1. (100 pts) This project is about constructing a random cortical network of spiking neurons and stabilizing its activity in the range 0-50 Hz.

Begin by constructing a network of 1000 neurons, 80% of which are randomly chosen to be excitatory neurons and the remaining inhibitory neurons. Now assign 100 synapses (each neuron receives inputs from 100 neurons) to each neuron and randomly choose the pre-synaptic neurons for each such postsynaptic neuron. Next assign 50 variables to each neuron to store the times at which the neuron spiked in the past 100 msec (Naturally, the time bound for the horizon is 100 msec.)

Finally, build a spike response model for each neuron as follows. The total potential at the soma of a neuron is the sum of the excitatory and inhibitory post-synaptic potentials generated by each spike at each synapse, and the after-hyperpolarization potentials generated by the spikes emitted by the neuron itself. Randomly assign to each synapse a time in the range [0.4,0.9] msec that accounts for the time it takes for a spike to travel down the axon and reach that synapse.

Use the following functions to model the postsynaptic potentials at the synapses

Excitatory:
\[ \frac{Q}{\alpha \sqrt{t}} e^{\alpha^2 / t} \exp(-\alpha^2 / t) \exp(-t/\tau) \]
set \( Q \) in the range [5,10], set \( \alpha \) in the range [1,2] and \( \tau = 20 \) msec.

Inhibitory:
\[ \frac{-Q}{\alpha \sqrt{t}} e^{\alpha^2 / t} \exp(-\alpha^2 / t) \exp(-t/\tau) \]
set \( Q \) in the range [30,60], set \( \alpha \) in the range [1,1.1] and \( \tau = 20 \) msec.

AHP:
\[ -1000 \times \exp(-t/1.2) \]

Start with different thresholds (5 mV for excitatory and 10 mV for inhibitory) for each neuron and play with the thresholds so as to stabilize the system to sustained recurrent activity.

Plot the spike trains for 250 neurons in the population, as well as the total number of spikes generated in the system over the past 100 msec.