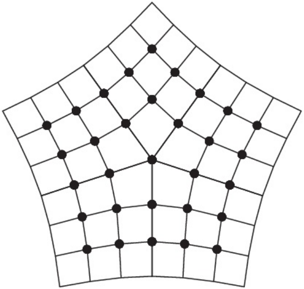
Refinable bi-quartics for design and analysis

Kęstutis Karčiauskas
Vilnius University

Jörg Peters
University of Florida

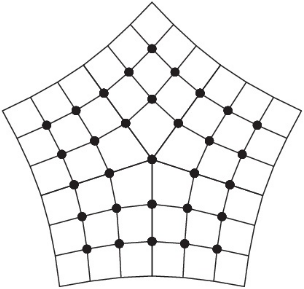
Outline

Input – Catmull-Clark (CC) nets



B-spline (CC) net

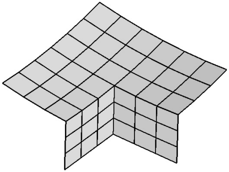
Input – Catmull-Clark (CC) nets



B-spline (CC) net

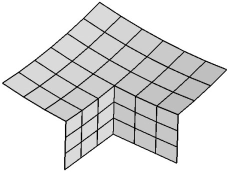
bicubic ring + tensor-border of degree 3

Guided subdivision



CC-net, $n = 5$

Guided subdivision



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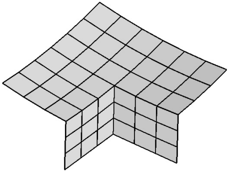
bicubic ring

Guided subdivision

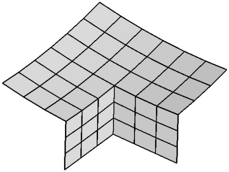


CC-net, $n = 5$

bicubic ring

guide

Guided subdivision



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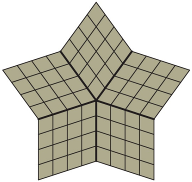
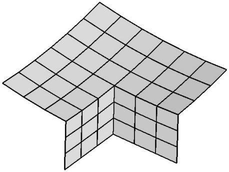


of degree bi-4

Guided subdivision



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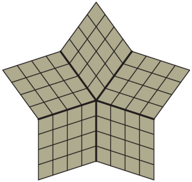

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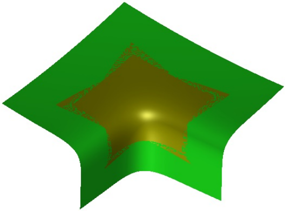
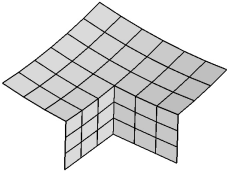


bicubic ring + guide

Guided subdivision



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bicubic ring

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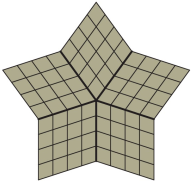

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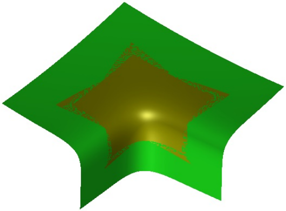
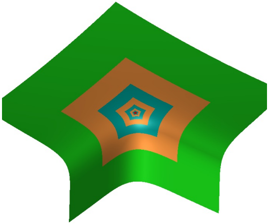
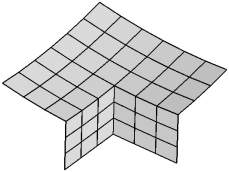


