Modeling of Noisy Quantum Circuits using Random Matrix Theory

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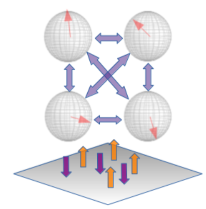
***Abstract*—A major challenge in exploiting the principles of quantum information is the influence of noise which tends to work against the quantum features of a system. The traditional quantum gate error model provides a general framework to study noisy quantum circuits, but may fail to capture intricate microscopic interactions between qubits in addition to the environment. This can significantly overestimate the feasibility of implementing useful quantum algorithms on physical quantum hardware. In this paper, we study a noise model driven by random matrix theory that captures an effective microscopic interaction, and assess the suitability of the quantum gate error model. Our approach assumes noise arises due to interactions with many physical sources, and modeled using random matrix theory and applied to the quantum circuit model.**

I. Introduction

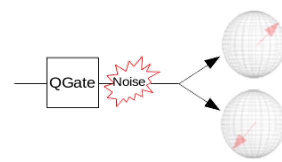
Development of quantum algorithms critically depends on evaluation using classical simulation and access to a quantum computer. Beyond fifty qubits, classical simulation to verify a theoretical quantum algorithm becomes infeasible. The only option left is to test the algorithm on a physical noisy quantum computer, and assume that studies from smaller algorithms scale with larger size. A major bottleneck in developing quantum algorithms today is how to realistically predict quantum noise using classical computers.

Engineering a real quantum computer first requires a fundamental quantum mechanical model that follows Shrödinger's equation [1]. In the last century, natural scientists have methodically developed and studied models of various quantum systems, thus bringing major insight to exploiting small quantum systems. However, for larger systems, which include the environment, modeling is strictly difficult and is further limited by computation – where, for example, 2^{50} complex floating-point numbers need to be stored and evolved. Although modeling systems via Shrödinger's equation brings the most insight, an alternative form of modeling is necessary for making feasible progress towards the development of large quantum systems.

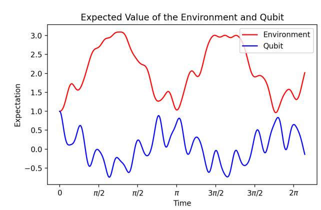
The naive alternative is to utilize the results from simple quantum systems (excluding the environment) modeled by Schrödinger's equation and introduce classical uncertainty to these results. For example, instead of yielding a final state $|\psi\rangle\,|E\rangle$ to describe a quantum device and environment, the model would imply a discrete probability of the quantum device being in the state $|\psi_1\rangle$, or in $|\psi_2\rangle$, or $|\psi_i\rangle$. Physically, these uncertainties can be attributed to unknown interactions between the system components as well as with the environment, but conceptually they arise from preparation, evolution and measurement of quantum systems as shown in Figure 1b. To actually discover these sets of probabilities, or to define the *quantum channel* from a quantum device, a thorough experiment must take place known as quantum process tomography. However, such experiments grow exponentially in resources as the system size, or number of qubits, is increased [2]. When only small subsets of the quantum devices undergo these experiments, they scale linearly in resources. These *simplified* quantum channels provide a starting-point in modeling noise, but do not completely describe the whole system nor the interactions with an environment.



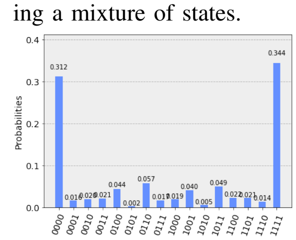
(a) The origins of noise can be studied by including the environment in the model.



(b) Without understanding the origins of noise (or without modeling environment), the effect of noise can be viewed as producing a mixture of states.



(c) An example system-environment model where classical simulation can provide information about the complete dynamics of both the system (qubit) and the environment.



(d) Results from real quantum devices is a set of probabilities corresponding to each basis state. Before measurement, the time dynamics is difficult to model, or even unknown. Following from (b), noise will change expected probabilities.

