Homework #2
(due midnight, Tuesday, October 7, 2003)

1. **Canny edge detector:** Show that the product of the Canny detection and localization criteria

\[
\text{SNR}(f) \times \text{Loc}(f) = \frac{|\int_{-W}^{W} G(-x)f(x)dx| |\int_{-W}^{W} G'(-x)f'(-x)dx|}{n_0 \sqrt{\int_{-W}^{W} f^2(x)dx} n_0 \sqrt{\int_{-W}^{W} f'^2(x)dx}}
\]

is maximized by \( f(x) = G(-x) \).

2. **Canny edge detector:** Rewrite the detection and localization criteria for a filter \( f_w(x) = f(x/w) \). Show that the product of the detection and localization criteria is invariant to \( w \).

3. **Level sets:** Assume that a particle is moving in 1D according to the equation of motion

\[
\frac{dx}{dt} = -\frac{x + t}{t + 1}.
\]

What is the corresponding \( \psi(x, t) \) function? [Hint: Use \( \frac{\partial \psi}{\partial t} = -\frac{\partial \psi}{\partial x} \frac{dx}{dt} \).] Plot \( \psi(x, t) \) in the interval \([0, 1]^2\) showing the level sets. Explain the relation between the equation of motion and the level sets of \( \psi \). Please realize that this is a reduced 1D motion embedded in a 2D function.