Algorithm for Nodes Localization in Wireless Ad-Hoc Networks Based on Cost Function

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***Abstract*— A new distributed algorithm is described in this paper for localizing sensor network nodes in Wireless Sensor Networks (WSN). The algorithm shares a common three-phase structure: (1) determine node-beacon distances, (2) build a cost function for every sensor node, and (3) compute the coordinate of the node. The coordinates of the sensor nodes are computed using either the Davidson-Fletcher-Powell Quasi-Newton algorithm or the Direct Method. The algorithm is based on the capacity of the nodes to calculate the Time-of-Arrival (ToA) and Angle-of-Arrival (AoA) with their neighbors. The accuracy of the measurements rely on the use of Ultra Wide Band (UWB) technology. It has been shown that the properties of the UWB signal allow more accurate ToA measurements. In order to simulate the range measurement noise in ToA and AoA measurements, two independent random variables are introduced with two different variances $(\sigma_{ToA}, \sigma_{AoA})$ for the ToA and AoA measurements.**

An extensive set of simulations is performed to assess the accuracy and performance of the algorithm. Also, we demonstrate its ability to localize the nodes in a large WSN.

I. Introduction

The main features of new ad-hoc networks are large number of unattended nodes with varying capabilities, and lack of impracticality of deploying supporting infrastructure. Recent advances in microsensor technology allow the mass production of cheap and low power consumption sensor nodes. Many applications with Wireless Sensor Networks (WSN) have been proposed such as habitat monitoring, health caring, battle field surveillance, environment observation and forecasting [1]. Most of those applications require the knowledge on the position of every nodes in the WSN.

In the past 5 years, a large number of algorithms have shown up on the market to localize sensor nodes in WSN. A pioneer work is the Ad Hoc Positioning System (APS) algorithm proposed by D. Niculescu [2]. Positioning [1] algorithms are classified in either centralized or distributed. Centralized means that the calculation is performed by a server whereas in distributed algorithms all the nodes are able to compute their own position.

This paper deals with positioning issues proposing an algorithm GPS-free using few beacons to initialize the positioning mechanism. The proposed algorithm is fully distributed. It follows the idea given in [2] and more recently [3], where all nodes compte the number of hops away from the beacons and then transform it in meters using either a rough approximation of the one hop-distance or other techniques. Our approach projects the hop-distance in Euclidean distance using the ability of nodes to calculate the Time-of-Arrival (ToA) and Angle-of-Arrival (AoA) with their neighbors within their transmission range. Hence a node computes its Euclidean distance to at least three beacons in 2D, either the Davidson-Fletcher-Powell (DFP) algorithm or the Direct Method (DM) is applied to find the unknown coordinates.

From the overall view of the proposed algorithm, one may underline the expensiveness in term of number of messages to be exchanged before nodes' position is established. But our algorithm shows a robustness to ToA and AoA measurements error, and indeed the information update between nodes may be used to refine the position of the nodes.

The rest of the paper is organized as follow: in section II reviews some popular methods used in positioning. In the section III, the method to localize nodes is explained. Then DFP and DM will be sum up in section IV. The results are given in section V, and section VI ends this article.

II. Background and Previous Work

The WSN is a collection of ad hoc deployed nodes such that any node can communicate with their neighbors within its transmission radius. Techniques usually used to estimate the one-hop distance are Time-of-Arrival (ToA), Time-Difference-Of-Arrival (TDOA), Received Signal Strength (RSSI), or Angle-of-Arrival (AoA). Recently, some studies investigate the gain of accuracy using two technologies like ToA and AoA for example [4].

Indeed, since that FCC allows the use of Ultra Wide Band (UWB) technology in civil communication systems in February 2002, the accuracy of the measurements is improved by the fine time delay resolution of the UWB signal. This technology was previously applied in RADAR systems at the beginning of the 50's [5]. Moreover, accurate AoA measurements is more difficult to obtain than using ToA for example. One way is to embed a compass in every node, which is not a suitable solution for tiny low cost sensors. In addition to the orientation of the compasses copes with erratical behavior in the vicinity

[1]Here, the authors do not take into account the special case of cluster-based algorithms

of large metal objects or electrical fields. The general way considers in WSN is only beacon nodes embedded a compass and the angles between two neighbors nodes is recursively calculated using basic triangle rules as done in [6]. Due to extensive literature about how to find the AoA between two neighbors nodes ([6], [7], [8], [9], [3]), this study does not take into account this problem. We consider that two neighbors nodes are able to find the AoA between them with an error.

