Quantum Steering of Surface Error Correcting Codes

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***Abstract*—Surface codes provide a promising path towards large-scale fault-tolerant quantum computers. However, outside the difficulty in engineering qubits, their theoretical realization is hindered by a number of technical implementation details, including the initialization of an encoded quantum state on contemporary quantum computers. We propose a solution to overcome these challenges by utilizing recent theoretical developments in measurement-induced quantum steering. An encoded quantum state is prepared by repeatedly performing the following steps: (1) entangling qubits via a specifically chosen operation, (2) performing measurement on some of the qubits, and (3) resetting the measured qubits' states. We demonstrate our results using numerical simulations of surface codes, noting convergence of state fidelity, and commenting on choices for parameter selection.**

I. Introduction

Quantum computers can solve a number of key problems exponentially faster than their classical counterparts. Examples include the famous Shor's algorithm for factoring prime numbers [1], Grover's search for finding a needle in a haystack [2], as well as the quantum algorithm for solving a system of linear equations [3]. Qubits (quantum bits) can be in an arbitrary combination of states (superposition), and entangled states can not be expressed in terms of individual qubit states. The enhanced computational ability is driven by the key quantum mechanical property of entanglement. Unfortunately in reality, qubits also entangle with unwanted degrees-of-freedom (the environment), leading to decoherence and a loss of information [4], [5]. In other words, the quantum computer must satisfy two conflicting requirements. Qubits need to be externally controlled, measured, and entangled. On the other hand, qubits must be isolated from their environment to avoid unwanted entanglement. As a result of these conflicts, quantum computers will be noisy, where errors propagate and grow during the execution.

A. *State-of-the-Art*

Fortunately, the invention of quantum error-correcting codes (ECC) provides a realistic path towards fault-tolerant quantum computing. ECC have three main requirements: (1) provide an encoding of physical qubits to logical qubits, (2) the ability to detect when an error has occurred, and (3) a mechanism to correct the logical qubit. In other words, by creatively entangling several physical qubits as one logical qubit, errors can be detected and corrected. As a particular promising example, surface codes (Figure 1) build logical qubits by assuming topological features of physical qubits [6], [7]. Specifically, surface codes assume the physical layout of qubits is given by a lattice. Experimentally realizing an engineered

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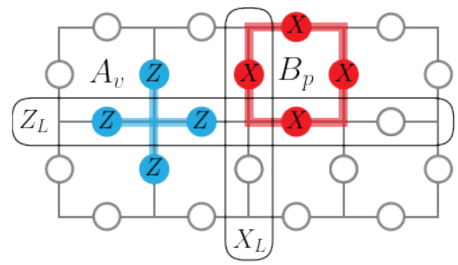


Fig. 1: The surface code is defined on a square lattice. Two key operators, A_v and B_p, define the codespace of the lattice. A logical qubits is encoded by spreading the information across all the qubits on the lattice. Logical gates X_L and Z_L are defined as non-trivial loops around the lattice, and act on the protected logical qubits.

system that satisfies the surface code is a difficult and ongoing problem. However, recent experiments have successfully demonstrated surface code implementation by "simulating" the code on programmable quantum processors, such as on Rydberg atoms [8] and superconducting qubits [7]. Two key steps are performed to prepare the computer for surface code simulation. The first step is to reset the computer to a fiducial, all-zeros, state – either by waiting for the qubits to naturally decay, or by measuring, classically reading, and correcting the qubits. The second key step is to apply highly calibrated (single and entangling) gates to the all-zeros state to prepare the simulated state of a surface code. Specifically, a quantum circuit $\mathcal{U}_{\text{prep}}$ is applied to n-qubits in the fiducial state $|0\rangle$ which then simulates a surface code state $|G\rangle = \mathcal{U}_{\text{prep}} |0\rangle^{\otimes n}$. Throughout this paper, we will refer to these initial steps as preparing the surface code. Existing approaches – including "repeat until success" [9] – have two practical limitations: (i) they require many gates (area overhead) to initialize the surface code, and (ii) they require many measurements (timing overhead) to ascertain high fidelity of the initial state.

