Small Worlds in Wireless Networks

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***Abstract*—In this study, the concept of small worlds is investigated in the context of wireless networks. Wireless networks are *spatial* graphs that tend to be much more clustered than random networks and have much higher path length characteristics. We observe that by adding a few short cut links, path length of wireless networks is reduced drastically. More interestingly, such short cut links need *not* be random but may be confined to a limited number of hops; a fraction of the network diameter. This facilitates the design of practical distributed algorithms, based on *contacts*, to improve performance of resource discovery in wireless networks.**

***Index Terms*—Small worlds, wireless networks.**

I. Introduction

THE concept of small worlds was studied in [1], during which experiments of mail delivery using acquaintances resulted in an average of "six degrees of separation." Recent research in [2] and [3] has shown that, in *relational* graphs, adding a few number of random links to regular graphs results in graphs with low average path length and high clustering. Such graphs are called small world graphs.

Emerging multi-hop wireless networks, such as ad hoc and sensor networks, do not belong to the category of *relational* graphs. Rather, they belong to the category of *spatial* graphs, where the links between nodes depend on the radio range, which in turn is a function of the distance between the nodes. The applicability of small worlds to spatial graphs has not been established by [2], [3].

In this paper, we attempt to develop a better understanding of the small world concept in the context of wireless networks. Specifically, what are the path length and clustering characteristics of wireless networks? Can we create networks that have reduced path length by adding a small number of short cuts? Do these short cuts have to be random? What do these short cuts represent given the limited radio connectivity in wireless networks? It may represent physical links (of larger distance but at lower bit rate) or logical links that translate into multiple physical hops. Finally, given reasonable answers to the above questions, can we develop network architectures, for resource discovery, that incur low degrees of separation between nodes in wireless networks? Under the assumptions of our study, results in this paper suggest that we can actually reduce the path length of wireless networks drastically by adding a few random links (resembling a small world). Furthermore, these random links need not be totally random, but in fact may be confined to a small fraction of the network diameter, thus reducing the overhead of creating such network. Based on these results a new architecture is introduced that attempts to create a small world in large-scale wireless networks. The architecture is based on defining *contacts* for network nodes. The contacts are to be used during resource discovery without flooding.

The rest of the paper is outlined as follows. Section II provides background on small worlds. Section III presents our experiments and results. Section IV provides a resource discovery protocol in wireless networks. Section V concludes.

II. Small Worlds

The *small world* phenomenon comes from the observation that individuals are often linked by a short chain of acquaintances. Milgram [1] conducted a series of mail delivery experiments and found that an average of 'six degrees of separation' exists between senders and receivers. Small worlds were also observed in the context of the Internet and the world wide web [7]–[9]. To understand network structures that exhibit low degrees of separation, Watts & Strogatz [2], [3] conducted a set of rewiring experiments on graphs, and observed that by rewiring a few random links in regular graphs, the average path length was reduced drastically (approaching that of random graphs), while the clustering[1] remains almost constant (similar to that of regular graphs). This class of graphs was termed *small world graphs*, and it emphasizes the importance of random links acting as *short cuts* that contract the average path length of the graph. The experiments were conducted for relational graphs, in which links are not restricted by the distance between nodes. It was noted that for spatial graphs, in which links are a function of distance, the small world phenomenon does not exist (i.e., path length and clustering curves almost match)[2].

Multi-hop wireless networks—including ad hoc and sensor networks—are spatial graphs, where the links are determined by the radio connectivity, which is a function of distance, among other factors. Hence, we expect that such networks, by their own nature, do not lend themselves to small worlds. We also expect high clustering in wireless network due to the locality of the links, since many of a node's neighbors are also neighbors of each other. Also, due to this locality we expect the average path length for such networks to be high as compared to random networks.

In this work, we conduct further experiments on spatial graphs in the context of multi-hop wireless networks, and investigate the applicability of the small world concept to these

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[1]C*lustering coefficient* is the fraction of nodes' neighbors that are also neighbors of each other. Clustering relates to the structure of the network.

[2][2] assumed 1-dimentional spatial graphs, in which k links, on average, originate from every node. Links for all nodes were chosen within a distance d. Path length and clustering exhibited similar dynamics as d increased.

TABLE I
SUBSET OF TOPOLOGIES USED IN THIS STUDY

Topology	Range (m)	Links	Topology	Range(m)	Links
55-random	55	4785	35-grid	35	1936
65-random	65	6850	50-grid	50	3811
65-normal	65	10790	75-grid	75	9310
55-skewed	55	10051	100-grid	100	12872

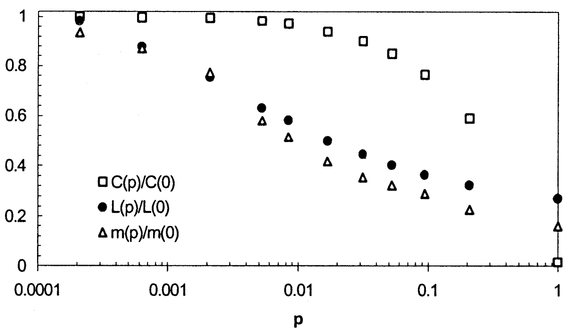

Fig. 1. Reduction of path length and clustering versus probability of relinking.