In the last past years, various approaches have been proposed for positioning in WSN. In [10], a centralized scheme is proposed which collects the entire topology in a server and then solves a large system to minimize the positioning errors. A location system based on an uniform grid of beacons is proposed in [11] while ultrasound signals are used in [12]to measure the distances of known beacons. Localization in ad hoc networks based on the known position of few nodes in the network are proposed in [13], [12]. In [6] the authors build a distributed mechanism for GPS free positioning in mobile ad hoc networks. Beside, nodes can also be considered as terminal nodes in wireless ad-hoc networks where the hardware resources are more important and sectorial antennas will be rather employed in this case instead of in wireless ad-hoc sensor networks [14].

The present algorithm derives a cost function based on the network topology. This point of view of node localization has been successfully applied by [15] in UWB position estimation based on ToA measurements with pre-existing infrastructure. Hence, the cost function is built, either a gradient-based algorithm like the DFP quasi-Newton algorithm will be used to estimate the position of the node, or DM algorithm can be applied.

III. Calculating Relative Positions

Primarily, the current algorithm works through two dimensional triangulation, using three or more beacons available in the network. In the first phase, the beacon broadcast their coordinates. Then, all nodes will set up the shortest path with all the beacons. The shortest path is defined by the Moore-Dijkstra algorithm. Then, all nodes belonging to this path will calculate the ToA and AoA with the next node in the path. As soon as a node received the data to calculate the Euclidean distances between it and at least 3 beacons, the DFP or DM are used to compute the coordinates of the node.

A. The Shortest path

After the initial phase where all the beacons broadcast their coordinates, nodes have to set up a ”shortest” path with all the beacons. Many well-known algorithms are developed to solve this issue. Moore-Dijkstra, Bellman-Ford or Floyd-Warshall algorithm to cite only the famous one, can be used. Here, we use the first cited above. At the end of this phase, the node will have at least the coordinates of three beacon nodes and the path to reach them. The figure 1gives an overview of the wireless sensor network. In this figure, node N1 set up a path between the beacon B1, B2 and B3. Then, it stores in its data

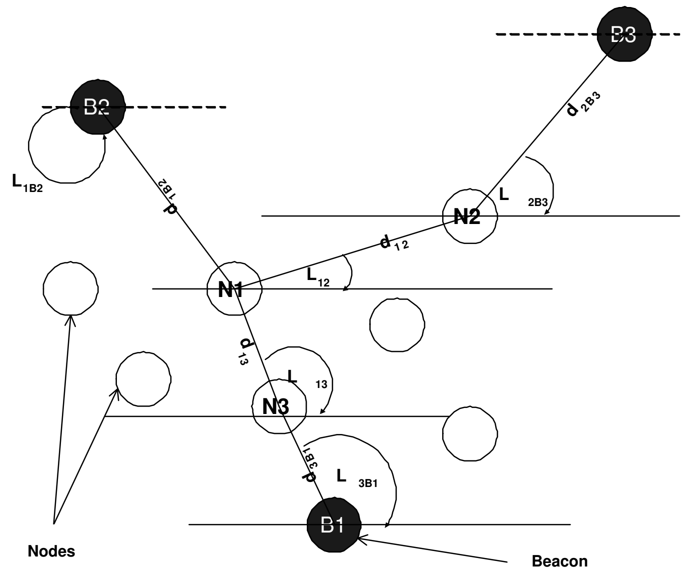

Fig. 1. Scheme of Wireless Sensor Network

base (1).

$$\begin{aligned} N_1 \rightarrow B_1 \quad &: \quad \Phi_1 = \{(N_1, N_3, B_1), (\hat{\alpha_{13}}, \hat{\alpha_{3B_1}}), (\hat{d_{13}}, \hat{d_{3B_1}})\} \\ N_1 \rightarrow B_2 \quad &: \quad \Phi_2 = \{(N_1, B_2), (\hat{\alpha_{1B_2}}), (\hat{d_{1B_2}})\} \\ N_1 \rightarrow B_3 \quad &: \quad \Phi_3 = \{(N_1, N_2, B_3), (\hat{\alpha_{12}}, \hat{\alpha_{2B_3}}), \\ &\qquad (\hat{d_{12}}, \hat{d_{2B_3}})\} \end{aligned} \tag{1}$$

$\hat{\alpha}_{13}$ and $\hat{d}_{13}$ are respectively the estimated angle and distance between node 1 and node 3.