B. *Research Contributions*

We propose a mechanism to prepare the surface code by exploiting the behavior that arise from quantum measurement. Quantum Steering (QS), as first coined by Schrödinger, is a puzzling phenomenon in quantum mechanics whereby measurements on one system influences the state of another entangled system [6]. Recent works theoretically define under which physical conditions a quantum-mechanical system may be steered [10] and experimentally demonstrated on contemporary quantum computers [11]. We utilize these advancements to construct a *steering protocol* that prepares a surface code. As a brief visualization, Figure 2 represents the steering protocol as a quantum circuit, where a special entangling operation

$\mathcal{U}$ is repeatedly applied alongside measurement. The end result is that from an arbitrary initial state, a desired state is prepared. In this paper, we develop the necessary ingredients to realize the steering protocol and prepare surface codes. Specifically, this paper makes the following major contributions:

1) Proposes an algorithm that can generate quantum circuits $\mathcal{U}$ to satisfy conditions for quantum steering.
2) Presents two methods to construct the groundstate of the surface code (quantum steering and hybrid approach).

The rest of the paper is organized as follows. Section II provides the relevant background on quantum steering and surface codes. Section III provides the problem formulation. Section IV describes our proposed framework for quantum steering of surface codes. Section V presents the experimental results. Finally, Section VI concludes the paper.

II. Theoretical Background

In this section, we first introduce quantum computing and noise models. Next, we provide background on quantum error correction and surface codes.

A. Quantum Computing

Qubits are the fundamental building blocks of quantum computers. The state $|\psi\rangle$ of a qubit lives in a 2-dimensional complex-Hilbert space and can be expressed as $|\psi\rangle = \alpha|0\rangle + \beta|1\rangle$ where $|\alpha|^2 + |\beta|^2 = 1$. Measuring a qubit yields "0" or "1" with probability $|a|^2$ or $|b|^2$ respectively. Several qubits may be combined to form a larger state space of size 2^n where n is the number of qubits. A quantum gate U (and in general a quantum circuit $\mathcal{U}$) acts on a collection of qubits to transform it to another state $U|\psi\rangle$ ($\mathcal{U}|\psi\rangle$). The Solovay-Kiteav theorem guarantees that any (ϵ-close) quantum circuit can be built using a small set of *universal* quantum gates [12]. The main challenge, unfortunately, is that real quantum computers are prone to noise that scales with the number of qubits – resulting in unreliable physical qubits and gates [13].

B. Noise Models

Quantum noise can be generally labeled as either coherent, incoherent, or decoherent. Coherent noise occurs when an intended gate U is perturbed, resulting in $\tilde{U} = U \cdot \overline{U}$ where $\overline{U}$ is a *small* unitary offset. Similarly, incoherent noise occurs when the intended gate U is stochastically perturbed, resulting in many possibilities for $\tilde{U} = \{U \cdot \overline{U}_0, U \cdot \overline{U}_1, \cdots, U \cdot \overline{U}_n\}$ depending on a classical probability distribution governing the chance of $\overline{U}_i$ occurring. Both coherent and incoherent noise can be mitigated using advanced (optimal) control techniques, such as dynamical decoupling [14]. Decoherent noise, however, occurs when information is lost to the environment, making the error irreversible. Unlike coherent and incoherent noise, decoherent noise requires advanced techniques to mitigate the errors, i.e. quantum error correction.

To represent a general quantum process, including the effect of noise, it is convenient to use the density matrix representation of quantum states:

$$\rho = \sum_i p_i |\psi_i\rangle \langle\psi_i|,$$

where p_i gives the probability of $|\psi_i\rangle$ occurring. A general quantum process (e.g. quantum noise), $\mathcal{E}$, is then given as a map on density matrices $\rho' = \mathcal{E}(\rho)$. As an example, in this formalism a quantum gate U acting on a density matrix ρ is expressed as $\mathcal{E}(\rho) = U\rho U^\dagger$.

C. Quantum Error Correction

Two key facts make traditional classical codes inapplicable for quantum computers: (a) measurement collapses the state of a qubit, and (b) by the no-cloning theorem, information cannot be copied. Instead, quantum error correcting codes spread the information of one logical qubit across many high-entangled physical qubits. Errors are detected using syndrome measurements, where many-qubit measurements are performed such that it does not destroy the information encoded in the logical qubit. If an error is detected, operations corresponding to the type of error is used to revert the error. In other words, an error-correcting code E, will ideally undo the action of a noisy process $\mathcal{E}$ and return the original density matrix: $E(\rho') = E \circ \mathcal{E}(\rho) = \rho$.

Quantum error correcting codes depend on a number of key parameters, such as: the number of physical qubits necessary, the number of encoded logical qubits, and the maximum physical error rates threshold [15], [16]. It is an open and active area of research to find the optimal set of parameters needed for error correction – particularly with respect to optimizing the physical error rate thresholds. Currently, surface codes are considered as an uncontested leader in terms of error correction [17].

