Refinable bi-quartics for design and analysis

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Input – Catmull-Clark (CC) nets

B-spline (CC) net
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B-spline (CC) net

bicubic ring + tensor-border of degree 3
Guided subdivision

CC-net, $n = 5$
Guided subdivision

CC-net, $n = 5$

bicubic ring
Guided subdivision

CC-net, $n = 5$

bicubic ring

guide
Guided subdivision

- CC-net, \( n = 5 \)
- Bicubic ring
- Guide
- Of degree bi-4
Guided subdivision

CC-net, \( n = 5 \)

bicubic ring

guide

of degree bi-4

bicubic ring + guide
Guided subdivision

CC-net, $n = 5$

bicubic ring

guide

of degree bi-4

bicubic ring + guide

guided rings
Guided subdivision

CC-net, $n = 5$

bicubic ring

guide

of degree bi-4

bicubic ring + guide

guided rings

highlight lines
Two crossing beams
Two crossing beams

mesh

CC refinement
Two crossing beams
Two crossing beams

mesh  CC refinement  layout  highlight lines
Two crossing beams

- Mesh
- CC refinement
- Layout
- Highlight lines

- CC-net
- 6 rings + cap
Two crossing beams

mesh
CC refinement
layout
highlight lines

CC-net
6 rings + cap
guided
Two crossing beams

mesh  CC refinement  layout  highlight lines

CC-net  6 rings + cap  guided  Catmull-Clark
Assembling Bézier patches from corner jets

\[
\begin{pmatrix}
\frac{\partial^2 f}{\partial v^2} & \frac{\partial u \partial^2 f}{\partial v} & \frac{\partial^2 u \partial^2 f}{\partial v^2} \\ 
\frac{\partial^2 f}{\partial v} & \frac{\partial u \partial^2 f}{\partial v} & \frac{\partial^2 u \partial^2 f}{\partial v} \\ 
\frac{\partial f}{\partial v} & \frac{\partial u \partial f}{\partial v} & \frac{\partial^2 u \partial f}{\partial v}
\end{pmatrix}
\rightarrow
\text{Hermite data in Bernstein-Bézier form}
\]
Assembling Bézier patches from corner jets

\[ \begin{pmatrix} \frac{\partial^2 f}{\partial v^2} & \frac{\partial u \partial^2 f}{\partial v \partial u} & \frac{\partial^2 f}{\partial u^2} \\ \frac{\partial f}{\partial v} & \frac{\partial u \partial f}{\partial v \partial u} & \frac{\partial^2 f}{\partial u^2} \\ f & \frac{\partial f}{\partial u} & \frac{\partial^2 f}{\partial u^2} \end{pmatrix} \rightarrow \text{Hermite data in Bernstein-Bézier form} \]
Assembling Bézier patches from corner jets

\[
\begin{pmatrix}
\frac{\partial^2 f}{\partial v^2} & \frac{\partial u \partial^2 f}{\partial v \partial u} & \frac{\partial^2 f}{\partial u^2} \\
\frac{\partial v f}{\partial v} & \frac{\partial u \partial v f}{\partial u \partial v} & \frac{\partial^2 v f}{\partial u^2} \\
f & \frac{\partial v f}{\partial u} & \frac{\partial^2 v f}{\partial u^2}
\end{pmatrix}
\rightarrow
\]

Hermite data in Bernstein-Bézier form

bi-4

assembled tensor-border
Characteristic map of Catmull-Clark subdivision
Characteristic map of Catmull-Clark subdivision as sampling tool
Characteristic map of Catmull-Clark subdivision as sampling tool

guide

sampled rings
Guide of degree bi-4

preguide of total degree 4: piecewise $C^1$; $C^2$ at central point
Guide of degree bi-4

preguide of total degree 4: piecewise $C^1$;
$C^2$ at central point

increasing flexibility with linear shear $L$: 
Guide of degree bi-4

preguide of total degree 4: piecewise $C^1$; $C^2$ at central point

increasing flexibility with linear shear $L$: preguide $\circ L$
Guide of degree bi-4

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$3 \times 3$
Guide of degree bi-4

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increasing flexibility with linear shear $L$:
preguide $\circ L$
$3 \times 3 \bullet$
$13n + 6$ dof
Guide of degree bi-4

preguide of total degree 4: piecewise $C^1$; $C^2$ at central point

increasing flexibility with linear shear $L$: preguide $\circ L$
$3 \times 3 \bullet$
$13n + 6$ dof
$6n + 1$ of CC
Characteristic parameterization
Characteristic parameterization
Characteristic parameterization for sampling
Characteristic parameterization for sampling

\[ L^{-1} \]

sampled \( C^1 \)
Characteristic parameterization for sampling

- Sampled $C^1$
- $C^2$ correction
Characteristic parameterization for sampling
Assembling bi-4 rings

tensor-border → $C^1$

The quality of $C_1$ and $C_2$ surfaces is alike; $C_1$ has more analysis functions, more sparse analysis matrix. By contrast, in regular bi-3 case: more $C_1$ functions, more dense analysis matrix.

