Isosurface Visualization of Data with Nonparametric Models for Uncertainty

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Outline

- Formulation of topology prediction in Marching Cubes Algorithm (MCA) as an optimization problem
- Resolving topological ambiguities in presence of uncertainty
- Inverse linear interpolation with nonparametric densities
- From non-local means to non-local distributions
Isosurface Visualization

- Extract surfaces from a scalar field corresponding to interesting values
- Isosurface visualization is used in many disciplines e.g., biomedical imaging, climate studies etc.
Related Work: Uncertainty Visualization Techniques

- Color and opacity mapping [Rhodes et al., 2003]
- Point-based displacement proportional to uncertainty [Grigoryan and Rheingans, 2004]
- Techniques such as "Uncertainty Bands" and the color-mapping of bands to visualize spatial variations in isocontours [Osorio and Brodlie, 2008]
- Contour boxplots to visualize statistical quantities related to uncertain isocontours [Whitaker et al., 2013]
Related Work: Uncertainty Quantification Techniques

- Fuzzy isosurface visualization by exploiting histograms of uncertain data [Thompson et al., 2008]
- Study of structural variability of isosurfaces by visualizing global correlation structures [Pfaffelmoser and Westermann, 2012]
- Quantification and visualization of cell-crossing probabilities for uncertain isosurfaces in nonparametric statistical framework [Pöthkow and Hege, 2013]
- Closed-form quantification of the spatial variations in isosurfaces for parametric density models [Athawale and Entezari, 2013]
State of the art: Direct volume rendering of the level-crossing probabilities (LCP)

Our work: Uncertainty visualization for nonparametric models while not shifting to direct visualization paradigm.
Marching Squares Algorithm (MSA) in Uncertain Data
MSA Step 1: Isocontour Topology

- Determine cell configuration and isocontour topology within each cell.
- 16 configurations reduce to only 4 configurations using symmetry.

Basic cell configurations

- Marked Vertices: Positive (data > isovalue)
- Unmarked Vertices: Negative (data < isovalue)
- Green Edges: Edges crossed by the isosurface
MSA Step 2: Isocontour Geometry

Determine the isocontour-crossing locations on the edges using inverse linear interpolation.

\[ v_c = (1 - z) v_1 + z v_2 \text{, where } z = \frac{c - x_1}{x_2 - x_1} \]

Isocontour geometry for isovalue \( c = 30 \)
Characterizing Data Uncertainty

- Field with independent random variables
- Characterization of uncertainty using probability density function
- Propagation of data uncertainty into marching cubes algorithm
Topology Prediction in Uncertain Data
Topological Uncertainty

Isovalue $c = 30$

24 ± 2  
29 ± 2  
28 ± 1  
33 ± 2  

26  
27  
35  

22  
27  
31  
33
Cell configuration (classification of vertices as positive/negative) determines isosurface topology.
Isosurface Topology Prediction Methods

- Vertex-based Classification
- Edge-based Classification
Method 1: Vertex-based Classification
Vertex-based Classification

- Process each vertex independently
- If $Pr(X > c) > 0.5$, classify vertex as positive and vice versa.

Shaded areas show most probable vertex sign for iso-value $c$. 
Vertex-based Classification

- If \( Pr(X > c) > 0.5 \), classify vertex as positive and vice versa.
- Approach doesn’t consider signs of neighboring vertices!

Shaded areas show most probable vertex sign for isovalue \( c \).
Method 2: Edge-based Classification
Edge-crossing probability for isosurface with isovalue $c$ for independent random variables $X$ and $Y$: $1 - Pr(X > c) \cdot Pr(Y > c) - Pr(X < c) \cdot Pr(Y < c)$
Edge-based Classification

- When edge-crossing probability is relatively high, we expect opposite signs and vice versa.
Edge-based Classification

\[ W_{ij} = \Pr_{\text{crossing}} \]

\[ S_i = \pm 1 \quad \text{and} \quad S_j = \pm 1 \]

- \( W_{ij} \) : Edge-crossing probability (known)
- \( s_i, s_j \) : vertex signs (unknown)
- \( W_{ij} \) relatively high \( \Rightarrow \) \( s_i \) and \( s_j \) opposite
- \( W_{ij} \) relatively low \( \Rightarrow \) \( s_i \) and \( s_j \) same
Optimization Problem

\[ s^* = \arg \min_{s_n=\pm 1} s^T W s. \]