D. Surface Codes

Surface codes are defined on a plane lattice. As depicted in Figure 1, qubits are placed on each edge of a square in the lattice. The encoding of logical qubits are based on two important operators that perform simple local operations. These are known as vertex and plaquette operators, and are defined as follows:

$$A_v = \prod_{i \in v} Z_i \text{ and } B_p = \prod_{i \in p} X_i,$$

where A_v enacts the Pauli-Z rotation on each qubit around a vertex v, and B_p enacts the Pauli-X rotation on each qubit that make up a plaquette p. All these terms mutually commute, $[A_s, B_p] = 0$, and so a state $|\psi\rangle$ can be defined to be a simultaneous eigenstate of both terms

$$\forall s : A_s|\psi\rangle = |\psi\rangle \qquad \forall p : B_p|\psi\rangle = |\psi\rangle.$$

The state $|\psi\rangle$ defines the codespace of the code. Logical operators are then defined to operate within the codespace, and hence logical qubits are realized.

To complete the description of logical states, we can inspect the surface code in a physical sense. The Hamiltonian for a surface code is defined as

$$\hat{H}_{\text{surface}} = -\sum_{i \in v} A_i - \sum_{i \in p} B_i. \tag{1}$$

The purpose of the Hamiltonian is to measure the energy of a system. The groundstate $|G\rangle$, which has the lowest energy, can be defined in terms of projection operators

$$|G\rangle = \frac{1}{4}\left\{\prod_{i\in v}(\mathbb{I}+A_i) + \prod_{i\in p}(\mathbb{I}+B_i)\right\}|\psi\rangle\,. \tag{2}$$

This tells us that for any state $|\psi\rangle$ we may bring the system to the groundstate by appropriately applying the projection operators $(\mathbb{I}+A_i)$ and $(\mathbb{I}+B_i)$. By definition, the groundstate is an encoding for a logical qubit. The space of the logical qubits is then defined as

$$|00\rangle_L = |G\rangle\,, \quad |01\rangle_L = \bar{X}_1\,|00\rangle_L \tag{3}$$

$$|10\rangle_L = \bar{X}_2\,|00\rangle_L\,, \quad |11\rangle_L = \bar{X}_1\bar{X}_2\,|00\rangle_L \tag{4}$$

where $\bar{X}_1$ and $\bar{X}_2$ are logical operators which are defined in terms of non-trivial "cycles" on the lattice as depicted in Figure 1.

III. Problem Formulation

We first describe the theory of measurement-induced quantum steering. Next, we discuss how to apply quantum steering for error correction using surface codes.

A. Measurement-induced Quantum Steering

One of the key features of quantum mechanics is entanglement. An interesting situation occurs when two particles are maximally entangled and one of the particles is measured. The result of measuring the second particle is then predictable. Bell's inequality gives a mathematical constraint to how the outcomes of the two measurements are correlated [18]. Quantum steering refers to a situation where measurements are conducted on part of an entangled state, and *steer* the other part of the state [19]. The theory of measurement-induced quantum steering specifies the form of entanglement necessary to prepare a state through measurements of another system [10], [20], [21].

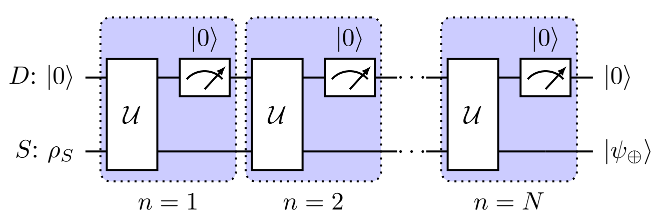


Fig. 2: The circuit visualization of the quantum steering protocol. Detectors are prepared in a fixed state, $|0\rangle$, and system qubits may be in arbitrary (mixed) states. Repeatedly applying an operation $\mathcal{U}$, measuring, and resetting the detector, causes the system qubit to converge to a desired state $|\psi_\oplus\rangle$.