bicubic ring + guide



guided rings

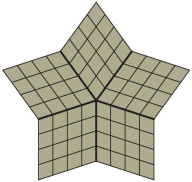
Guided subdivision



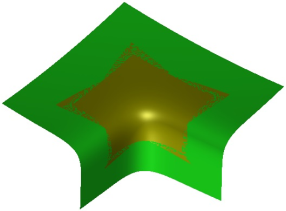
CC-net, $n = 5$

bicubic ring

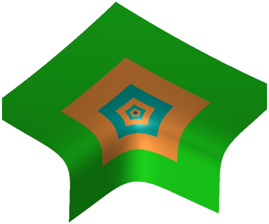
guide



of degree bi-4



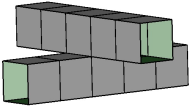
bicubic ring + guide



guided rings

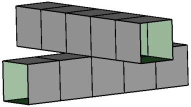
highlight lines

Two crossing beams

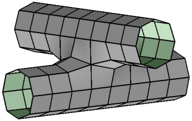


mesh

Two crossing beams

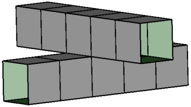


mesh

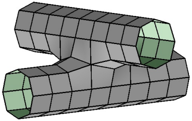


CC refinement

Two crossing beams



mesh



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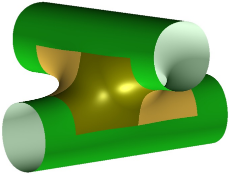
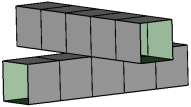
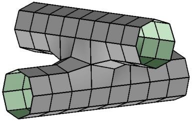


layout

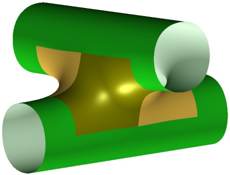
Two crossing beams



mesh



CC refinement



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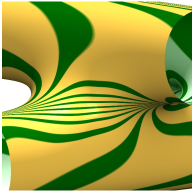
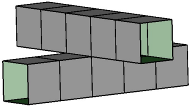
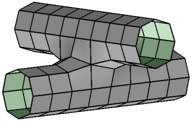


highlight lines

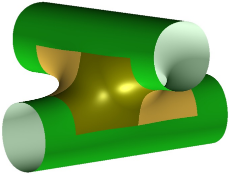
Two crossing beams



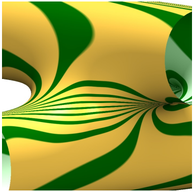
mesh



CC refinement



layout



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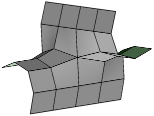
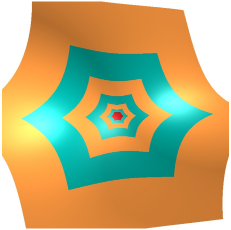
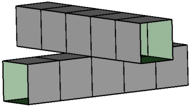


CC-net

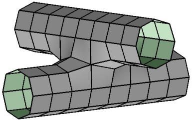


6 rings + cap

Two crossing beams



mesh



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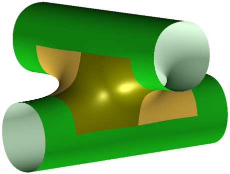
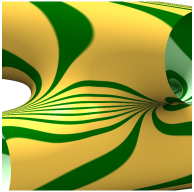


layout



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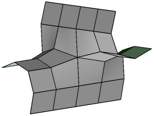

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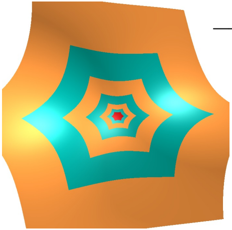

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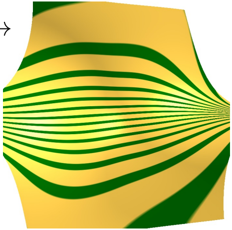
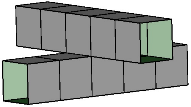
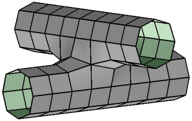


guided

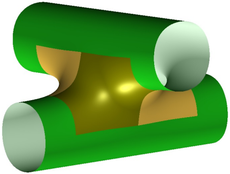
Two crossing beams



mesh



CC refinement



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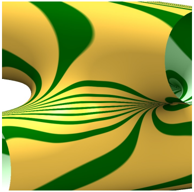
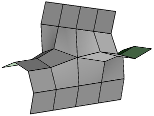
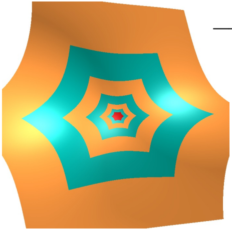