Fig. 1: Figure (a) and (c) demonstrate the advantage of a full system and environment model which is governed by Schrödinger's equation. All proprieties, including noise, are known and well-defined. The disadvantage is the exponential cost required to classically simulate these models. Figure (b) and (d) introduce a simpler viewpoint, where the environment is not modeled, and that random errors produce discrete sets of output probabilities. The advantage is that this model can be feasibly simulated, at the cost of not completely understanding noise dynamics and scalability.

From an algorithm development perspective, the interest is to continue using the comfortable and well-established framework of the quantum circuit model, but now with an included prediction of noise. Many leading quantum computing software toolchains [3] apply simplified quantum channels, such as the standard Pauli error model, to a quantum circuit to simulate noise in quantum algorithms. In particular, a quantum circuit is first synthesized to an approximately equivalent circuit using a set of universal quantum gates. The universal quantum gate set is given by the specification of a quantum device, and experiments have already been performed to yield simplified quantum channels for each of the gates, and even subsets of pairwise interacting gates, within the universal gate set. Figure 1d shows a sample result of this approach, where simulated noise clearly introduced probabilities in unwanted states.

Although these tuned simplified noise models provide an important starting point for developing algorithms and noise mitigation techniques, it remains unclear to what extent this noise prediction reflects the real quantum devices [4], [5]. In addition, recent experiments including Google's Quantum Supremacy [6] result as well as IBM Q benchmarks [7] have shown that the usual noise model are insufficient. A noise simulation may yield results that indicate successful performance of a quantum algorithm [8], yet after running it on a quantum device the results may be embarrassingly inaccurate.

This paper attempts to progress the methodology in software toolchains to start addressing the uncertainty between noise simulation and real quantum computation. The general approach is to use an approximation, as studied by natural scientists, to model an environment and system interactions, and apply it to simulations of quantum circuits. This presents a different mindset from the simplified quantum channels – the results from noise simulation now have a concrete interpretation and connection with Schrödinger's equation. The approach is also connected via proofs establishing equivalency between the viewpoints of environment modeling and quantum channels [9]–[11]. Specifically, in this paper, we make the following major contributions:

1) We model many-qubit and arbitrary gate operations under the physical assumption that there are many independent noise sources.
2) We introduce the noise model form the viewpoint of a natural scientist, via the Schrödinger's equation, and incorporate it with the quantum circuit representation.
3) We run the quantum Fourier transform algorithm showcasing our noise model, comparing it to the popular simplified quantum channels.

The rest of the paper is organized as follows. Section II introduces some concepts used in this paper and presents related efforts. Section III describes our proposed modeling of noise in quantum circuits. Section IV presents the experimental results. Finally, Section V concluding the paper.

II. Background and Related Work

In this section, we first define relevant notions for analyzing noise in a quantum circuit. Next, we present the related research to highlight the novelty of our approach.

A. Background and Preliminaries

1) Quantum Evolution: The time evolution of a pure quantum state $|\psi\rangle$, given a Hamiltonian H, and setting $\hbar = 1$ (this is commonly done in quantum mechanics for simplicity), is:

$$i\frac{\mathrm{d}}{\mathrm{d}t}|\psi(t)\rangle = H|\psi(t)\rangle \tag{1}$$

For a given time τ, the time evolution can be solved for a unitary operator U:

$$U = e^{-iH\tau} \tag{2}$$

This will give the resulting evolution for $|\psi(\tau)\rangle$ as:

$$|\psi(\tau)\rangle = U|\psi(0)\rangle = e^{-iH\tau}|\psi(0)\rangle \tag{3}$$

2) Quantum Circuit: A quantum circuit with a unitary quantum gate U acting on n-qubits, all initialized to $|0\rangle$, is expressed as:

$$|0\rangle^{\otimes n} \;-\!\boxed{U}\!-\; U(|0\rangle^{\otimes n}) \tag{4}$$

The final state of a quantum circuit can be expressed as a result of a time evolution from Equation 3:

$$|\psi(\tau)\rangle = e^{-iH_S\tau}(|0\rangle^{\otimes n}) \tag{5}$$

where the system Hamiltonian H_S can now be found from the quantum gate U by:

$$H_S = i\frac{\log U}{\tau} \tag{6}$$

Equation 5 and Equation 6 provide a mechanism to view a quantum circuit as the action of a physical quantum evolution driven by an effective system Hamiltonian.