Figure 2 shows the measurement-induced quantum steering protocol as a circuit. Suppose we have detector qubits initialized to the state $|0\rangle$ and system qubits in an arbitrary state ρ_S. We wish to steer ρ_S to a desired state, particularly the groundstate of a surface code $|G\rangle$. Measurement-induced quantum steering involves the following:

1) Couple the detector qubits and system qubits with a composite unitary operator $\mathcal{U}$. The state of the detector-system after the n-th application of the unitary evolution is $\rho^{n+1} = \mathcal{U}\left(|0\rangle\langle 0| \otimes \rho_S^n\right)\mathcal{U}^\dagger$.
2) The detector qubits are then decoupled from the system – the statistics of measurement are averaged out – giving the density state of the system as:

$$\rho_S^{n+1} = \mathrm{Tr}_A\left[\rho^{n+1}\right] = \mathrm{Tr}_A\left[\mathcal{U}\left(|0\rangle\langle 0| \otimes \rho_S^n\right)\mathcal{U}^\dagger\right] \tag{5}$$

3) The detector qubits are reinitialized to their initial states, $|0\rangle$, and the steps are repeated.

The goal is to steer the system state to a desired state $|\psi_{S\oplus}\rangle$, and hence the dynamics of $\mathcal{U}$ should be chosen such that with each iteration the state moves closer to the desired state:

$$\langle\psi_{S\oplus}|\,\rho_S^{n+1}\,|\psi_{S\oplus}\rangle \geq \langle\psi_{S\oplus}|\,\rho_S^{n}\,|\psi_{S\oplus}\rangle \tag{6}$$

Throughout this paper we will be deriving $\mathcal{U}$ such that inequality is satisfied to steer to the groundstate $|G\rangle$.

B. Preparation of Surface Codes using Quantum Steering

As outlined in Section II-D, the groundstate of the surface code Hamiltonian defines the encoding of our logical qubits. As shown in Equation 2, the groundstate may be initialized by applying projection operators to an arbitrary state. However, the realization of these projection operators is not so clear. A straightforward approach is to assume that qubits are already initialized to $|0\rangle^{\otimes n}$. Then Equation 2 simplifies to

$$|G\rangle = \prod_{i\in p}\frac{\mathbb{I}+B_i}{\sqrt{2}}|00\ldots 0\rangle = \prod_{i\in p}U_i|00\ldots 0\rangle.$$

In other words, we now have a unitary operator U_i which can be performed on a quantum computer, to prepare the groundstate for a surface code – and importantly, encode our logical qubits. To visualize the action of U_i, consider a single group of four qubits, hence the groundstate is

$$U\,|0\rangle^{\otimes 4} = \frac{(\mathbb{I}+X_1X_2X_3X_4)}{\sqrt{2}}\,|0\rangle^{\otimes 4} = \frac{1}{\sqrt{2}}\left(|0\rangle^{\otimes 4} + |1\rangle^{\otimes 4}\right).$$

This generalized Greenberger–Horne–Zeilinger (GHZ) state can be generated using Hadamard and CNOT gates. However, this only works if the initial state of the quantum computer is known, i.e. the initial state is $|0000\rangle$. In general this condition may not be true, hence requiring careful initialization of the quantum computer. In other words, if a quantum computer is in some arbitrary state $|\psi\rangle$, then the steps necessary to prepare the surface code looks like: $|\psi\rangle \xrightarrow{\text{reset}} |0\rangle^{\otimes n} \xrightarrow{U_i} |G\rangle$. We show in Section IV a method for performing groundstate initialization by going directly from $|\psi\rangle \xrightarrow{\text{steer}} |G\rangle$, without relying on the condition that the initial state is known.

IV. Quantum Steering of Surface Codes

Figure 3 provides an overview of our proposed approach compared to the existing approach. Our objective is to implement a mechanism to prepare the groundstate $|G\rangle$ which defines the logical state $|00\rangle_L$. Our implementation consists of two major tasks. The first task is outlined in Algorithm 1 that performs detector initialization. Line 1-4 compute an orthogonal space to the surface code's groundstate $|G\rangle^{\perp}$. Subsequently lines 5-11 produce projection operators that connect to the orthogonal groundstate and are used to derive the quantum circuit to implement the steering. The second task performs quantum steering to prepare the groundstate $|G\rangle$, as outlined in Section III-A. In a repeated fashion, detector and system qubits are acted upon by the quantum circuit, detector qubits are measured, and then the detector qubits are reinitialized.

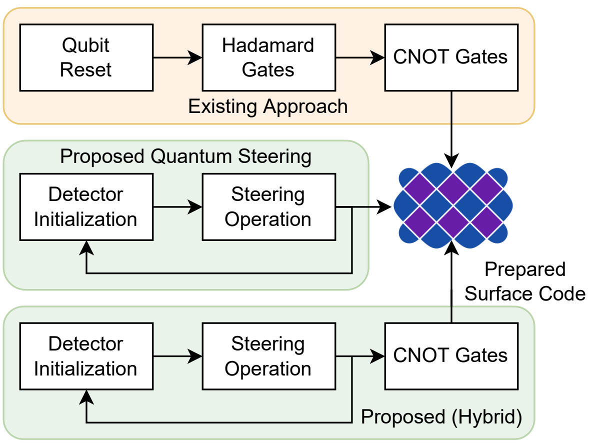


Fig. 3: Surface code preparation using the existing approach versus our proposed quantum steering approach. We also explore a hybrid approach that requires only two qubits for steering followed by application of CNOT gates.