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Assembling bi-4 rings

tensor-border

$C^1$

$C^2$

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Assembling bi-4 rings

tensor-border

$C^1$

$C^2$

macropatches internally $C^3$
Assembling bi-4 rings

quality of $C^1$ and $C^2$ surfaces is alike;
Assembling bi-4 rings

tensor-border

\[ \rightarrow \]

\( C^1 \)

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Reformulation towards traditional subdivision

Original dof

New structure
Reformulation towards traditional subdivision

Original dof contains almost all data for assembling bi-4 rings.

New structure

⊿ Fewer arithmetic operations ⇒ faster evaluation;

⊿ new refinement is akin to traditional subdivision;

⊿ considerably larger precalculated stencils;

⊿ considerably better quality.
Reformulation towards traditional subdivision

Original dof contains almost all data for assembling bi-4 rings.

of tensor-border: ○ are defined by ● and ●;

Completion

Fewer arithmetic operations ⇒ faster evaluation;

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Reformulation towards traditional subdivision

Original dof contains almost all data for assembling bi-4 rings. of tensor-border: ◦ are defined by ● and ○; averaging;

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 jScrollPane少吃 arithmetic operations ⇒ faster evaluation;
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Central $G^1$ bi-4 cap

well-defined curvature at eop;
$C^1$ connection to last guided ring
Central $G^1$ bi-4 cap

well-defined curvature at eop; $C^1$ connection to last guided ring

$$\partial \hat{f}_v + \partial \hat{f}_v - (2c(1 - u) + \frac{2}{3} cu) \partial \hat{f}_u = 0$$

$$\partial \hat{f}_v + \partial \hat{f}_v - \frac{2}{3} c(1 - u)^2 \partial \hat{f}_u = 0$$
Homogeneous functions I

Homogeneous function of degree $d$: $F(\lambda x) = \lambda^d F(x)$

$d = 4$
Homogeneous functions I

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d = 4
Homogeneous functions I

Homogeneous function of degree $d$: $F(\lambda x) = \lambda^d F(x)$

$d = 4$

$d = 3$
Homogeneous functions $I$

Homogeneous function of degree $d$: \( F(\lambda x) = \lambda^d F(x) \)
Homogeneous functions II
Homogeneous functions II

$\mathbf{d} = 4$

$\mathbf{d} = 5$

$\mathbf{d} = 6$

$\mathbf{d} = 7$

$\mathbf{d} = 8$
Homogeneous functions II

\[ d = 4 \]
\[ d = 5 \]
\[ d = 6 \]
\[ d = 7 \]
\[ d = 8 \]
Eigenfunctions

Top row: $d = 2$ (hyperbolic shape); bottom row: $d = 3$
Eigenfunctions

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eigen-guide
eigen-ring
Eigenfunctions

Top row: $d = 2$ (hyperbolic shape); bottom row: $d = 3$
Eigenfunctions

Top row: \( d = 2 \) (hyperbolic shape); bottom row: \( d = 3 \)

eigen-guide  
eigen-ring  
eigen-cap  
surface:

eigen-ring scaled by \( \lambda^s \), \( s = 0, 1, \ldots, m - 1 \) and eigen-cap scaled by \( \lambda^m \); \( \lambda \) is subdominant eigenvalue of Catmull-Clark subdivision.
Convex shape

CC-net, $n = 5$
Convex shape

CC-net, $n = 5$ layout
Convex shape

CC-net, $n = 5$

layout

Gauss curvature

highlight lines
Convex shape

CC-net, $n = 5$

CC-net, $n = 6$

Catmull-Clark

layout

Gauss curvature

highlight lines
Convex shape

CC-net, $n = 5$

CC-net, $n = 6$

Catmull-Clark

guided after one CC refinement

layout

Gauss curvature

highlight lines
Convex shape

CC-net, $n = 5$

CC-net, $n = 6$

Catmull-Clark guided after one CC refinement default
Exotic shape (Mitsubishi logo)

CC-net, $n = 9$
Exotic shape (Mitsubishi logo)

CC-net, \( n = 9 \)

6 guided rings

+ cap
Exotic shape (Mitsubishi logo)

CC-net, \( n = 9 \)

6 guided rings + cap

8 guided rings + cap
Examples

Exotic shape (Mitsubishi logo)

CC-net, $n = 9$

6 guided rings + cap

8 guided rings + cap

highlight lines
Dominant multi-sided surfaces

mesh, $n = 6$
Dominant multi-sided surfaces

mesh, $n = 6$

layout
Examples

Dominant multi-sided surfaces

mesh, \( n = 6 \)

layout

highlight lines
Examples

Dominant multi-sided surfaces

mesh, $n = 6$

layout

highlight lines
Examples

Dominant multi-sided surfaces

mesh, $n = 6$

layout

highlight lines
Dominant multi-sided surfaces

mesh, $n = 6$

layout

highlight lines
Examples

Refinability: embossing the details

CC-net, $n = 8$  
Catmull-Clark  
default
Refinability: embossing the details

CC-net, $n = 8$

Catmull-Clark

default
Summary

New class of smooth high quality bi-4 surfaces using

• subdivision ⇒ refinable $C^1$ ($C^2$) surfaces;
• guided subdivision + $G^1$ central cap ⇒ good highlight line distribution.

Thank you!
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- Built-in eigen-structure characterized and determined by the guide.
New class of smooth high quality bi-4 surfaces using

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