3) Gate Accuracy: If the gate operation U_S is instead implemented by unitary $\widetilde{U_S}$ then the accuracy is defined as:

$$E(U_S,\widetilde{U_S}) = \max_{|\psi\rangle}\left\|(U_S - \widetilde{U_S})|\psi\rangle\right\| \tag{7}$$

The defined accuracy captures the maximum difference in probabilities for an outcome of measurement outcomes between U and $\widetilde{U}$ [12].

Moreover, if we have a series of quantum gates such as U_2U_1 which is implemented by $\widetilde{U_2U_1}$ then the following holds:

$$E(U_2U_1,\widetilde{U_2}\widetilde{U_1}) \leq E(U_2,\widetilde{U_2}) + E(U_1,\widetilde{U_1}) \tag{8}$$

The consequence is that accuracy deteriorates in a series of gates since the overall error depends on the summation of the respective gate errors.

In addition to gate accuracy, we also use fidelity to quantify how close two quantum states are. The fidelity between quantum states $|\psi\rangle$ and $|\phi\rangle$ is defined as:

$$F(\psi,\phi) = |\langle\psi|\phi\rangle|^2 \tag{9}$$

An approximation of fidelity is achieved by measuring in several basis on a quantum computer, and classical fidelity is computed over the expected and obtained probability distributions. If two states are equivalent, the fidelity $F = 1$. On the other hand, if two states have absolutely no overlap, the fidelity is $F = 0$.

4) Quantum Gates and Matrix Representation: Given a basis, any quantum gate U can be represented as a unitary matrix. Here we assume the standard computational basis for a qubit: $|0\rangle = \begin{bmatrix}1 & 0\end{bmatrix}^T$ and $|1\rangle = \begin{bmatrix}0 & 1\end{bmatrix}^T$. Hence the matrix to describe a single-qubit gate is simply the action of the gate on the two basis elements.

For representing multiple qubits we follow the standard convention. The basis for a multiple-qubit system arises from the tensor products of the single qubit-basis. For instance, an example state of three qubits $1, 2$, and 3 could be: $|0\rangle_1 \otimes |1\rangle_2 \otimes |1\rangle_3 = |011\rangle$.

To algorithmically construct the matrix representation of multiple-qubit gate, we use projection operators. For a controlled-not gate operating on 3-qubits, where the first qubit is control and third qubit is target, the construction is as follows:

$$\begin{aligned} &|0\rangle_1 \langle 0|_1 \otimes (|0\rangle_1 \langle 0|_1 + |1\rangle_1 \langle 1|_1) \otimes (|0\rangle_2 \langle 0|_2 + |1\rangle_2 \langle 1|_2) \\ &\qquad\qquad + \\ &|0\rangle_1 \langle 0|_1 \otimes (|0\rangle_1 \langle 0|_1 + |1\rangle_1 \langle 1|_1) \otimes (|0\rangle_2 \langle 1|_2 + |1\rangle_2 \langle 0|_2) \end{aligned} \tag{10}$$

5) Open Quantum Systems: Equation 5 describes a perfect unitary evolution given by a Hamiltonian H_S for a closed system. In practice, the overall time evolution may be imperfect. Hence, H_S will have additional interactions with the environment, which gives rise to a general Hamiltonian:

$$\widetilde{H} = H_S + H_B + H_{SB} \tag{11}$$

where H_B is the evolution of the bath (environment), and H_{SB} is the interaction between the system and the bath. Equation 11 defines an open quantum system.