We investigate two different approaches of applying the Quantum Steering (QS) implementation. The first implementation is a hybrid approach, utilizing QS to steer a single qubit into a superposition and then applying traditional CNOT gates to obtain $|G\rangle$. Our second implementation directly applies QS to all the qubits of the surface code to prepare the groundstate.

A. Hybrid Quantum Steering Protocol

We begin by investigating the simplest form of quantum steering which consists of two qubits: the detector qubit and the system qubit. Unlike the existing approach, where the qubits are prepared to $|0\rangle$ followed by an application of a Hadamard gate and CNOT gates, our simple protocol replaces the $|0\rangle$ initialization and Hadamard gate but still relies on a subsequent application of CNOT gates.

The target state for the system qubit is $|+\rangle = \frac{1}{\sqrt{2}}(|0\rangle + |1\rangle)$. We utilize active reset to prepare the detector qubit in the state $|0\rangle$. To construct the steering Hamiltonian, we (a) have the operator $U_s^{\dagger}|+\rangle = |-\rangle = \frac{1}{\sqrt{2}}(|0\rangle - |1\rangle)$ that maps to the orthogonal space of $|+\rangle$ and (b) the operator $O_d|0\rangle = |1\rangle$ that maps to orthogonal space of the detector's state $|0\rangle$. Now, the two operators $U_s^{\dagger} = |-\rangle\langle +|$ and $O_d = |1\rangle\langle 0|$ produce

$$\hat{H} = |1\rangle\langle 0| \otimes |+\rangle\langle -| + \text{h.c.}. \tag{7}$$

The Hamiltonian describes the dynamics of the two-qubit system and, in this case, is swapping the detector's qubit space with the system's qubit space. The two-qubit quantum circuit is then given as a matrix-exponential of the Hamiltonian $\mathcal{U} = \exp\left(-iJ\hat{H}\right)$ for some coupling strength J.

We are now ready to prepare the groundstate of the surface code. First, we employ quantum steering, which consists of: (a) entangling a detector and system qubit with the quantum circuit $\mathcal{U}$, (b) measuring the detector qubits, and (c) reinitializing the detector qubit. After N repetitions, the state of the system qubit converges, yielding the final detector-system state as $|0\rangle_D |+\rangle_S$. Because only two qubits are used, we can simultaneously prepare several qubits on the surface code via QS, and therefore, prepare a general state in the form $|\psi_{\text{surface}}\rangle = |0\rangle|+\rangle \otimes |0\rangle|+\rangle \ldots |0000\rangle$. After several qubits are prepared, we apply a sequence of CNOT gates to entangle all qubits on the surface code to obtain the groundstate $|\psi_{\text{surface}}\rangle = |G\rangle$.

B. Proposed Quantum Steering Protocol

In the previous section, we replaced the Hadamard gate and initial state preparation of $|0\rangle^{\otimes n}$ by a simple two-qubit quantum steering protocol. However, we still required CNOT gates to finish the entanglement. In this section, we develop the steering protocol that removes the explicit CNOT step, and instead inherently entangles the system and directly prepares the groundstate $|G\rangle$. The target state for the surface code is the generalized GHZ state:

$$|\psi_{\oplus}\rangle = |G\rangle = \frac{1}{\sqrt{2}}\left(|0\rangle^{\otimes n} + |1\rangle^{\otimes n}\right).$$

The states orthogonal to the target state are represented as $|G\rangle^{\perp}$, which is a subspace of dimension $2^n - 1$. Similar to the previous section, we utilize active reset to prepare detector qubits in the state $|0\rangle$. To construct the Hamiltonian we have the following: (a) operators $U_s^{k\dagger}$ that connect the desired groundstate $|G\rangle$ to a k-th state in the orthogonal subspace $|G\rangle^{\perp}$, and (b) the operator $O_d|0\rangle = |1\rangle$ that connects the detector's state to the orthogonal state. The Hamiltonian is

$$\hat{H} = \sum_k |1\rangle\langle 0| \otimes U_s^k + \text{h.c.}. \tag{8}$$

Consequently, the unitary operator is given as $U = \exp\left(-iJ\hat{H}\right)$, which is compiled into a quantum circuit $\mathcal{U}$ using standard techniques such as [22].

