ISOGEOMETRIC ANALYSIS TOWARD SHAPE OPTIMIZATION IN ELECTROMAGNETICS

NGUYEN DANG MANH⋆, ANTON EVGRAFOV⋆, JENS GRAVESEN⋆, JAKOB SØNDERGAARD JENSEN†

⋆Department of Mathematics and †Department of Mechanical Engineering
Technical University of Denmark
e-mail: ⋆{D.M.Nguyen,A.Evgrafov,J.Gravesen}@mat.dtu.dk, †jsj@mek.dtu.dk

Key words: Isogeometric analysis, shape optimization, electromagnetics.

Summary. We use NURBS-based isogeometric analysis to investigate dependences of magnetic energy on geometry for some two-dimensional scattering problems.

1 INTRODUCTION

We consider two-dimensional electromagnetic (EM) scattering problems as depicted in Fig. 1(a) where the incident magnetic field intensity is given as: \( \mathbf{H} = (0, 0, H_z) \). The governing equations of the problem, c.f. [1], are

\[
\nabla \cdot \left( \frac{1}{\varepsilon_{cr}} \nabla H_z \right) + k_0^2 \mu_r H_z = 0 \quad \text{in } \Omega, \quad (1a)
\]
\[
\frac{1}{\varepsilon_{cr}} \frac{\partial H_z}{\partial n} - jk_0 \sqrt{\frac{\mu_s}{\varepsilon_{cr}}} H_z = 0 \quad \text{on } \Gamma_s, \quad (1b)
\]
\[
\frac{\partial H_z}{\partial n} + (jk_0 + \frac{1}{2r_t})H_z - \frac{\partial H_z^i}{\partial n} - (jk_0 + \frac{1}{2r_t})H_z^i = 0 \quad \text{on } \Gamma_t, \quad (1c)
\]

Figure 1: Various models of scattering problems

---18---
where \( \varepsilon_{cr}, \mu_r \) are the relative complex permittivity and the relative permeability of the dielectric material to the corresponding constants of free space, respectively; \( \varepsilon_i^{cr}, \mu_i^{cr} \) are the relative complex permittivity and the relative permeability of the scatterer, respectively; \( r_t \) is the radius of the circular truncation boundary; \( k_0 \) is the wavenumber of free space. As a results of the above equations, the weak form of the scattering problem reads: Find \( H_z \in H(\text{div}, \Omega) \) such that for every \( \phi \in H(\text{div}, \Omega) \):

\[
\int_{\Omega} \frac{1}{\varepsilon_{cr}} \nabla H_z \cdot \nabla \phi \, dV - k_0^2 \int_{\Omega} \mu_r H_z \phi \, dV - jk_0 \int_{\Gamma_s} \eta_s^{cr} H_z \phi \, d\partial + \left( jk_0 + \frac{1}{2r_t} \right) \int_{\Gamma_t} \varepsilon_{cr} H_z \phi \, d\partial = \int_{\Gamma_t} \frac{1}{\varepsilon_{cr}} \left( \frac{\partial H_z^i}{\partial n} + (jk_0 + \frac{1}{2r_t}) H_z^i \right) \phi \, d\partial.
\]

In particular, if \( H_z = H_0 e^{-jkx} \) and geometry of the problem is symmetric about the line \( y = 0 \), e.g. the model in Fig. 1(b), the problem can be solved in a half of the truncation domain with the following boundary condition along the boundary \( y = 0 \): \( \frac{\partial H_z}{\partial y} = 0 \).

2 NUMERICAL EXAMPLES

2.1 Comparison between numerical and exact solutions

We consider the problem sketched in Fig. 1(c), in which the scatterer is a perfect electric conductor. The exact solution of the problem, cf. [3], is

\[
H_z = \sum_{n=\infty} J_n(k\rho) - \frac{J'_n(kr_s)H_n^{(2)}(kr_s)}{H_n^{(2)}(kr_s)} e^{jn\phi}.
\]

To apply isogeometric analysis to the problem, we model the problem by two patches as showed in Fig. 2(a). According to Fig. 2(d), the numerical and exact solutions agree up to 2%.

![Figure 2: Comparison between numerical and exact solutions of the scattering problem in Fig. 1(c). (a): The truncation domain comprised of two patches. (b): Exact solution. (c): Isogeometric analysis-based solution. (d): Relative error.](image)

2.2 Magnetic resonator

We now want to examine dependences of magnetic energy, in terms of the quantity \( W_m = \int_{\Omega_m} \log \left( \frac{\|H(u)\|^2}{\|H_0\|^2} \right) \, dV \), on geometry of the scattering problem sketched in Fig. 1(b). To solve the
problem, three patches are used to model its truncation domain, see Fig. 3(left). To compute \( W_m \), we use an extended trapezoidal rule and an optimization problem to find preimages of the integrating points of the rule in the physical domain is invoked.

We first examine the dependence on the distance \( d \) between the origin and the scaterer center. The results, depicted in Fig. 3(right), shows that magnetic energy depends inversely on \( d \).

\[ W_m = -1.5562 \] \hspace{1cm} \[ W_m = -1.5204 \] \hspace{1cm} \[ W_m = -1.5821 \] \hspace{1cm} \[ W_m = -1.347 \]

(a) \hspace{1cm} (b) \hspace{1cm} (c) \hspace{1cm} (d)

Figure 4: Magnetic energy in the presence of various magnetic resonators with different shapes

Moreover, we choose the model with circular magnetic resonators as a reference model and compare models whose magnetic resonators have different shapes to the reference model. In comparison to magnetic energy of the reference model (Fig. 4(a)), magnetic energies of the models whose resonators with one deformed upper part (Fig. 4(b)) or lower part (Fig. 4(d)) are stronger.

\[ \text{maximize} \quad W_m \] \hspace{1cm} \[ \text{where} \quad K(\rho_b)u = f(\rho_b), \]

3 \textbf{FUTURE WORK}

We consider the following shape optimization problem

\[ \text{maximize} \quad W_m \] \hspace{1cm} \[ \text{where} \quad K(\rho_b)u = f(\rho_b), \]
where $p_b$ are the control points that govern the shape of the scatterer and the equation (4b) is the discretized form of the weak form (2). This is not a new problem. It originates from recent attempts for improving wireless power transfer via coupled magnetic resonances [4]. Recently, Sigmund and his coworkers [5] have used topology optimization to find spatial distributions of two magnetic resonators. Some of their results are depicted in Fig. 3.

![Results from a previous work](image)

Figure 5: Results from a previous work [5], which were obtained by topology optimization, the initial designs are similar to the model in Fig. 1(b).

Our future work is to utilize advantages of isogeometric shape optimization [6] to enhance their results.

REFERENCES


