Root Finding Using Bézier Clipping

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Abstract

Finding the solutions to polynomial equations is a ubiquitous problem with applications to curve and surface intersection, collision detection, geometric modeling, and ray tracing, to name just a few. To solve a polynomial, we desire methods which converge quickly and unconditionally, have a fast running time, and find *all* the roots of the polynomial. Unfortunately, many approaches, such as Newton's method, fail at least one of these criteria.

The method of Bézier Clipping solves these issues by exploiting the geometric properties of the Bernstein-Bézier basis. Given only the coefficients of the polynomial, we find an interval in which all the roots of interest are guaranteed to exist and then repeatedly "clip" away sections of the domain which are guaranteed to have no solutions. This method is quadratic in both running time and, in the case of a simple root, convergence rate.