6) Noisy Quantum Circuits: In an ideal scenario, a quantum circuit – defined as a series of quantum gates – will take an initial state to a final desired state. In other words, a final quantum state $|\psi_f\rangle = U|\psi_i\rangle$ is produced by applying a quantum circuit U to an initial quantum state $|\psi_i\rangle$. Of course, in reality, quantum gates are implemented as *real* quantum systems that undergo time evolution, and are subject to the cruel interactions with an environment. Specifically, a gate and the environment evolve as an open quantum system: $\widetilde{U} = e^{-i\widetilde{H}t}$. Tracing out the environmental degrees-of-freedom from the composite evolution $\widetilde{U}$ yields the effective noisy quantum gate. Therefore, in general, the final state of the system $\widetilde{U}|\psi_i\rangle$ differs from the ideal case $|\psi_f\rangle$. The *amount* by which the ideal and realistic gate differ is given by average gate fidelity, or gate accuracy as introduced in Section. II-A3.

Studying $\widetilde{U}$ has given rise to various general situations that are found in quantum computers today. For example, a common assumption is that the gate's interaction with the environment is *Markovian* – that is, the evolution with an environment is described in terms of a memoryless dynamics which leads to an irreversible loss of characteristic quantum features. Within this example of open quantum systems, noisy quantum effects, such as *decoherence* can be concretely defined as an irreversible loss of purity of the quantum gate.

B. Related Work

Standard noise models studied during the mid-90s are suitable for small scale, computationally insignificant, quantum computers. These models were first studied from a physics point of view, where notions such as a dissipative environment were introduced. Examples include the treatment of two-level systems interacting with an effective harmonic oscillator potential [13]. Over time, physicists developed better techniques for studying quantum decoherence. These methods include the master equation [14], the stochastic Schrödinger equation [1], and the Belavkin equation [15]. These techniques allow for a study of a continuous time evolution of a system with an environment. They include some assumptions about the physical interaction, predominantly assuming that the time evolution of the system will depend solely on the previous state (Markovian). Even with such thorough analysis of quantum decoherence, the complex interaction between many independent two-state systems and their interactions with an environment remains a challenging problem.

In the late-90s, as quantum information matured, models of small-scale noise on quantum algorithms were introduced in terms of quantum channels [16]. For example, error correction schemes, such as the Stabilizer code discussed by *Shor* [17], assume a quantum channel where discrete errors are from a Pauli group [18]. Recent work attempts to unite the different viewpoints from physics and quantum information, such as the work by *Aurell* [19].

Research on random matrix theory is vast. *Stöckmann* succinctly covers the subject matter within the context of quantum chaos [20]. There are initial efforts in using random matrix theory to study quantum noise [21]–[23]. *Roland et al.* study adiabatic quantum computing with noise modeled using random matrix theory [24]. Entangled two qubit dynamics coupled to a random matrix environment has been recently studied by *Braus et al.* [25]. The main novelty of this paper is to directly apply quantum noise approximation from random matrix theory to the quantum circuit model.

III. Modeling of Noisy Quantum Circuits

Within the physical sciences, modeling and defining a complete quantum system with addition to the environment is a difficult task. Such a task can be infeasible for a complex system. However, such an approach can yield a clear explanation to the behavior of the system. On the other hand, using existing simplified quantum channels to model noise is feasible, but limits the understanding of the actual system, thus hindering development in algorithms and noise mitigation. This section is organized as follows. First, we provide a brief overview of random matrix theory and the motivation for this work. Next, we describe our proposed noise model. Finally, we discuss modeling of errors in a single-qubit gate and multi-qubit gates using the proposed noise model.

A. Random Matrix Theory Overview and Motivation

A random matrix is a matrix in which all elements are random variables. In the context of modeling noisy quantum circuits, the random matrices are symmetric matrices of size $N \times N$ obtained as $(A + A^T)/2$ where the entries for A are sampled from the Gaussian distribution: $\mathcal{N}(0, \sigma^2)$. An ensemble of such matrices is referred to as the Gaussian orthogonal ensemble (GOE). An important property of GOE is that each matrix in the ensemble has orthogonal invariance. Namely, for each random matrix, the norm will remain invariant under orthogonal transformation. Therefore, given orthogonal transformations Q_1 and Q_2, and a random matrix A, $||Q_1 A|| = ||A|| = ||A Q_2||$.