TABLE II
CLUSTERING, PATH LENGTH, AND MAX LENGTH FOR THE ORIGINAL NETWORKS

	Rand graph	55-rand	Normal	Skewed	35-grid	50-grid
C(0)	0.009	0.58	0.568	0.567	0	0.45
L(0)	3.3	12.3	6.9	8.92	21.1	14.8
m(0)	5	31	21	32	62	31

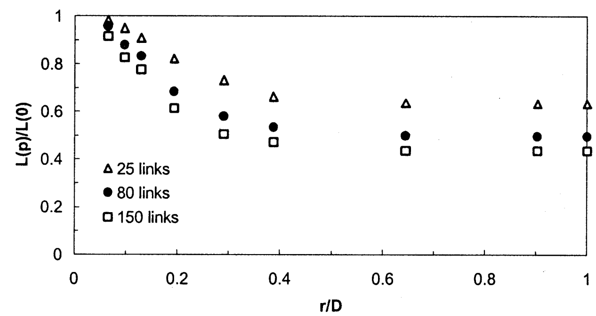

Fig. 2. Path length reduction versus max contact distance r.

networks. Our study takes a practical perspective in which we hope to utilize small worlds in designing efficient protocols for ad hoc and sensor networks. In particular, we briefly propose a novel contact-based architecture for resource discovery in large-scale ad hoc and sensor network. In such architecture, our goal is to reduce the number of queries during the search for a target node or resource.

III. SMALL WORLDS SIMULATION AND ANALYSIS

We start our experiments by investigating several layouts of wireless networks. Without loss of generality, we choose a setting of 1000 nodes over a 1 km $\times$ 1 km area. We investigate various node distributions, including random, normal, skewed, and grid. Several values of radio ranges were chosen to provide different number of links and average node degrees. Table I shows a subset of the topologies studied.

Link rewiring and link addition experiments were conducted on the above networks. For link rewiring a node is chosen at random, then the link to one of its neighbors is removed and relinked to a random node. For link addition, two nodes are chosen at random and a link is added between them. This was performed for various numbers of links (or probability of rewiring/addition). For every probability of rewiring or link addition p, the average path length L, the maximum path length m, and the clustering coefficient C are measured. For the original case, where $p = 0$ (without rewiring), these values are denoted as $L(0)$, $m(0)$, and $C(0)$, respectively. For other values of p we get $L(p)$, $m(p)$, and $C(p)$, respectively. We plot the ratios $L(p)/L(0)$, $m(p)/m(0)$, and $C(p)/C(0)$ on a semi-log plot. These ratios represent reduction in length or clustering with increased probability of rewiring or link addition. Results are shown in Fig. 1.

We note several observations on these results. First, values for clustering and path length of the original graphs ($p = 0$) are quite high as compared to those of random graphs. These values are shown in Table II. One exception is for 35-grid, where $C(0) = 0$. Aside from this exception, wireless networks, in general, tend to be highly clustered due to locality of the links, which increases the probability that a node's neighbors are also neighbors of each other. Second, from Fig. 1, we observe a very consistent trend among all the experiments and across all topologies. There is a clear distinction between the reaction of the path length and clustering to rewiring or link addition. The path length reduction occurs quite drastically for 0.2% to 20% of rewiring. Further rewiring does not contribute much to reducing the path length. For example, rewiring or addition of 0.2% of the links results in 25% reduction in L. On the other hand, to achieve 25% reduction in C we need around 9% rewiring, i.e., two orders of magnitude difference for the needed rewiring. This suggests that *by rewiring a very small number of random links the path length is drastically reduced without affecting the structure of the network.* These results are *consistent* with the small world graph phenomenon.

The previous analysis shows that, it is possible to achieve significant reduction in degrees of separation in wireless networks by adding a few (0.2%–2%) *random* links (or *short cuts*). What has not been clear this far is how random should the *random* links be? From a practical point of view, choosing random *short cuts* for a node in wireless networks may result in unpredictable overhead. So in our next set of experiments we investigate the effect of limiting the distance of the random links (such that the overhead is more predictable).