B. Position Estimation

Once a node is able to estimate its distance to each of the beacons, it can estimate its position. Each beacon monitored by a node yields a cost function of the form (2). Where $\hat{\alpha}_i$ and $\hat{d}_i$ are respectively the estimated angle and distance between two nodes belonging to the path, N_B is the number of beacon available in the network, and $N_{hop}(k)$ is the number of hops between the node and a given beacon.

Finally, the coordinates of the nodes are estimated by minimizing (2), the cost function. Two algorithms are used to decrease it: DFP and DM. For the DFP algorithm, the measurement update between two nodes in the path may be used to refine the initial position of the node iteratively. In practice, it may exist more than three beacons in the network. One way is to follow the study done in [15] which underlines the more beacons are used to position a node, the better the accuracy is. An other way is to weight (2) depending on how far in number of hops the beacon is from the unknown node. In this study, all the beacon available in the WSN are used.

IV. Overview of Positioning Algorithms

DFP and DM have been recently considered in positioning systems for the emerging UWB technology [16], [15]. In particular, the DFP algorithm was recently proposed and applied

$$\mathbf{E}(x,y) = \sum_{k=1}^{N_B}\left(\sqrt{\left(x_k-(x_k+\sum_{i=1}^{N_{hop}(k)}\hat{d}_i\cdot\cos(\hat{\alpha_i}))\right)^2+\left(y_k-(y_k+\sum_{i=1}^{N_{hop}(k)}\hat{d}_i\cdot\sin(\hat{\alpha_i}))\right)^2}-\sqrt{\left(x_k-x\right)^2+\left(y_k-y\right)^2}\right)^2 \quad (2)$$

to the localization of UWB sensors. Given that this algorithm is still relatively unknown, its principles are quickly described below, together with the DM. A more comprehensive study is found in [15], [17].

A. The DFP algorithm

The DFP algorithm belongs to the family of quasi-Newton methods, which are based on the gradual construction of the matrix of curvature information of a certain objective function (Hessian matrix), from the gradient calculated during the iterations of the algorithm. As previously said, DFP algorithm decreases the objective function in (2). It can be rewritten in

$$f(p) = \sum_{i=1}^{N_B}[\sqrt{(x_e-x_i)^2+(y_e-y_i)^2}-\hat{d}_{ei}]^2,$$

Where $\hat{d}_{ei}$ is the estimated distance between the node e and the i-th beacon. The *position vector* that minimizes the above objective function is defined as

$$\mathbf{p} = [x,y]^{\mathrm{T}}. \quad (3)$$

Starting from initial parameters $\hat{\mathbf{p}}_0 = [x_0, y_0]^{\mathrm{T}}$ chosen as the limit of the monitored area, the solution of this optimization problem is found by iteratively computing

$$\hat{\mathbf{p}}_{k+1} = \hat{\mathbf{p}}_k - \gamma\mathbf{B}_k\cdot\mathbf{g}_k, \quad (4)$$

where γ is the iteration step size, $\hat{\mathbf{p}}_k$ is the estimate of the position vector, and $\mathbf{g_k}$ and $\mathbf{B}_k$ are, respectively, the gradient and the inverse Hessian of $f(p)$ at the k-th iteration.