The distribution of eigenvalues of the random matrices follow *Wigner's Semicircle Law* as the size $N \to \infty$ [26]. This is an important result as one can study the behavior of the eigenvalues without ever having to compute the eigenvalues directly. The density of eigenvalues of GOE matrices is:

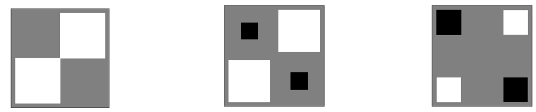
$$\rho(E) \xrightarrow[N \to \infty]{} \frac{1}{4\sigma^2 \pi} \sqrt{4\sigma^2 N - E^2} \tag{12}$$

Within the quantum evolution setting, if we combine many physical sources of errors, as a consequence of the central limit theorem, we can view noise as sampling a random Hamiltonian from a GOE. This is the primary motivation of our proposed work in utilizing GOE to model noise within a quantum circuit.

B. Noise Model

Suppose we have a quantum gate that performs the unitary transformation U. From Equation 5 we can view U as being part of a physical time evolution given by a Hamiltonian H_S for some time τ. To capture a physical noisy process, H_S is included as part of an open quantum system. To model the imperfection, we assume a coupling with a GOE defined by a diagonal matrix $\boldsymbol{\theta}$ where each diagonal entry represents the amount coupling for a given basis with the GOE. Since the square-matrix elements in the GOE can be of arbitrary size, additional zeros are added to the diagonal of $\boldsymbol{\theta}$. This diagonal matrix representation of $\boldsymbol{\theta}$ is used for convenience, but is not a unique representation. Within GOE, all matrix entries are independently and identically Gaussian distributed, hence the probability of each column is only dependent on the column vector's norm. Since the norm of each column is preserved due to the GOE property of orthogonal invariance, the matrices are then basis-independent. Thus, rearranging the columns of $\boldsymbol{\theta}$ will be an equivalent representation. In block-matrix form, the Hamiltonian is hence modeled as:

$$\widetilde{H} = \left(\begin{array}{c|c} H_S & \boldsymbol{\theta} \\ \hline \boldsymbol{\theta} & GOE \end{array}\right) \tag{13}$$



(a) $U = X$ (b) $\mathrm{Re}(\widetilde{U})$ and $\mathrm{Im}(\widetilde{U})$

Fig. 2: Hinton plots of the original gate X (2a) and of the real and imaginary components of the noisy gate $\widetilde{U}$ (2b). The difference between (a) and (b) is the introduced noise.

With Equation 2, we now evolve $\widetilde{H}$ to obtain a unitary operator that captures the evolution of both the original quantum gate as well as the environment:

$$\widetilde{U_{SB}} = e^{-i\widetilde{H}\tau} \tag{14}$$

To view the new effective unitary operator $\widetilde{U}$, we disregard the GOE subspace from $\widetilde{U_{SB}}$. The new operator will now be unnormalized with the magnitude depending on both the coupling strength and size of the random matrix.

C. Example: Single Qubit, Bit-Flip Gate

Assume we operate on a single-qubit with a bit-flip gate, represented as follows:

$$U = X = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} \tag{15}$$

For time $\tau = 1$, $H_S = i \log X$, and assuming the size of matrices in GOE are, for example, 3×3 then the effective $\widetilde{H}$ is:

$$\widetilde{H} = \left(\begin{array}{ccc|ccc} & & & \theta_0 & 0 & 0 \\ & i\log X & & 0 & \theta_1 & 0 \\ & & & 0 & 0 & 0 \\ \hline \theta_0 & 0 & 0 & & & \\ 0 & \theta_1 & 0 & & GOE & \\ 0 & 0 & 0 & & & \end{array}\right) \tag{16}$$

The evolution $e^{-i\widetilde{H}}$ leads to interaction between the single-qubit gate and environment. By ignoring the GOE subspace, we obtain the noisy single-qubit gate $\widetilde{U}$ as shown in Figure 2b.