To prepare the groundstate of the surface code we now have a straightforward procedure: (a) perform the quantum circuit $\mathcal{U}$, (b) measure the detector qubits, and (c) reinitialize the detector qubits to $|0\rangle$ via active reset. After N repetitions the overall state of the surface code will converge to the groundstate, $|\psi_{\text{surface}}\rangle \rightarrow |G\rangle$.

V. Experiments

In this section we experimentally investigate our approach in preparing the groundstate $|G\rangle$ of the surface code. We first outline the experimental setup. Next, we present the results.

A. Experimental Setup

We explore the two implementations via simulations of surface codes, one with 17 qubits, and one with 4 qubits. Modern quantum toolchains, particularly Qiskit [23] and Cirq

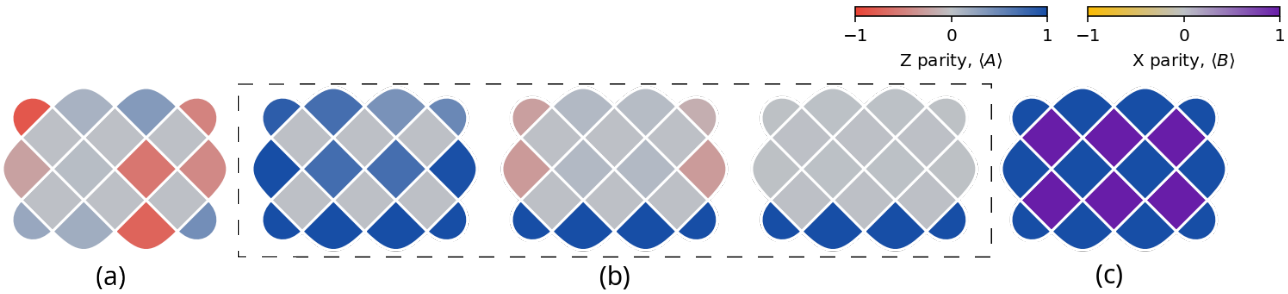


Fig. 4: Utilizing single-qubit quantum steering, and applying CNOT gates after, we prepare the groundstate $|G\rangle$ of the surface code. (a) The qubits in the surface code are given in a random initial state. (b) Three steps in the steering are shown where the qubits in the surface code converge to a superposition $\frac{1}{\sqrt{2}}(|0\rangle + |1\rangle)$. (c) Entangling CNOT gates are applied to finalize the groundstate of the surface code.

Algorithm 1: Steering Circuit

Output: Quantum circuit $\mathcal{U}$
Input: Surface code groundstate $|G\rangle$
Input: Detector state: $|D\rangle$

Find $|G\rangle^{\perp}$**:**
- Prepare projection operator: $\mathrm{P} = \mathbb{I} - |G\rangle\langle G|$
- Define the space: $S = \mathbb{I} - \mathrm{P}$
- Solve for the nullspace: $|G\rangle^{\perp} = \mathrm{null}(S)$

Prepare $\mathcal{U}$**:**
- Find operators that connect to orthogonal spaces
- **for** $k = 1$ **to** $\dim(|G\rangle^{\perp})$ **do**
  - $O_d^k = |D\rangle^{\perp}\langle D|$
  - $U_s^k = |G\rangle\langle G^k|^{\perp}$
- $\hat{H} = \sum_k O_d^k |D\rangle\langle D| \otimes U_s^k + \mathrm{h.c.}$
- Solve for $\mathcal{U} = \exp\left(-iJ\hat{H}\right)$

Done

[24], are used to prepare the quantum circuits $\mathcal{U}$ for steering, as well as to conduct simulations. Full state-vector simulations are performed, correctly evolving density matrices through measurement and state re-initialization.

The groundstate of a surface code is such that the expectation values of all vertex and plaquette operators equal to one, $\langle A_v\rangle = \langle B_p\rangle = 1$. Hence, we calculate the expectation values of all the operators and record them to track the evolution throughout simulation. ReCirq [25] is used to visualize the expectation values on a surface code.

B. Results for Two-Qubit Protocol

We construct a surface code with 2 rows and 3 columns, consisting of 17 qubits. Representative qubits are chosen, as done in [7], to be prepared into the state $|+\rangle$. Neighboring qubits to the representative qubits are selected to act as detectors. Equation 7 is used to construct a quantum circuit that will steer the representative qubits to $|+\rangle$ with the help of the detector qubits. All the representative qubits are steered simultaneously. Figure 6a visualizes the convergence of a single steered qubit on the Bloch sphere.

