CIS6930 Mathematics for Intelligent Systems II Spring 2009
Home Work Assignment 1: Due Thursday 01/29/09 before class

1. Let V be a finite dimensional vector space with basis elements $v_1, v_2, ...v_n$. Consider the dual vector space V^* of all linear functions $f : V \rightarrow \mathbb{R}$. Write down the natural basis for V^*. Now consider the dual vector space V^{**} of all linear functionals $G : V^* \rightarrow \mathbb{R}$. Work out the natural isomorphism between V and V^{**}.

2. Let U, V, W be 2-dimensional vector spaces with basis elements u_1, u_2, v_1, v_2, and w_1, w_2. Let $T : U \times V \times W \rightarrow \mathbb{R}$ be a 3-tensor represented as $T(u_i, v_j, w_k) = t_{ijk}$. Consider now basis transformations, $\tilde{\mathbf{u}} = A\mathbf{u}$, $\tilde{\mathbf{v}} = B\mathbf{v}$, and $\tilde{\mathbf{w}} = C\mathbf{w}$. What does the tensor representation transform to (i.e., assuming that the tensor remains the same)? Write it in as compact a form as you can.

3. Argue why a signed volume element can be represented as a rank n alternating tensor. Show its relationship with the determinant of a matrix whose columns are the vectors on which the volume is computed.

4. Find the extremals of the functionals: $\int_a^b \frac{y'^2}{x^3} dx$ and $\int_a^b (y^2 + y'^2 + 2ye^x) dx$

5. Consider a smooth function $f : \mathbb{R} \times \mathbb{R} \rightarrow \mathbb{R}$, whose first derivative is 0 at $\langle 0, 0 \rangle$. The second derivative of the function can be represented compactly as a bilinear form. Why? Next, show that a sufficient condition for f to have a local minima at $\langle 0, 0 \rangle$ is to have the corresponding quadratic form be positive definite.