Now, increasing the GOE matrix size, beyond 3×3, brings forth Wigner's Semicircular Law. This limit conforms to several observed and studied phenomena in physics [20]. The relationship between the gate error $E(X, \widetilde{U})$ and coupling strength, where the GOE matrix size is large ($\approx 10,000$), is shown in Figure 3. Moreoever, Figure 3 also compares with one an often used noise model, the depolarizing channel,

$$\mathcal{E}(\rho, p) \mapsto (1 - p)\rho + \frac{p}{2} I \tag{17}$$

which use the density matrix ρ to represent uncertainty in measurement outcome.

D. Multi-Qubit Gates

The primary focus of this section is to define the construction of U, which represents the matrix that is applied to the state of all qubits. This is not straightforward, since

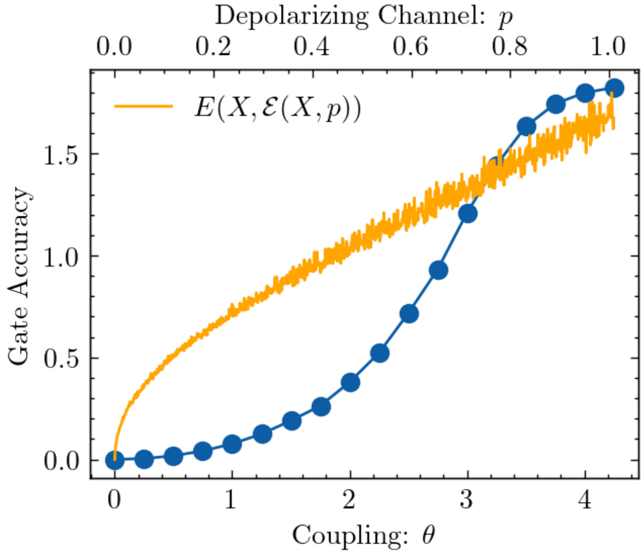


Fig. 3: Gate error, using Equation 7, of X subject to the GOE noise model with varying coupling parameter as well as the accuracy subject to the simplified depolarizing noise channel. Smaller values means better gate accuracy. Hence, the GOE model predicts a larger gate error at low coupling strength, while the depolarizing channel gives better accuracy at the lower parameters.

restrictions resulting from time-based ordering, or assumptions imposed by physical devices, lead to limitations. For example, an operation on one qubit might restrict simultaneous operation on another qubit. After imposing specific assumptions, the matrix U will then be used to find a system Hamiltonian H_S, followed by the procedure outlined in Section III-B to introduce noise and obtain an accuracy.

Within the context of the circuit model, we may have gates in sequence, in parallel, or in a combination of both. For simplicity, we assume that the qubits are fully-connected – any arbitrary qubit can interact with any other arbitrary qubit. Although at the moment this is not a physically realistic assumption, the results remain insightful, as quantum compilers can remap a circuit description to a limited-connectivity qubit layout via an additional overhead of gates. In addition, we decompose any multi-qubit gate into the action of single-qubit unitary gate with two-qubit entangling controlled-not (CX) gates. This is a realistic decomposition as quantum processors are usually fixed to a basis set consisting of single-qubit gates and one or two entanglement gates. We use *staq* to synthesize a circuit written in openQASM to a circuit that only contains single-qubit gate operations and CX gates.