Figure 4 visualizes the expectation value of the surface code throughout the steering procedure. The initial state of all the qubits on the surface code is chosen at randomly (Haar random), and therefore may contain both separable and entangled states. The steering procedure is then invoked, preparing all the representative qubits to the state $|+\rangle$. Finally, CNOT gates are applied to complete the surface code initialization.

C. Results for Many-Qubit Protocol

To showcase the steering of many qubits, we simulate a four-qubit surface code. The quantum circuit that drives the steering is defined by several operators as shown in Equation 8, and steer the overall state of the surface code to the groundstate. The orthogonal space, $|G\rangle^{\perp}$, has dimension $2^4 - 1 = 15$, and hence we require 15 U_s operators to connect the state $|G\rangle$ to each of the 15 states in the orthogonal space. We prepare a random four-qubit mixed state and simulate the state until it convergences to the groundstate. Figure 5a shows the evolution of a mixed state, where the elements in the orthogonal space vanish, and the groundstate is maximized. Figure 5b visualizes the expectation values of the surface code. Figure 5c showcases the convergence of the diagonal elements of the density matrix, $\langle G|\rho|G\rangle$.

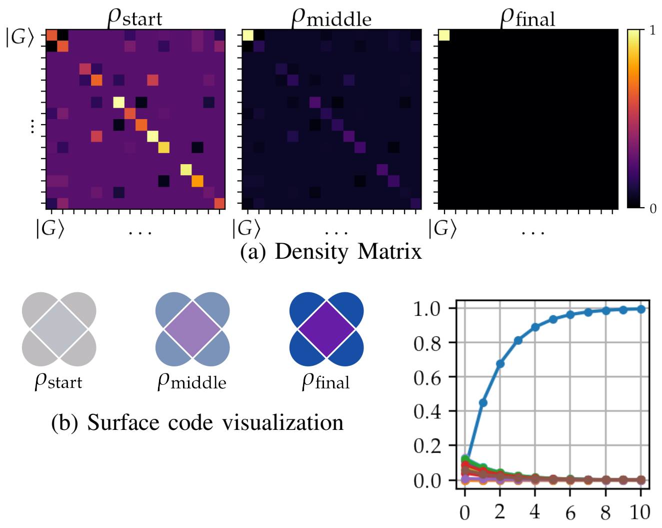
For various number of qubits, Figure 6b shows the number of iterations necessary to obtain a fidelity of $\mathcal{F} > 0.999$ given the coupling strength J. Because the desired groundstate is pure, the fidelity is computed as $\mathrm{Tr}(\rho \cdot |G\rangle\langle G|)$ for a given ρ. We note that the fastest convergence occurs when $J = \pi/2$, requiring only one iteration of the protocol. However, we note that the number of iterations is correlated with the strength of entanglement induced by the circuit $\mathcal{U}$. Therefore, depending on the underlying quantum hardware, it may be beneficial to lower the entanglement strength and perform several repetitions of $\mathcal{U}$. Examples would include photonic-based quantum computers where entangling operations are difficult to implement, but qubits have long coherence times [26], [27].

D. State Preparation Time

The time it takes to run K circuits and gather N shots (experiment repetitions) for each circuit is [28]

$$\tau^{(x)} = NK\left(\tau_{\text{reset}}^{(x)} + \tau_{\text{delay}}^{(x)} + \langle\tau_{\text{circ}}\rangle + \tau_{\text{meas}}\right),$$

where $\tau_{\text{reset}}^{(x)}$ and $\tau_{\text{delay}}^{(x)}$ are reset and post-measurement delay times. The superscript (x) indicates standard (s) or quantum

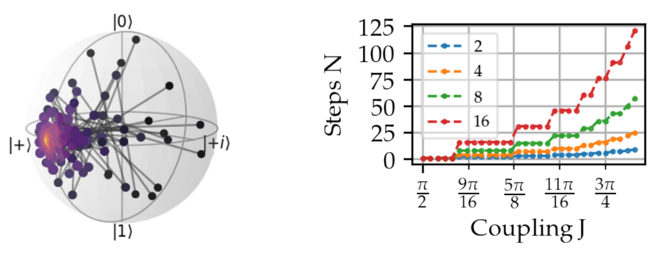


(a) Density Matrix

(b) Surface code visualization

(c) Evolution

Fig. 5: Subsystem of four qubits that are steered directly to the groundstate $|G\rangle$. (a) The density matrix of the four qubits in the basis spanned by the desired groundstate $|G\rangle$ and the orthogonal complement $|G\rangle^{\perp}$ as a colormap. The orthogonal elements decay, while the groundstate is maximized. (b) Visualization of the corresponding expectation values of A_v and B_p for the four-qubit surface code. (c) The evolution of the diagonal elements of the density matrix shows convergence to the groundstate $|G\rangle$.