**W**: weight matrix of edge-crossing probabilities

**s**: sign vector

**s_n**: n’th entry of matrix **s**

- **Solution**: Combinatorial approach (Not practical!)
Relaxed Optimization Problem

Original:

\[ s^* = \arg \min_{s_n = \pm 1} s^T W s. \]

Relaxed:

\[ s^* = \arg \min s^T W s. \]

\[ s^T s = 1 \]

\( W \): weight matrix of edge-crossing probabilities
\( s \): sign vector
\( s_n \): \( n \)'th entry of matrix \( s \)

▶ **Solution:** Eigenvector of \( W \) with largest (negative) eigenvalue
▶ Fast randomized solution using Nyström method [Fowlkes et al., 2004]
▶ Computationally expensive compared to vertex-based classification
Gaussian Mixture Example

Magenta: Positive Vertices, White: Negative Vertices

Groundtruth  Vertex-based  Edge-based
Uncertain Midpoint Decider
Ambiguous Topology: Decider Uncertainty

Isovalue $c = 30$

Vertex-based/Edge-based Classification

$29.5 \pm 2$

$24 \pm 2 \quad 35 \pm 2$

$33 \pm 2 \quad 26 \pm 2$

$28$

$31$
Ambiguity Resolution for Uncertain Data

Midpoint sign corresponding to underlying data?
B-Splines

- Random variable corresponding to 1-d cell midpoint: \( M = \frac{X_1 + X_2}{2} \)
- Sum of random variables → Convolution of densities.

Box spline with non-uniform knots
Figure: Convolution of uniform kernels with unequal bandwidths

- Face midpoint random variable: $M = \frac{X_1 + X_2 + X_3 + X_4}{4}$
- Face midpoint density ($Pdf_M$): Cubic univariate box-spline with non-uniform knots
- Body (3-d cell) midpoint random variable: $M = \frac{X_1 + \cdots + X_8}{8}$
- Body (3-d cell) midpoint density ($Pdf_M$): Degree 7 univariate box-spline with non-uniform knots
Uncertain Midpoint Decider

- **Step 1**: Compute density function in closed form at midpoint (using convolution).
- **Step 2**: Find most probable vertex sign for midpoint

Shaded areas show most probable vertex sign for isovalue $c$. 
Geometric Uncertainty Quantification
Isovalue $c = 30$
Density of the random variable corresponding to inverse linear interpolation formula

\[ Z = \frac{c - X_1}{X_2 - X_1} \]
Cumulative density function can be obtained by integrating polynomials falling within red region.
Green's Theorem

Integration of polynomial $P_1$ over enclosed region ABC using line integrals
Results and Conclusion
Ensemble Visualization: The Tangle Function

Tangle function: Well-known for its complexity in isosurface reconstruction

Groundtruth  Mean  Density Propagation

Low  Error  High
Ensemble Visualization: Temperature Field

- **Mean**
- **Vertex-based**
- **Edge-based**

Ratio Variance

- 1
- 0
Ensemble Visualization: The Teardrop Function

Base kernel: Uniform, Classification: Edge-based
Uncertain Scalar Field: Non-local Techniques

- Non-local Means [Buades and Antoni, 2005]: Weighted average of the neighboring intensities
- Non-local Distribution: Density estimation using weighted sum of the kernels associated with each of the intensities
Conclusion

- Impact of uncertainty on the marching cubes algorithm
- Isosurface topology prediction using vertex-based and edge-based classification
- Uncertain midpoint decider for resolving topological ambiguities
- Geometric uncertainty quantification in closed-form for nonparametric models.
- Expected isosurface visualization with color-mapped uncertainties.
Limitations and Future Work

- Characterization of uncertain asymptotic decider for resolving topological ambiguities.
- Study of isosurface extraction for dependent random variables.
Uncertain Scalar Field: The Bonsai Tree Dataset \((c = 84)\)

**Base kernel:** Uniform

![Local Parametric](image1)
![Local Nonparametric](image2)

![Non-local Nonparametric](image3)
![Non-local Nonparametric](image4)
Uncertain Scalar Field: The Fuel Dataset ($c = 96.74$)

Figure: (a) Mean field, (b) Local Parametric, (c) Local Nonparametric, (d) Non-local Means, (e) Non-local Nonparametric, (f) Colormapped Uncertainties.