In order to accommodate the different scenarios seen in a quantum circuit, we view the absence of a gate as the action of the identity matrix. We couple all gates, including the identity, if they are applied in parallel. This now yields a set $\{U^{(i)}\}_{i=0}^{L-1}$, which contains L-instances of Equation 5, each denoting a distinct step in the time when applying a set of gates in parallel. After doing so, we use Equation 13 to construct a set of system Hamiltonians $\{H_S^{(i)}\}$ for gates from time $t = \tau_i$ to $t = \tau_{i+1}$, where i denotes an ordered slice of the circuit sequence. For example, following from Figure 4, there are 3-sequences: $\{U_l, I \otimes U_i \otimes U_j, I \otimes I \otimes U_k\}$ with the ordering i = 1, 2, 3. Each element in the sequence is then made into a system Hamiltonian.

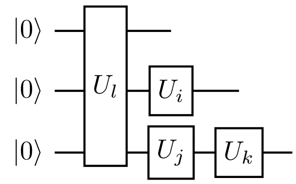


Fig. 4: An example configuration consisting of both sequential and parallel application of gates.

Figure 5 shows the overall framework. The set of system Hamiltonians (as discussed above) are combined with GOE for validation and evaluation of our proposed model, as described in the next section.

IV. Case Studies

A one-qubit case study was performed in the previous section by comparing GOE with the standard depolarizing channel. This section looks at a specific algorithm to assess several qubits and multi-qubit gates. The study highlights some of the differences between our model and built-in noise models as found in popular software packages. First, we describe our experimental setup. Next, we outline the limitation of existing noise models using QFT algorithm.

A. Experimental Setup

To represent a quantum circuit we use openQASM [27], an intermediate- to low-level language with similarities to classical hardware description languages. It provides a small number of programming features: declaration of registers, definitions of unitary gates, gate application, and measurement. Specifically, we use *staq* [28] – a C++17 toolkit that is designed to be minimalistic and includes the latest state-of-the-art methods in quantum compiling. We use *staq* to parse openQASM, perform desugaring and inlining, and traverse the abstract syntax tree to inject noise. In addition, we use Qiskit [3] for high-level circuit generation as well as simulating circuits with built-in noise models. Figure 5 outlines the overall approach.

B. Quantum Fourier Transform

The quantum Fourier transform (QFT) follows from the familiar Fourier transform, but makes use of superposition to encode the phase information directly in qubits. As with the Fourier transform, QFT performs the mapping:

$$|x\rangle \mapsto \frac{1}{\sqrt{N}} \sum_{k=0}^{N-1} y_k \omega_N^{xk} |k\rangle \tag{18}$$

where ω_N^n is the Nth root of unity. Via standard treatment, the mapping is then decomposed into a quantum circuit consisting of Hadamard gates and controlled-phase gates. Using *staq* the circuit is then further transformed into single and

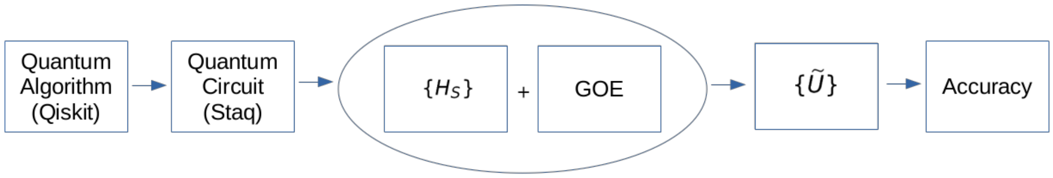


Fig. 5: Qiskit is used to generate initial openQASM code from a high-level description of a quantum algorithm. The code is then passed to modified libraries from Staq, which produces an array of Hamiltonians. The Hamiltonians are then coupled with GOE and evolved in time. An array of effective unitaries is then extracted, and the overall accuracy is calculated.

two qubit gates. Since the quantum gates cannot be applied all at once, using the fully-connected layout assumption and procedure outlined in Section III-D, the number of time steps required to perform all the gates is 26.

Using *staq*, the gates are then mapped to their corresponding matrix representation, leading to 26-matrices of size $2^4 \times 2^4$. These matrices are then coupled with GOE, and the noisy gate is then stored. A state is then initialized and passed through the noisy quantum circuit. The fidelity of the state is computed by comparing with a noiseless simulation. Figure 6 shows the fidelity, between the expected state and the noisy state, after each matrix from the 26-matrices.