For this set of experiments we perform link addition. We choose three values for the added links: 25, 80, and 150 links (these values achieved 40%–60% reduction in L previously). We limit the maximum distance, 'r' (in hops) from which short cuts maybe chosen. A short cut is chosen randomly from a distance d, where $2 < d < r$. We vary r from 2 to the network diameter, D. The results (shown in Fig. 2) provide clear and consistent trends. $L(p)/L(0)$ decreases with the increase of r up to a certain fraction of D, then it saturates. We measure the fraction r/D after which further increase leads to less than 3% reduction. The average of such fraction for the topologies studied was $\sim 45\%$ (with min 35% and max 50%). Since the short cut is chosen from $[2, r]$ hops, we get the expected short cut distance,

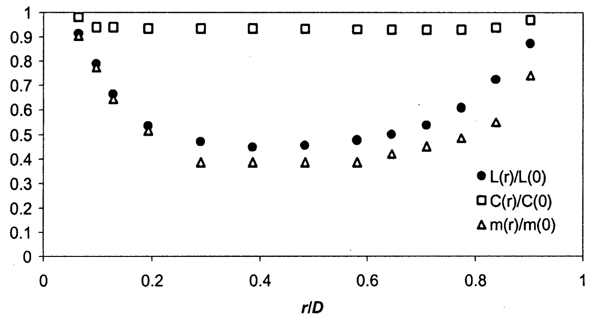

Fig. 3. Contacts at r hops. Max decrease in path length is at $r/D \sim 25\%–40\%$.

$r_{\mathrm{av}} \sim 2 + (r-2)/2$ (i.e., $0.2D$ to $0.35D$) in order to achieve the most effective reduction in L. Note that the probability distribution is weighted by the number of nodes at each hop, i.e., r_{av} is slightly higher than shown above.

The next set of experiments involved choosing the short cuts from exactly r hops away and investigating the trends in path length and clustering. Results are shown in Fig. 3. Unlike our previous experiment, the curve does *not* saturate after a certain distance. What is observed is that at a certain distance $(r/D \sim 25\%–40\%)$ the maximum reduction in path length is achieved, after which the path length *increases*. This result has significant practical impact. This clearly indicates that *by limiting the short cuts distance to a fraction of the network diameter* $(\sim D/4)$ *we can achieve maximum reduction in path length* or degrees of separation. This result motivates our contact-based architecture in Section IV.

Discussion: Previous work in [4] shows that, in a 2-D grid, the minimum number of forwarding steps (i.e., degrees of separation) is achieved when the short cuts are chosen with probability inversely proportional to their distance. For example, for two nodes A and B separated by distance $d(A,B)$ hops, the probability of link A-B being a short cut is $d(A,B)^{-2}$. However, to obtain this probability in a wireless network each node needs to know locations of all other nodes in the network (or the hop-node distribution with respect to each node). In many cases, this may not be achievable.

However, if this probability is per hop, a node may identify itself as d hops away from another node (using TTL or similar), and can compute this probability independently with only a rough estimate of the network diameter, D, in a decentralized way. So, we study the distance (in hops) that achieves the best performance. Note that d^{-2} as used in [4] is per node (not per hop). We want to obtain the expected number of hops for such probability distribution. Given a 2-D grid and given a general node near the center of the grid, the number of nodes that can be reached at exactly d hops away is $4d$. Taking the probability of each node to be $1/d^2$ and that $4d$ nodes are at hop d we get the hop probability distribution as $1/d$ (i.e., *the probability of choosing the short cut at hop* h *is* $1/h$). To get the probability distribution we normalize by $\sum 1/d$, for all hops up to the network diameter, D. Hence, the expected hop value is $D/(\sum 1/d) = D/H(D)$, where $\sum 1/d$ is the harmonic series, $H(D)$. $H(D) = \ln(D) + O(1)$. For 30–70 hops this ratio is $\sim 0.2D$ to $0.25D$, consistent with our results above. These results were also validated through simulations.

IV. Small Worlds for Resource Discovery

Inspired by the above results we propose the concept of *contacts* to improve the efficiency of *search* and *query* techniques in large-scale wireless networks. Contacts act as short cuts to transform the wireless network into a small world. Instead of flooding, a node queries its contacts for the resource sought. Above, we have obtained an estimate of the number and distance of contacts to achieve significant path length reduction. So, how are contacts established? They may either be physical or logical. Physical contacts may be achieved by increasing the radio range (using higher transmission power or lower bit rates). In common-channel networks, however, this may have negative effects on utilization of the spectrum. Frequency division, CDMA or ultra wide band may be used to establish these contacts. Contacts may also be logical links that translate into several physical hops. In this case the aim is to reduce the logical path length and in turn reduce the number of queries during resource discovery. This concept has been studied in [5], [6]. Detailing these protocols is out of scope of this paper.

V. Conclusions and Future Work

We have established a relationship between small world graphs and wireless networks. Our findings indicate that by adding a few short cuts, with only a small fraction (25%–40%) of the network diameter, the degrees of separation may be reduced drastically. Based on this finding we propose a contact-based architecture for resource discovery in ad hoc and sensor networks, subject to further research.

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