(a)

(b)

Fig. 6: (a) Bloch sphere visualization of a single qubit starting in random initial states (black) and converging to $|+\rangle$. (b) Number of iterations, N, needed to achieve fidelity of $\mathcal{F} > 0.999$ given coupling strength J for various number of qubits.

steering (qs) approach. The speedup is therefore calculated as $\tau^{(s)}/\tau^{(qs)}$ which is independent on K and N.

We use data given by IBM Quantum to approximate the associated times respectively. The standard reset time is approximated as $\tau^{(s)}_{\text{reset}} = 0\mu\text{s}$, whereas $\tau^{(qs)}_{\text{reset}} = 4\mu\text{s}$ since we utilize an active-reset to reset a qubit. The repetition delay is $\tau^{(s)}_{\text{delay}} = 250\mu\text{s}$, which is the default value that is roughly twice the lifetimes of qubits. For the quantum steering, we approximate the waiting time as $\tau^{(qs)}_{\text{delay}} = 1\mu\text{s}$. Using these approximations, Figure 7 approximates the time necessary to initialize the groundstate $|G\rangle$. The simple quantum steering protocol based on two qubits has slightly shorter time compared to the existing approach. Most of the overhead in time arises from the post-application of CNOT gates. The proposed steering protocol is expensive at first, but begins to outperform the existing approach for larger quantum computers. This is due to the fact that existing approach requires many CNOT gates to initialize the surface code, while the QS approach directly applies the steering operation in one direct step (we assume the fastest version with N=1). Furthermore, the existing approach requires many measurements to ascertain high fidelity in the initial state $|0\rangle^{\otimes n}$.

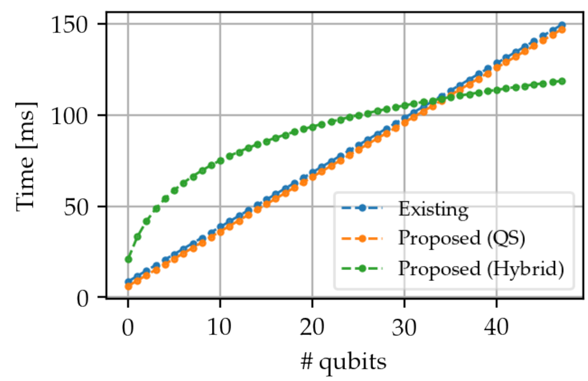


Fig. 7: Comparison of the three different approaches to initializing the surface code. The existing approach utilizes expensive resets, Hadamard gates, and CNOT gates. The two-qubit quantum steering (hybrid) slightly reduces the cost of reset, and replaces the Hadamard gate. Finally, the proposed quantum steering directly prepares the state.

VI. Conclusion and Future Work

While surface codes are a promising error correcting code for large-scale fault-tolerant computation, their encoding heavily relies on fine-tuned calibrations of a quantum computer to correctly orchestrate the state. This directly translates to significant area (requires many CNOT gates) and timing overhead (requires a computer reset). We proposed two quantum algorithms for initializing the states in a surface code via quantum steering. Both algorithms prepare the groundstate and logical states of a surface code irrespective on the initial quantum computer's state. Experimental results using simulations of surface codes, and starting in random initial states, demonstrated that our approach is scalable and correctly prepares the encoding for logical qubits. This work can be used as a stepping stone for preparing key states in other quantum error correction schemes for reliable quantum computing.

In the future, we aim to refine these algorithms, boost their efficiency and reduce overheads. We plan to widen the scope of our research to explore the application of quantum steering-based algorithms across various types of error-correcting codes, thus broadening our methodology's potential. Additionally, we are keen on assessing the viability of implementing these algorithms on actual quantum hardware. An exciting possibility we foresee is leveraging steering as a new quantum error-correcting code, much like the Petz fully recovery map [29]. This could even extend to the implementation of quantum gates via steering across a parameter space [30]. We plan to delve deeper into the theoretical basis of our approach, with the aim of better understanding the quantum dynamics involved and unearthing more avenues for optimization.

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