CIS6930 Mathematics for Intelligent Systems II Spring 2009
Home Work Assignment 1: Solution sketch

1. First show that the natural basis for V^* is $f_i(v_j) = \delta_{ij}$ and therefore V^* is n-dimensional (work out the linear independence and span of these vectors). Likewise show that the natural basis for V^{**} is $G_j(f_i) = \delta_{ij}$ and therefore V^{**} is also n-dimensional (once again work out linear independence and span). Consider now the functions $v_j(f) \triangleq f(v_j)$ where $f \in V^*$. Define vector addition and scalar multiplication for this space accordingly. Finally, consider the basis mapping $\Phi(v_j) = G_j$, and show that $\Phi : V \to V^{**}$ is bijective.

2. $\tilde{T}_{jkh} = \sum_{l=1}^{2} \sum_{p=1}^{2} \sum_{q=1}^{2} a_{jl} b_{kp} c_{hq} T_{lpq}$.

3. Its easy to argue for multi-linearity by geometrically showing that $V(u_1 + u_2, v, w) = V(u_1, v, w) + V(u_2, v, w)$ and $V(\alpha u, v, w) = \alpha V(u, v, w)$ and then arguing the rest through symmetry. Next, $V(u, u, w) = 0$ can be imposed by assuming that the tensor is alternating. Finally, determinant computation can be shown to be equivalent to assuming that in addition to being alternating $V(i, j, k) = 1$.

4. Its just a matter of working thru the algebra. In the first case the EL simplifies to $\frac{d}{dx} f_{y'} = \frac{d}{dx} \frac{y'}{x^3} = 0$, which when solved gives $y = C_1 x^4 + C_2$. In the second case the EL is $y'' = y + e^x$, which when solved gives $y = \frac{1}{2} x e^x + C_1 e^x + C_2 e^{-x}$.

5. This question is mostly conceptual. As discussed in class, it is difficult to define a quadratic form in a basis independent manner. Note that what you want is a definition like: $f(v + \delta v) = f(v) + L(\delta v) + Q(\delta v) + \epsilon(\delta v)$, where $\lim_{||\delta v|| \to 0} \epsilon(\delta v)/||\delta v||^2 = 0$. $L(\cdot)$ can be described in a basis independent manner to be a covector. Likewise, $Q(\cdot)$ can be described in a basis independent manner to be *derived* from a bilinear form with both inputs set to be the same. Of course one has to run through the argument that both L and Q are unique. Finally standard open-neighborhood arguments can be used to show that Q has to be positive definite for f to be reach a local minima at the stated point.