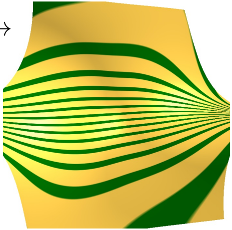
highlight lines



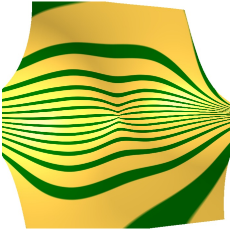
CC-net



6 rings + cap



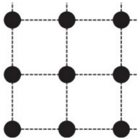
guided



Catmull-Clark

Assembling Bézier patches from corner jets

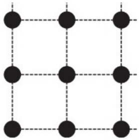
$$\begin{pmatrix} \partial_v^2 f & \partial_u \partial_v^2 f & \partial_u^2 \partial_v^2 f \\ \partial_v f & \partial_u \partial_v f & \partial_u^2 \partial_v f \\ f & \partial_u f & \partial_u^2 f \end{pmatrix} \to$$



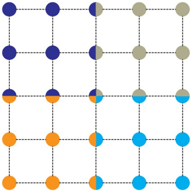
Hermite data   in Bernstein-Bézier form

Assembling Bézier patches from corner jets

$$\begin{pmatrix} \partial_v^2 f & \partial_u \partial_v^2 f & \partial_u^2 \partial_v^2 f \\ \partial_v f & \partial_u \partial_v f & \partial_u^2 \partial_v f \\ f & \partial_u f & \partial_u^2 f \end{pmatrix} \rightarrow$$



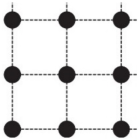
Hermite data   in Bernstein-Bézier form



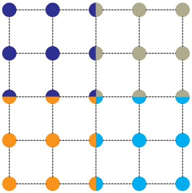
bi-4

Assembling Bézier patches from corner jets

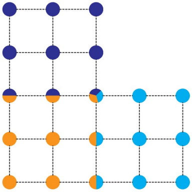
$$\begin{pmatrix} \partial_v^2 f & \partial_u \partial_v^2 f & \partial_u^2 \partial_v^2 f \\ \partial_v f & \partial_u \partial_v f & \partial_u^2 \partial_v f \\ f & \partial_u f & \partial_u^2 f \end{pmatrix} \rightarrow$$



Hermite data in Bernstein-Bézier form

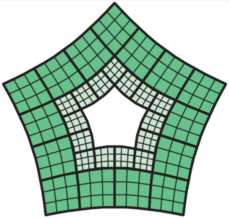


bi-4

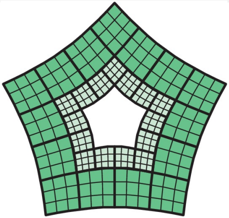


assembled tensor-border

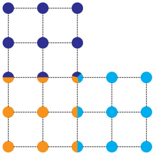
Characteristic map of Catmull-Clark subdivision



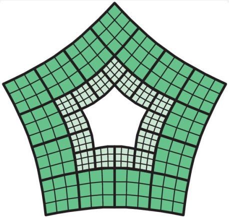
Characteristic map of Catmull-Clark subdivision as sampling tool



