1. [Markov model] (20)
Consider the following Web site for e-commerce with 4 states: S for the start state, B for a browsing state, C for a check-out state, and E for an end-state.

1) From the start state S the user goes to state B with probability 0.9 or to state E with probability 0.1.
2) Given that a user is in state B, the user next goes to state C with probability 0.1, to state E with probability 0.3, or has a 0.6 probability of staying in state B.
3) Given that a user is in state C, the user next goes to state B with probability 0.2, to state E with probability 0.7, or has a 0.1 probability of staying in state C.
4) Once a user reaches an end state, the user stays there forever.

Define a Markov model using a fully labeled diagram with the probability matrix (where row 1 = S, row 2 = B, row 3 = C, row 4 = E).
2. **[Definitions + Explanations]** (30)
   
a. What are three of the most fundamental concepts in system modeling? Identify and define them. (6)

   \[\text{State: describes the system for an interval of time} \]
   \[\text{Event: a point in time that designates a change in state} \]
   \[\text{Time: denoted by either an integer or a real number} \]

b. What is an autonomous system? Specify it in terms of the model components. (5)

   \[\text{System without an input} \]

c. Under what circumstances is an event scheduling method more efficient than a time slicing method? (6)

   \[\text{The event scheduling method is efficient where the values of inputs and outputs change irregularly.} \]

d. How does a random variable differ from a variable within a computer program? (7)

<table>
<thead>
<tr>
<th>Operation</th>
<th>Variable</th>
<th>Random variable</th>
</tr>
</thead>
<tbody>
<tr>
<td>get</td>
<td>X → value</td>
<td>Sample X → value</td>
</tr>
<tr>
<td>store</td>
<td>X ← value</td>
<td>X ← function with value range</td>
</tr>
</tbody>
</table>
e. In what way is a production system more powerful than an FSM? (6)

A production system maps regions, sub sets, or partitions of state space, whereas an FSM maps an element or point in state space or another.

3. [Hash table] (15)
Consider the following sequence of operations on a dynamic hash table that has 2 days in a year where each day takes 10 time units: ENQ 3; ENQ 12; ENQ 14; ENQ 28; ENQ 37; CANCEL 12; DEQ. The current time/date is 0. Draw the table after each operation. The three types of operations are ENQueue, DEQueue and CANCEL.

Hash function: \( h(T) = \lfloor (T/D) \mod Y \rfloor \), \( Y=2 \) and \( D=10 \)

<table>
<thead>
<tr>
<th>Day 0</th>
<th>Day 1</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Year 1</th>
<th>Year 2</th>
</tr>
</thead>
<tbody>
<tr>
<td>[0 - 10)</td>
<td>[20 - 30)</td>
</tr>
<tr>
<td>[10 - 20)</td>
<td>[30 - 40)</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Time = 0</th>
<th>Time = 0</th>
<th>Time = 0</th>
<th>Time = 0</th>
</tr>
</thead>
<tbody>
<tr>
<td>ENQ 3</td>
<td>ENQ 12</td>
<td>ENQ 14</td>
<td>ENQ 28</td>
</tr>
<tr>
<td>3</td>
<td>3</td>
<td>3</td>
<td>3 → 28</td>
</tr>
<tr>
<td>12</td>
<td>12 → 14</td>
<td>12 → 14</td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Time = 0</th>
<th>Time = 0</th>
<th>Time = 3</th>
</tr>
</thead>
<tbody>
<tr>
<td>ENQ 37</td>
<td>CANCEL 12</td>
<td>DEQ</td>
</tr>
<tr>
<td>3</td>
<td>3</td>
<td></td>
</tr>
<tr>
<td>28</td>
<td>28</td>
<td>28</td>
</tr>
<tr>
<td>12 → 14</td>
<td>14</td>
<td>14</td>
</tr>
<tr>
<td>37</td>
<td>37</td>
<td>37</td>
</tr>
</tbody>
</table>
4. [FSM] (35)
Consider the following vending machine. The vending machine accepts only nickels and dimes and dispenses bottled water or Coke (by pressing one of two selection buttons). Bottled water costs 15¢ and Coke costs 20¢.
The machine should behave in accordance with the following specifications:
1) A customer needs to deposit a sufficient amount of money before he/she selects an item for purchase. If the item costs more than the deposited amount, the machine waits for more coins to be inserted. We assume that a customer presses one of two selection buttons only when a sufficient amount of money for each item has been deposited.
2) When a coin is inserted and the deposit (including the last inserted coin) is greater than 20¢, the last inserted coin is immediately returned. For example, a dime is immediately returned when the deposit is 15¢.
3) The change is not returned even if the item costs less than the deposited amount. For example, a nickel is not returned if the customer selects bottled water when the deposit is 20¢.
4) The vending machine does not have a "return" button.
We'll model the state by how much money has been deposited. Define the FSM of this vending machine with both (a) a fully labeled diagram and (b) a mathematical definition using <I, O, Q, δ, λ> notation.

(a) Diagram (20pts)
- FSM (Each state is exactly identified): at least 15 pts
- FSM (understandable) or well-drawn Flow chart: 12 pts
- Others: 10 pts

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![Diagram of the FSM](image-url)
(b) Mathematical definition (15pts)
- 3pts for each notation
- I, O, Q have been graded based on the question, not your solution in (a)
- \( \delta \) and \( \lambda \) have been graded based on your solution in (a) or the question

- Identify the states
  - Each state is defined by how much money has been deposited.
  - Maximum deposit is 20¢.
  \[ Q = \{q_0, ..., q_4\} = \{0\,\text{¢}, 5\,\text{¢}, 10\,\text{¢}, 15\,\text{¢}, 20\,\text{¢}\} \]
  \( O = Q \)

- Identify the inputs: what causes a change in state?
  - Coke and water buttons also cause the amount of deposited money.
  \( I = \{\text{nickel, dime, coke button, water button}\} \)

- \( \delta: Q \times I \rightarrow Q \)
  
  \( \delta(q_i, \text{nickel}) = q_{i+1}, \delta(q_i, \text{dime}) = q_{i+2} \)
  \( \delta(q_3, \text{nickel}) = q_4, \delta(q_3, \text{dime}) = q_3, \delta(q_4, \text{nickel or dime}) = q_4, \)
  \( \delta(q_3, \text{water button}) = q_0, \delta(q_4, \text{water button}) = q_1, \)
  \( \delta(q_4, \text{coke button}) = q_0 \)

- \( \lambda: Q \rightarrow O \)
  
  \( \lambda(q_i) = q_i \)