CIS6930 Mathematics for Intelligent Systems II Spring 2009
Home Work Assignment 3: Due Tuesday 04/21/09 before class

1. A metric space is called *separable* if it contains a countable dense subset. Let X be a metric space in which every infinite subset has a limit point. Prove that X is separable.

2. Prove that the *Cantor* set is *Borel* measurable. Give an example of a set that is not *Borel* measurable (feel free to search the net) and prove that it is not.

3. Prove the following: If X is a random variable with $E(|X|) < \infty$, and A_n are disjoint sets with union A, then $\sum_{n=0}^{\infty} E(X; A_n) = E(X; A)$.

4. Let random variable $Y \geq 0$ with $EY^2 < \infty$. Prove that $P(Y > 0) \geq (EY)^2/EY^2$.

5. Prove that for random variable $X \geq 0$, $\sum_{k=1}^{\infty} P(X > k) \leq E(X) \leq \sum_{k=0}^{\infty} P(X > k)$