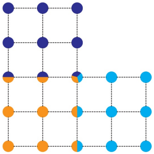
guide



Characteristic map of Catmull-Clark subdivision as sampling tool

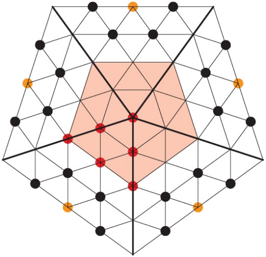


guide



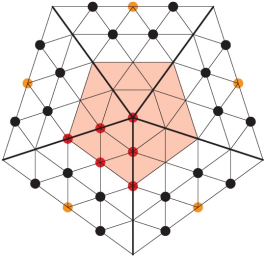
sampled rings

Guide of degree bi-4



preguide of total degree 4:
piecewise C^1;
C^2 at central point

Guide of degree bi-4



preguide of total degree 4:
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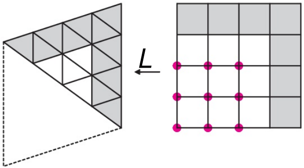
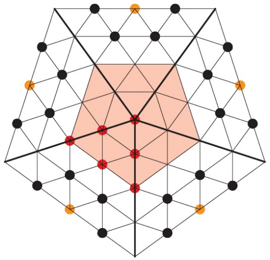
C^2 at central point

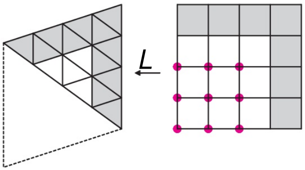


increasing flexibility
with linear shear L:

Guide of degree bi-4

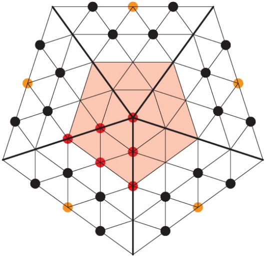


preguide of total degree 4:
piecewise C^1;
C^2 at central point

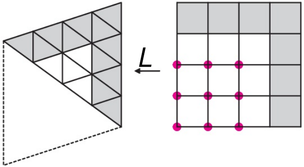


increasing flexibility
with linear shear L:
preguide $\circ$ L

Guide of degree bi-4

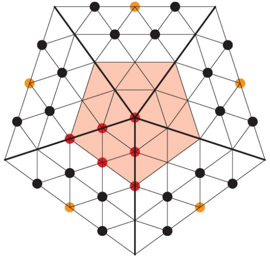


preguide of total degree 4:
piecewise C^1;
C^2 at central point

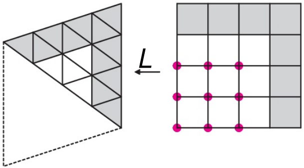


increasing flexibility
with linear shear L:
preguide $\circ$ L
3×3 •

Guide of degree bi-4

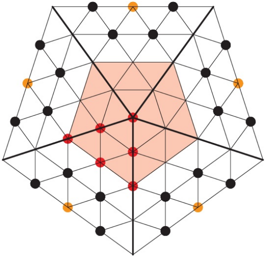


preguide of total degree 4:
piecewise C^1;
C^2 at central point

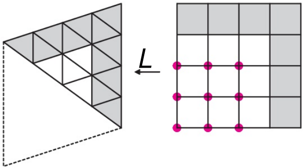


increasing flexibility
with linear shear L:
preguide $\circ$ L
3×3 •
$13n + 6$ dof

Guide of degree bi-4

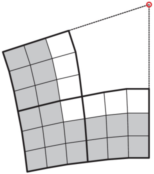


preguide of total degree 4:
piecewise C^1;
C^2 at central point

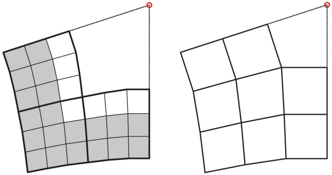


increasing flexibility
with linear shear L:
preguide $\circ$ L
3×3 •
$13n + 6$ dof
$6n + 1$ of CC