B. The Direct Method

Actually, the DM does not decrease directly the cost function as written in (2). Instead of, it solves the set of equations written in (5).

$$\sqrt{(x_e-x_k)^2+(y_e-y_k)^2}-\hat{d}_{ek} = 0 \qquad \forall k\in[1,2,\ldots,N_B] \quad (5)$$

Where $\hat{\alpha}_i$ and $\hat{d}_i$ are respectively the estimated angle and the estimated distance between two nodes belonging to the path, N_B is the number of beacon available in the network, and $N_{hop}(k)$ is the number of hops between the node and a given beacon. (x_e, y_e) are the coordinates of the node to estimate its position. Squaring and rearranging the equation system in(5) yields to

$$\left(\hat{d}_{e1}\right)^2-\left(\hat{d}_{ej}\right)^2 = \beta_{1j}-2(x_ex_1+y_ey_1)-2(x_ex_j+y_ey_j) \qquad \forall j\in[2,\ldots,N_B] \quad (6)$$

where

$$\beta_{1j} = (x_1^2+y_1^2)-(x_j^2+y_j^2) \quad (7)$$

By substitution in (IV-B), it yields

$$a_1x_e+b_1y_e = g1 \quad (8)$$

where

$$\begin{aligned} a_1 &= -2(x_j-x_1)\\ b_1 &= -2(y_j-y_1)\\ g_1 &= -\beta_{1j}+\left(\hat{d}_{e1}\right)^2-\left(\hat{d}_{ej}\right)^2 \end{aligned} \quad (9)$$

With 3 beacons, the solution is:

$$\begin{pmatrix} x_e\\ y_e\end{pmatrix} = \begin{pmatrix} a_1 & b_1\\ a_2 & b_2\end{pmatrix}^{-1}\cdot\begin{pmatrix} g_1\\ g_2\end{pmatrix}$$

The resolution of the system in 3D can be found in [15].

V. Simulation Results

The monitored area has a dimension of $100(w)\times100(l)$ m. 100 nodes are randomly positioned in the area together with a small number of beacon nodes. This number can take the value $\{5, 15 \text{ or } 20\}$ depending on the simulation scenario. The performance evaluation is performed by assuming that the ToA and AoA measurements error are two independent random variables Gaussian distributed with zero-mean and variance ${\sigma_{ToA}}^2$ and ${\sigma_{AoA}}^2$. At each test point (σ), 30 simulations are conducted with new random topology of the WSN. Then, the performance is averaged on the 100×30 simulations for the DM. In the case of the DFP algorithm, 50 iterations are used to update the estimated coordinates. The number of simulations is then equal to $100\times30\times50$. Note that the transmission radius of any node in this network is equal to 30 m and it is kept constant. Figures 2 and 3 show the corresponding root mean square error of the coordinates estimations using either the DM or DFP algorithm.

In general, the RMS error values for both algorithm are under two meters. The DFP algorithm shows more robustness to the AoA and ToA error measurements with a RMSE error value under 1.3m. We notice on the figures the low slope of the curve among the different value of AoA measurements error. It means that the algorithm, to localize the nodes, is more sensitive to the ToA measurements error.

Figures 4 and 5 show the RMS error for various σ_{ToA} and at a given σ_{AoA}. In this case, the slope of curve is high. The RMSE values become higher when the σ_{ToA} increases. This remark confirms the fact that the algorithm is more sensitive

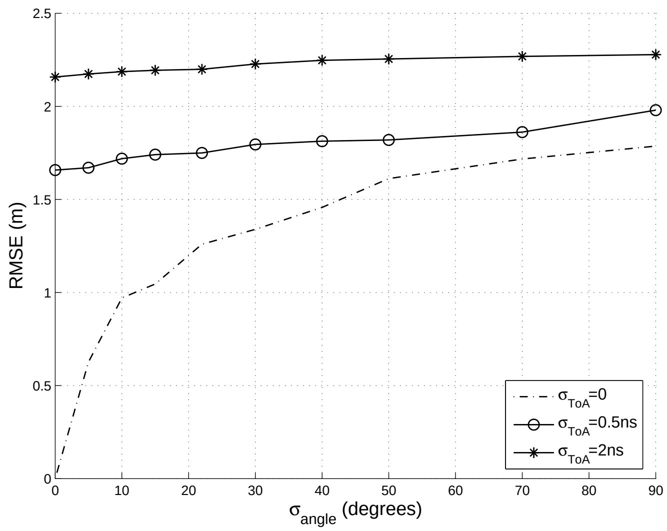

Fig. 2. Root Mean Squared error of position estimation with Direct Method. Only 5 beacons are chosen to position all the sensor nodes in the WSN. Three different values of the variance on the ToA error are chosen ($\sigma_{ToA} = \{0, 0.6, 2\}$ns).

