Fuzzy data. What simulation model components can be made fuzzy? It is not too surprising to find that most nonfuzzy mathematical structures can be made fuzzy simply by extending the appropriate definitions to encapsulate fuzzy, not crisp, sets. The extension principle permits the transfer of existing mathematical methods to incorporate fuzzy semantics. Here is a sample of how this relates to simulation. We can make fuzzy:

- A state variable value. This includes both initial conditions and values at a specific time.
- An event variable value.
- Parameter values. Time variant systems can use fuzzy functions for parameters.
- Inputs and outputs.
- Model structure.
- Algorithmic structure.

In the last item, we note that a simulation model is really just an algorithm at the lowest semantic level, and therefore methods in fuzzy algorithms can be utilized.

Considering the wide variety of applications of fuzzy set theory to simulation, we have designed and implemented procedures to deal with fuzzy-valued variables. We term a first order fuzzy number to be one that equals a single real value. As we increase the order, we obtain the following:

- An interval of confidence. Ex: $X = (1, 3)$, where $X$ is an interval fuzzy number with $\mu(1) = 1$ and $\mu(3) = 1$.
- A triangular number. Ex: $X = (1, 1.2, 3)$, where $X$ is a triangular fuzzy number with $\mu(1) = 0, \mu(1.2) = 1$, and $\mu(3) = 0$.
- A trapezoidal number. Ex: $X = (1, 2.5, 3, 4)$, where $X$ is a trapezoidal fuzzy number with $\mu(1) = 0, \mu(2.5) = 1, \mu(3) = 1$, and $\mu(4) = 0$.
- General discrete fuzzy number. $X = \sum_{i=1}^{n} \mu_i / x_i$.

A fuzzy number is defined as a fuzzy set that is both convex and normal. The simple types of fuzzy numbers are piecewise linear, so they can usually be abbreviated using the interval notation just delineated. With generalized fuzzy numbers we must, though, write down each domain value and its corresponding confidence level. The general form of the discrete fuzzy number (as shown above) is

$$\sum_{i=1}^{n} \mu_i / x_i$$

and the continuous fuzzy number has the associated denotation for a continuous domain:

$$\int_{X} \mu(x) / x$$

These two types of fuzzy numbers are similar in concept to the discrete probability mass function and continuous probability density function found in the theory of probability. It is natural to assign lexical values to fuzzy numbers, so that we can assign high to (200.0, 400.0), low to (10.0, 200.0), and hardly anything to (0, 10.0) for a given application.

Figure 2.5 defines a fuzzy variable, also known as a lexical variable since it can take on several natural language words as values. The fuzzy variable is temperature, and its possible values, using triangular fuzzy numbers, are (1) cold, (2) cool, (3) ambient, (4) warm, and (5) hot. It may be appropriate to use fuzzy numbers when there are few available data from which a probability distribution can be induced or when there are enough data,
but obtaining the data is costly or difficult. Fuzzy sets are a useful bridge between expert systems knowledge and quantitative knowledge since there is a direct mapping from quality to quantity.