We have also evaluated the same quantum circuit on a quantum computer *ibmq_valencia* [3]. The classical fidelity of the final output probability distribution is computed against the expected output probability. Similarly, the quantum circuit is simulated using the noise model fitted for *ibmq_valencia*. As shown in Figure 6, the quantum computer had close-to-perfect accuracy, while the associated noise model under-predicted the accuracy. Our proposed GOE model with $\theta = 0.01$ aligned with the real computation accuracy. Due to the inconsistency in existing noise models, these results motivate the need for a circuit-level noise model that follows from underlying assumptions of the quantum hardware. In our proposed model, with large GOE matrix size, the collective effect of noise is encoded via *Wigner's Semicircle Law* which represents the action of several random sources of quantum interaction. In other words, for quantum devices where the interaction with several environmental sources can be sufficiently sampled, the GOE model can quickly predict a gate error.

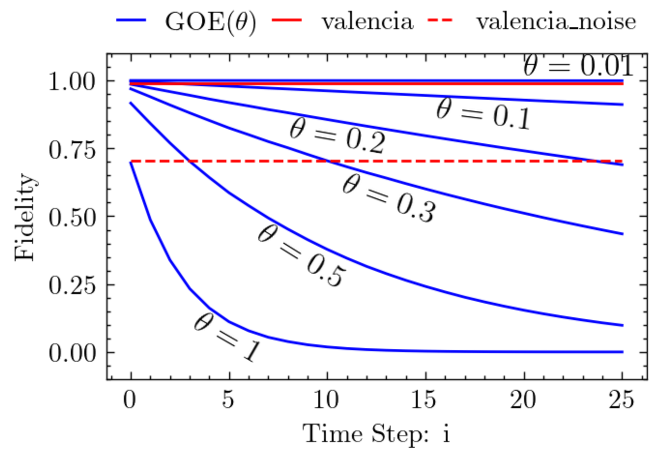


Fig. 6: The state fidelity over the course of the QFT algorithm with varying coupling strength θ to GOE. It compares with the final-state fidelity from the quantum computer *ibmq_valencia* as well as the final-state fidelity as produced by the noise model of *ibmq_valencia*. The *ibmq_valencia* uses simplified noise models that have been specifically fitted with large accuracy for single-qubit gates and a multi-qubit. The disadvantage is that when combined to a larger quantum system, the custom fitted models do not sufficiently cover other sources of noise. Instead quantum process tomography must be done to obtain a complete noise model, which requires exponential resources. Our proposed GOE model can provide a reasonable first guess to the extent of noise by fitting one parameter to an appropriate system.

V. Conclusion

Quantum computers provide a promising avenue to solve many real-world problems in a reasonable time that requires exponential time in conventional computers. Development of quantum algorithms critically depends on an accurate noise model of the physical quantum computer. Unfortunately, the current practice considers simplified noise models while developing quantum algorithms, which can lead to inconsistencies. In this paper, we developed a quantum noise model driven by random matrix theory that aims to compromise between the difficulty in modeling quantum systems and the simplified noise models used in quantum information science. Based on this noise model, we have developed error models for both single-qubit and multi-qubit gates in order to model noise in quantum circuits. Our case studies connected physical assumptions about noise behaviors in physical quantum computer with simplified noise model considered by algorithm developers today. Our work highlighted the weak approximations of existing noise models where modeling many-qubit noise is difficult, and motivated the need for developing realistic circuit-level noise models based on physical assumptions.

This work can be extended to analyze larger quantum systems. The noise modeling can be improved by employing quantum process tomography techniques to investigate the contribution of random sources of errors, and more generally, to discern the differences in simplified noise models with real quantum results. Future work includes investigating and benchmarking quantum algorithms under the influence of noise, and developing appropriate encodings for quantum error correction that optimize overhead with respect to the noise models.

Acknowledgments

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