitioning planes that are perpendicular to the same axis. Let $h_1, h_2 \in F_r$ with $r \in \{x, y, z\}$, then $h_1 < h_2$ if $p_1.r < p_2.r$ for any $p_1 \in h_1$ and any $p_2 \in h_2$. Next, we define three auxiliary lists that store partitioning faces regarding the order $<$:

$$\begin{aligned}
list_{left_right} &= \{\langle f_1, ..., f_{k'}\rangle \,|\, k' = |F_x|, \\
&\quad \forall 1 \leq i < j \leq k' \; \forall f_i, f_j \in F_x : f_i < f_j\} \\
list_{behind_front} &= \{\langle g_1, ..., g_{m'}\rangle \,|\, m' = |F_y|, \\
&\quad \forall 1 \leq i < j \leq m' \; \forall g_i, g_j \in F_y : g_i > g_j\} \\
list_{top_down} &= \{\langle h_1, ..., h_{n'}\rangle \,|\, n' = |F_z|, \\
&\quad \forall 1 \leq i < j \leq n' \; \forall h_i, h_j \in F_z : h_i > h_j\}
\end{aligned}$$

Further, for the partitioning faces $f \in F_r$ with $r \in \{x, y, z\}$, let the function $get_r(f)$ return the common value r of all points of the face f. An *objects interaction cubic cell* $c_{(i,j,l)}$ with $1 \leq i \leq k$, $1 \leq j \leq m$ and $1 \leq l \leq n$ is then defined as:

$$\begin{aligned}
c_{(i,j,l)} = \{(x,y,z) \in & OIGS(A,B) \,| \\
& get_x(f_i) \leq x \leq get_x(f_{i+1}) \\
& get_y(g_j) \leq y \leq get_y(g_{j+1}) \\
& get_z(h_l) \leq z \leq get_z(h_{l+1})\}
\end{aligned}$$

Figure 3(c) shows the visualization of the labeling of cubic cells in a $3 \times 3 \times 3$ OIC.

4.2 The Objects Interaction Cube Matrix (OICM)

The objects interaction cube for two volume objects A and B provides us with the valuable information which volume object intersects with cubic cell. Definition 5 provides the definition of the *interaction* between A, B, and a cubic cell.

Definition 5. Given $A, B \in volume$ with $A \neq \varnothing$ and $B \neq \varnothing$, let ι be a function that encodes the *interaction* of A and B with a cubic cell $c_{(i,j,l)}$, and checks whether no object, A only, B only, or both objects intersect a cubic cell. We define this function as

$$\iota(A,B,c_{(i,j,l)}) = \begin{cases} 0 \text{ if} & A^\circ \cap c^\circ_{(i,j,l)} = \varnothing \wedge \\ & B^\circ \cap c^\circ_{(i,j,l)} = \varnothing \\ 1 \text{ if} & A^\circ \cap c^\circ_{(i,j,l)} \neq \varnothing \wedge \\ & B^\circ \cap c^\circ_{(i,j,l)} = \varnothing \\ 2 \text{ if} & A^\circ \cap c^\circ_{(i,j,l)} = \varnothing \wedge \\ & B^\circ \cap c^\circ_{(i,j,l)} \neq \varnothing \\ 3 \text{ if} & A^\circ \cap c^\circ_{(i,j,l)} \neq \varnothing \wedge \\ & B^\circ \cap c^\circ_{(i,j,l)} \neq \varnothing \end{cases}$$

The operator $^\circ$ denotes the point-set topological *interior* operator and yields a volume without its boundary. The function ι encodes the interaction between two volumes and a cubic cell of their OIC. In order to represent a three dimensional $k \times m \times n$ cube with a two dimensional matrix, we introduce a *cubic column vector* $v_{i,j} = \langle c_{(i,j,1)}, \ldots, c_{(i,j,n)} \rangle$ with $1 \le i \le k$ and $1 \le j \le m$. It represents a list of cubic cells that are ordered along the z-axis in a top-down fashion. Further, we define a variant of the function ι, ι', that takes the objects A, B, and the cubic column vector $v_{i,j}$ as inputs, and returns a vector of encodings. So we have $\iota'(A,B,v_{i,j}) = \langle \iota(A,B,c_{(i,j,1)}), \ldots, \iota(A,B,c_{(i,j,n)}) \rangle$. Thus for each cubic column vector, we store the encoded interaction information in a vector element of an *objects interaction cube matrix* (*OICM*). In this way, we establish a mapping from the objects interaction cube, which is the geometric representation of the objects' interaction, to the matrix representation of the objects' interaction information. We obtain 27 possible objects interaction cubes of size $k \times m \times n$ since $k, m, n \in \{1,2,3\}$. These are mapped to 27 possible objects interaction cube matrices of size $k \times m$ with vector element length n. As an example, we show the objects interaction cube matrix $OICM(A,B)$ for a given $3 \times 3 \times 3$ objects interaction cube $OIC(A,B)$:

$$OICM(A,B) = \begin{pmatrix} \iota'(A,B,v_{1,1}) & \iota'(A,B,v_{1,2}) & \iota'(A,B,v_{1,3}) \\ \iota'(A,B,v_{2,1}) & \iota'(A,B,v_{2,2}) & \iota'(A,B,v_{2,3}) \\ \iota'(A,B,v_{3,1}) & \iota'(A,B,v_{3,2}) & \iota'(A,B,v_{3,3}) \end{pmatrix}$$

The elements in the matrix are vectors that store the coded interaction information. Figure 4 shows two matrices computed from the examples in Figure 2b and Figure 3a.

$$\begin{pmatrix} \langle 0,0,0\rangle & \langle 2,0,0\rangle & \langle 0,0,0\rangle \\ \langle 0,1,1\rangle & \langle 2,3,1\rangle & \langle 0,1,1\rangle \\ \langle 0,1,1\rangle & \langle 0,1,1\rangle & \langle 0,1,1\rangle \end{pmatrix} \begin{pmatrix} \langle 0,0,0\rangle & \langle 2,0,0\rangle & \langle 0,0,0\rangle \\ \langle 0,1,1\rangle & \langle 2,2,0\rangle & \langle 0,0,0\rangle \\ \langle 0,1,1\rangle & \langle 0,1,1\rangle & \langle 0,1,1\rangle \end{pmatrix}$$

(a) (b)

Figure 4: The OICM for the example in Figure 2b (a), and the OICM for the example in Figure 3a (b).

5. INTERPRETING DIRECTION RELATIONS WITH THE OIC MODEL

The second phase of the OICM model is the *interpretation phase*. It takes an objects interaction cube matrix obtained as the result of the representation phase as input and uses it to generate a set of cardinal directions as output. A new cardinal direction set with 27 basic cardinal directions is first defined. Then the interpretation phase based on an OICM is detailed.

5.1 Cardinal Direction Models for 3D Space

As we have seen, for any two complex volumes, an OIC with $k \times m \times n$ non-overlapping cubic cells can be generated, where $k, m, n \in \{1,2,3\}$. Any cubic cell $c_{(i,j,l)}$ ($1 \le i \le k, 1 \le j \le m, 1 \le l \le n$) can be located by its index triplet (i,j,l). Taking the triplets of two cubic cells, we aim at deriving their cardinal direction. For this, we need a cardinal direction model. We could use a popular model in the 2D space with the nine cardinal directions *north* (N), *northwest* (NW), *west* (W), *southwest* (SW), *south* (S), *southeast* (SE), *east* (E), *northeast* (NE), and *origin* (O) to denote the possible cardinal directions between cubic cells. We call the elements of the set $CD_{2D} = \{N, NW, W, SW, S, SE, E, NE, O\}$ *basic 2D cardinal directions*. However, this model ignores the different heights of spatial objects towards each other. The height is represented by the z-dimension and in addition enables us to distinguish whether an object is located *upper* (u), at the *same level* (s), or *lower* (l) than another object. Taken together, we obtain cardinal directions like SE_u, SE_s and SE_l which have the meaning of *upper southeast*, *same level southeast*, and *lower southeast*, respectively. In general, we obtain a refined version of CD_{2D} into the set $CD_{3D} = \{N_u, N_s, N_l, NW_u, NW_s, NW_l, \ldots, NE_u, NE_s, NE_l, O_u, O_s, O_l\}$ of 27 *jointly exhaustive and pairwise disjoint basic 3D cardinal directions* between two cubic cells in an objects interaction cube. A different underlying set of basic cardinal directions would lead to a different interpretation of cubic cell pairs. Definition 6 gives a specification of the basic cardinal directions in terms of an *interpretation function*.