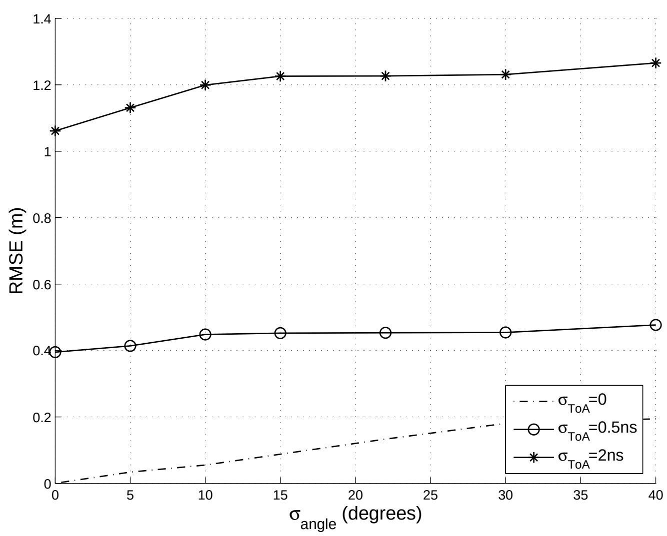

Fig. 3. Root Mean Squared error of position estimation with DFP algorithm. Only 5 beacons are chosen to position all the sensor nodes in the WSN. Three different values of the variance on the ToA error are chosen ($\sigma_{ToA} = \{0, 0.6, 2\}$ns).

to ToA measurements error than AoA ones. In addition to DFP algorithm performs slightly better than DM on figures 4 and 5 as we previously notice on the two first ones. Figures 6 and 7 show the performance of the present algorithm when increasing the number of beacon. In general, It has been shown in some studies ([15]) using DFP and DM algorithm that the higher the number of beacons is, the less the RMSE value is in centralized network. Figure 6 confirms this result. But with the DM, this result is no longer valid in the WSN scenario. As seen in the figure 7, the RMSE value with 15 and 20 beacons are higher than using only 5 beacons. This result can be explained taking account two parameters. First, the DM is not an iterative algorithm and then does not use the capacity

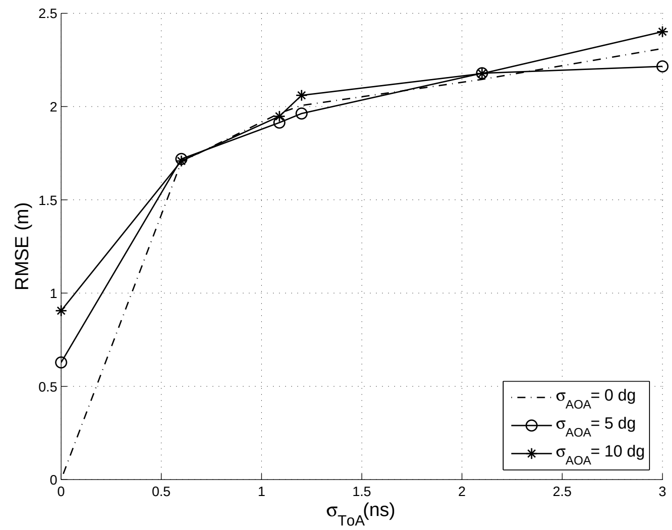

Fig. 4. Root Mean Squared error of position estimation with Direct Method. Only 5 beacons are chosen to position all the sensor nodes in the WSN. Three different values of the variance on the AoA error are chosen ($\sigma_{AoA} = \{0, 5, 10\}$degrees).

