CIS6930/4930 Intro to Computational Neuroscience Fall 2008
Home Work Assignment 3:
Due Thursday 10/30/08 before class

1. A complex number z is said to be *algebraic* if there are integers $a_0, a_1, ..., a_n$, not all zero, such that

$$a_n z^n + a_{n-1} z^{n-1} + ... + a_1 z + a_0 = 0$$

   Prove that the set of all algebraic complex numbers is *countable*.

2. Consider a metric space $(\mathcal{M}, d(\cdot,\cdot))$. Prove that every convergent sequence is also a Cauchy sequence.

3. Consider a finite dimensional inner product space over the field of complex numbers with the inner product defined through

$$\langle c_1 \mathbf{e}_i, c_2 \mathbf{e}_j \rangle = c_1 \overline{c_2} \quad \text{if} \quad i = j, \quad \text{and} \quad 0 \quad \text{otherwise}$$

   c_1, c_2 are complex numbers and $\mathbf{e}_i, \mathbf{e}_j$ are basis vectors.

   Prove that the induced norm satisfies the triangular inequality.

4. Consider the following function over the range $[0, 1]$

$$f(x) = -2 \times x \quad \text{if} \quad x \in [0, \frac{1}{3}]$$

$$f(x) = 1 \quad \text{if} \quad x \in (\frac{1}{3}, \frac{2}{3})$$

$$f(x) = 0 \quad \text{if} \quad x \in [\frac{2}{3}, 1]$$

   First translate and scale uniformly the domain of the function so that it now lies on $[-\pi, +\pi]$. All future references to $f(x)$ is this scaled and translated version. Your goal will be to find an approximation of this function as a Fourier series, and show the graphs of successive approximations overlayed on the actual function.

   Consider the Fourier basis e^{inx} for $n = -N, ..., +N$, and the corresponding sum

$$\sum_{n=-N}^{+N} c_n e^{inx}$$

   Calculate the values of c_n by numerically approximating the integral

$$\int_{-\pi}^{+\pi} f(x) e^{-inx} dx$$

   , that is, by dividing the range $[-\pi, +\pi]$, into small intervals and approximating the integral as a sum.

   Show graphs of how well $f(x)$ is approximated by overlaying the series over $f(x)$ for various values of N (for example, $N = 5, 10, 20, 50$).