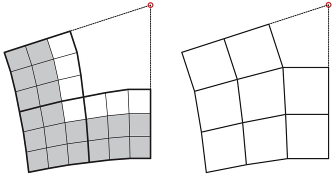
Characteristic parameterization

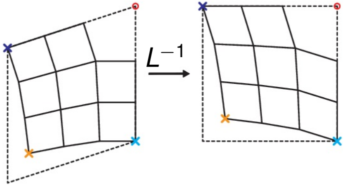


Characteristic parameterization

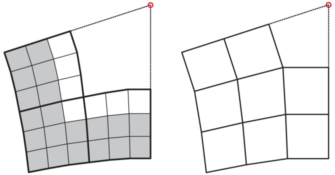


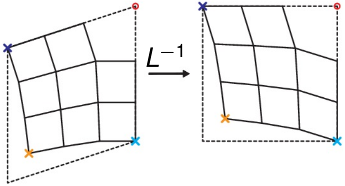
Characteristic parameterization for sampling

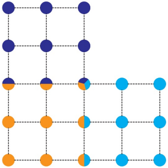




Characteristic parameterization for sampling

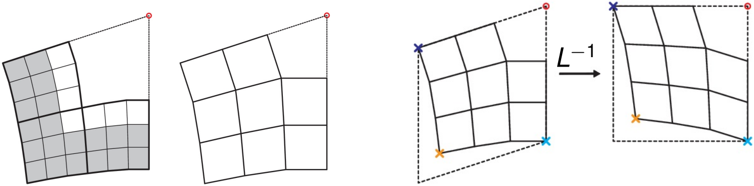


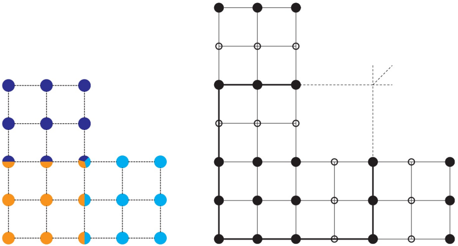




sampled C^1

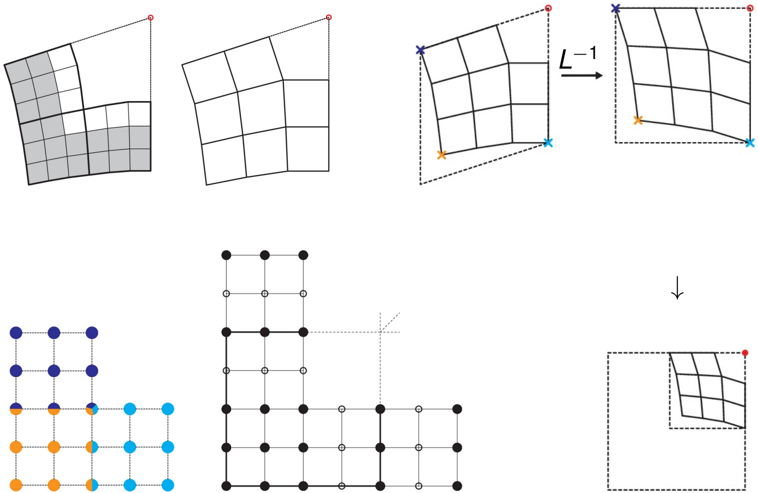
Characteristic parameterization for sampling