Definition 6. Let $A, B \in volume$ with $A \neq \varnothing$ and $B \neq \varnothing$, and let $c_{(i_1,j_1,l_1)}$ and $c_{(i_2,j_2,l_2)}$ denote the two cubic cells in the $k \times m \times n$ objects interaction cube $OIC(A,B)$ with $k, m, n \in \{1,2,3\}$, $1 \le i_1, i_2 \le k, 1 \le j_1, j_2 \le m, 1 \le l_1, l_2 \le n$. Then we define the following equivalences:

$$\begin{array}{lll} x_1 :\Leftrightarrow i_1 > i_2 & x_2 :\Leftrightarrow i_1 = i_2 & x_3 :\Leftrightarrow i_1 < i_2 \\ y_1 :\Leftrightarrow j_1 > j_2 & y_2 :\Leftrightarrow j_1 = j_2 & y_3 :\Leftrightarrow j_1 < j_2 \\ z_1 :\Leftrightarrow l_1 > l_2 & z_2 :\Leftrightarrow l_1 = l_2 & z_3 :\Leftrightarrow l_1 < l_2 \end{array}$$

Further, let $\psi((i_1,j_1,l_1),(i_2,j_2,l_2))$ denote the *interpretation function* that takes the location index triplets (i_1,j_1,l_1) and (i_2,j_2,l_2) of two cubic cells as input and yields the cardinal direction from cubic cell $c_{(i_1,j_1,l_1)}$ to $c_{(i_2,j_2,l_2)}$. Then Table 1 provides the definition of the interpretation function.

In Definition 6, a total of 27 basic cardinal directions between two cubic cells in 3D space are defined. For any given

Table 1: Interpretation table for the interpretation function ψ

ψ	condition	ψ	condition	ψ	condition
NW_u	$x_1 \wedge y_1 \wedge z_1$	NW_s	$x_1 \wedge y_1 \wedge z_2$	NW_l	$x_1 \wedge y_1 \wedge z_3$
N_u	$x_1 \wedge y_2 \wedge z_1$	N_s	$x_1 \wedge y_2 \wedge z_2$	N_l	$x_1 \wedge y_2 \wedge z_3$
NE_u	$x_1 \wedge y_3 \wedge z_1$	NE_s	$x_1 \wedge y_3 \wedge z_2$	NE_l	$x_1 \wedge y_3 \wedge z_3$
W_u	$x_2 \wedge y_1 \wedge z_1$	W_s	$x_2 \wedge y_1 \wedge z_2$	W_l	$x_2 \wedge y_1 \wedge z_3$
O_u	$x_2 \wedge y_2 \wedge z_1$	O_s	$x_2 \wedge y_2 \wedge z_2$	O_l	$x_2 \wedge y_2 \wedge z_3$
E_u	$x_2 \wedge y_3 \wedge z_1$	E_s	$x_2 \wedge y_3 \wedge z_2$	E_l	$x_2 \wedge y_3 \wedge z_3$
SW_u	$x_3 \wedge y_1 \wedge z_1$	SW_s	$x_3 \wedge y_1 \wedge z_2$	SW_l	$x_3 \wedge y_1 \wedge z_3$
S_u	$x_3 \wedge y_2 \wedge z_1$	S_s	$x_3 \wedge y_2 \wedge z_2$	S_l	$x_3 \wedge y_2 \wedge z_3$
SE_u	$x_3 \wedge y_3 \wedge z_1$	SE_s	$x_3 \wedge y_3 \wedge z_2$	SE_l	$x_3 \wedge y_3 \wedge z_3$

two cubic cells in the objects interaction cube, their directional relationship d belongs to the set CD_{3D} ($d \in CD_{3D}$). For example, in Figure 3c, the cardinal direction from cubic cell $c_{(2,2,1)}$ to $c_{(3,1,2)}$ is by definition $\psi((2,2,1),(3,1,2)) = SW_l$, which can be expressed as the cubic cell $c_{(3,1,2)}$ is to the *lower southwest* of the cubic cell $c_{(2,2,1)}$.

The objects interaction cube subdivides each volume object into a set of non-overlapping components that are located in different cubic cells. As a result, the cardinal direction between any two components is equivalent to the cardinal direction between the two cubic cells that hold them. This equivalence ensures the correctness of applying the set CD_{3D}, which is for two cubic cells, to symbolize the possible cardinal directions between *object components*.

5.2 Interpreting Direction Relations with OICM

As a first step of the interpretation phase, we define a function *loc* (see Definition 7) that acts on one of the volume objects A or B and their common objects interaction cube matrix and determines all locations of components of each object in the matrix. Let M denote an objects interaction matrix, then we use an index triplet (i, j, l) to represent the location of the lth element in the vector element $M_{i,j}$ and thus the location of an object component in the objects interaction cube matrix M.