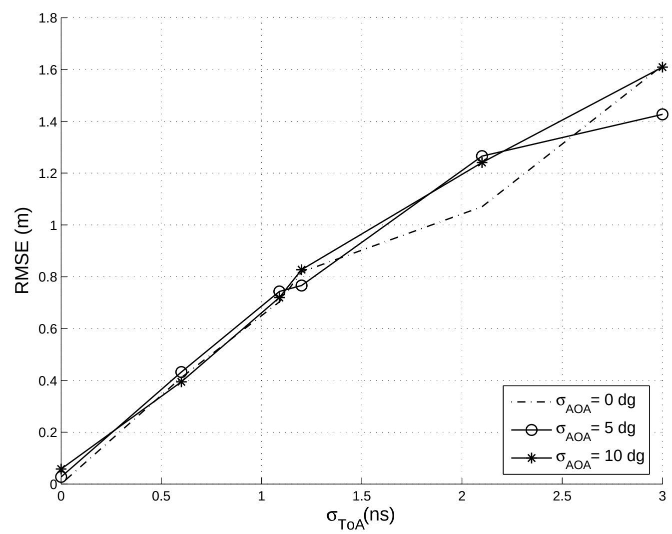

Fig. 5. Root Mean Squared error of position estimation with Direct Method. Only 5 beacons are chosen to position all the sensor nodes in the WSN. Three different values of the variance on the AoA error are chosen ($\sigma_{AoA} = \{0, 5, 10\}$ degrees).

of the nodes of calculating few times the ToA and AoA values with their neighbors. Secondly, in [15] and [18], they underline that the DM is easily diverging for high values of the ToA error. Or in this scenario, two errors are introduced: on the ToA and AoA measurements. Indeed, more beacons are taken into account to position the nodes. Then the DM has an higher probability to diverge in the wireless sensor networks scenario. The present study is based on only 30 simulations. The last result can be improved with more simulations.

VI. Conclusions

In this paper, a new algorithm is presented to position sensor nodes in WSN. First, a cost function is built based

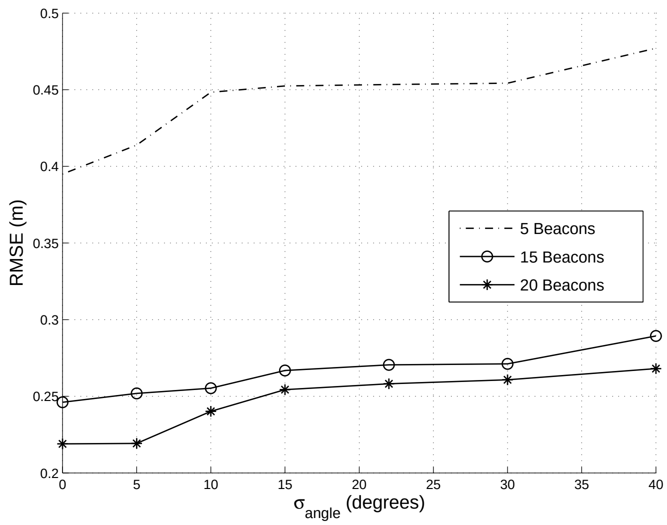

Fig. 6. Root Mean Squared error of position estimation with DFP using 5, 15 and 20 beacons. The σ_{ToA} is equal to $0,5$ ns.

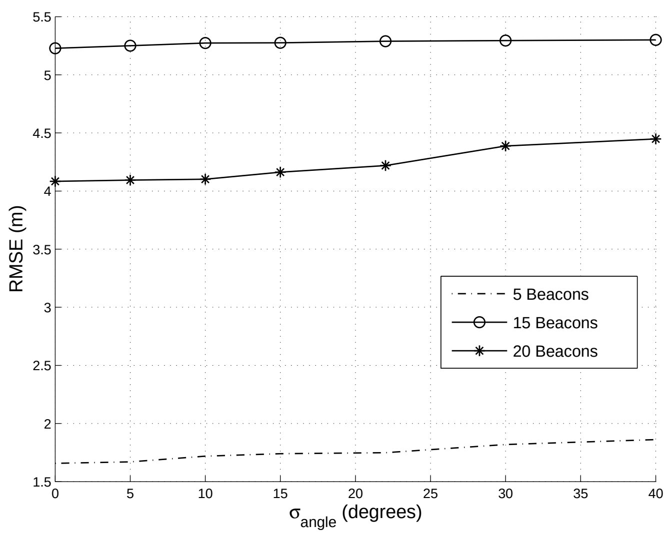

Fig. 7. Root Mean Squared error of position estimation with DM using 5, 15 and 20 beacons. The σ_{ToA} is equal to $0,5$ ns.

on the distance between the node and beacons available in the network. A couple of practical position estimation methods (DM and DFP) is used to decrease the cost function and find the coordinates of the node. The good position accuracy shown with the DFP algorithm is due to the ability of the nodes to measure few times the ToA and AoA. This paper does not consider the way to calculate accurately the AoA between two neighbor nodes. It remains for a future work.

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