Definition 7. Let M be the $k \times m$-objects interaction cube matrix of two volume objects A and B with the vector element length n. Then the function *loc* is defined as:

$$\begin{aligned} loc(A, M) = \{(i,j,l) \mid\ & 1 \le i \le k, 1 \le j \le m, 1 \le l \le n \\ & M_{i,j}[l] = 1 \ \vee \ M_{i,j}[l] = 3\} \\ loc(B, M) = \{(i,j,l) \mid\ & 1 \le i \le k, 1 \le j \le m, 1 \le l \le n \\ & M_{i,j}[l] = 2 \ \vee \ M_{i,j}[l] = 3\} \end{aligned}$$

Once the location of a component is computed, we can apply the interpretation function ψ, which takes the location index triplets of two components and produces the cardinal direction between them.

Finally, we specify the *cardinal direction function* named *dir* which determines the *composite cardinal direction* for two volume objects A and B. This function has the signature $dir : volume \times volume \to 2^{CD_{3D}}$ and yields a set of basic cardinal directions as its result. We now specify the cardinal direction function *dir* in Definition 8.

Definition 8. Let $A, B \in volume$. Then the *cardinal direction function dir* is defined as

$$\begin{aligned} dir(A,B) = \{\psi((i,j,l),(i',j',l')) \mid\ & \\ & (i,j,l) \in loc(A, OICM(A,B)), \\ & (i',j',l') \in loc(B, OICM(A,B))\}. \end{aligned}$$

Function *dir* yields the union of the basic cardinal directions between all component pairs of both objects. It determines all cardinal directions that exist between two volume objects. To illustrate the interpretation phase, we use our example in Figure 4b. From the objects interaction cube matrix $OICM(A,B)$, we can obtain $loc(A, OICM(A,B))$ and $loc(B, OICM(A,B))$ as follows:

$$\begin{aligned} loc(A, OICM(A,B)) = \{&(2,1,2),(2,1,3),(3,1,2), \\ &(3,1,3),(3,2,2),(3,2,3),(3,3,2),(3,3,3)\} \\ loc(B, OICM(A,B)) = \{&(1,2,1),(2,2,1),(2,2,2)\} \end{aligned}$$

The cardinal directions between the two volume objects A and B can then be derived with the cardinal direction function *dir*.

$$\begin{aligned} dir(A,B) = \{&\psi((2,1,2),(1,2,1)), \psi((2,1,3),(1,2,1)), \\ &..., \psi((3,3,3),(2,2,2))\} \\ = \{&N_s, N_u, E_s, E_u, NE_s, NE_u, NW_s, NW_u\} \end{aligned}$$

Similarly, we obtain the inverse cardinal direction as:

$$\begin{aligned} dir(B,A) = \{&\psi((1,2,1),(2,1,2)), \psi((2,2,1),(2,1,2)), \\ &..., \psi((2,2,1),(3,3,3))\} \\ = \{&S_s, S_l, W_s, W_l, SW_s, SW_l, SE_s, SE_l\} \end{aligned}$$

Finally we can say regarding Figure 4b that "Object B is partly *same level north*, partly *upper north*, partly *same level east*, partly *upper east*, partly *same level northeast*, partly *upper northeast*, partly *same level northwest*, and partly *upper northwest* of object A" and that "Object A is partly *same level south*, partly *lower south*, partly *same level west*, partly *lower west*, partly *same level southwest*, partly *lower southwest*, partly *same level southeast*, and partly *lower southeast* of object B", which is consistent.

6. CONCLUSIONS AND FUTURE WORK

In this paper, we have laid the foundation of a new concept, called the Objects Interaction Cube Matrix (OICM), for determining cardinal directions between volume objects in the 3D space. Our model overcomes a number of drawbacks of available models and takes into account the shapes of both volume objects and ensures the property of converseness of cardinal directions.

In the future, we plan to explore the impact that the percentage to which a 3D spatial object intersects a cubic cell of the objects interaction cube could have on our OICM model. Such a metric refinement could enable us to provide a more punctuated result of cardinal directions that would allow us to specify terms like "strictly north", "predominantly southwest", or "hardly south". Other research issues refer to an efficient implementation of our approach as well as the design of spatial reasoning techniques based on the OICM model.

7. REFERENCES

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