sampled C^1 C^2 correction

Characteristic parameterization for sampling

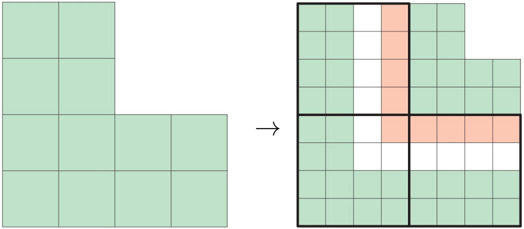


sampled C^1

C^2 correction

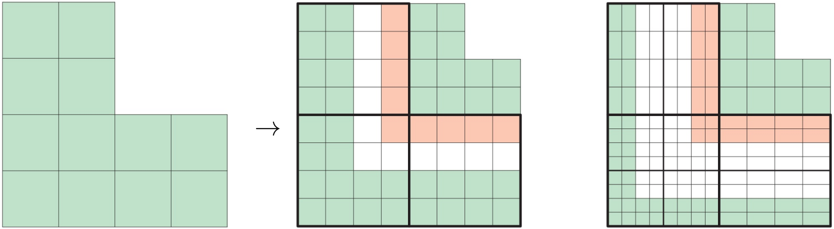
scaled sampling

Assembling bi-4 rings



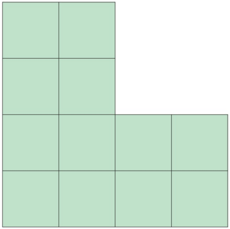
tensor-border $\rightarrow$ C^1

Assembling bi-4 rings



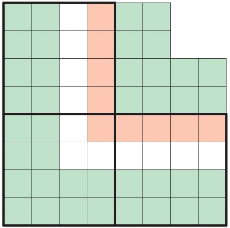
tensor-border $\qquad$ C^1 $\qquad$ C^2

Assembling bi-4 rings

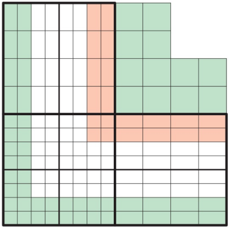


tensor-border

→

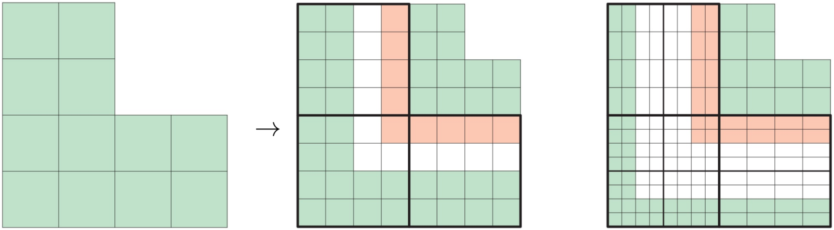


C^1



C^2
macropatches
internally C^3

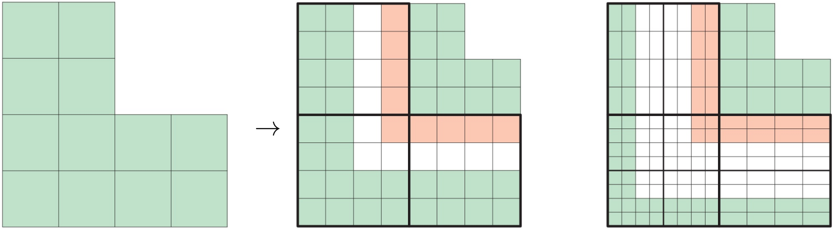
Assembling bi-4 rings



tensor-border $\rightarrow$ C^1 | C^2 macropatches internally C^3

quality of C^1 and C^2 surfaces is alike;

Assembling bi-4 rings

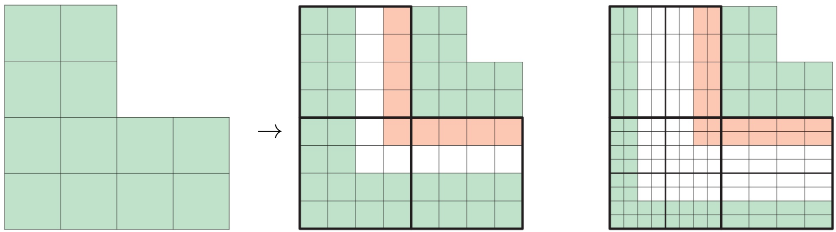


tensor-border → C^1

C^2 macropatches internally C^3

quality of C^1 and C^2 surfaces is alike;
C^1: more analysis functions, more sparse analysis matrix.

Assembling bi-4 rings



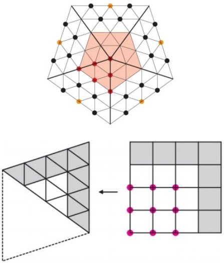
tensor-border

C^1

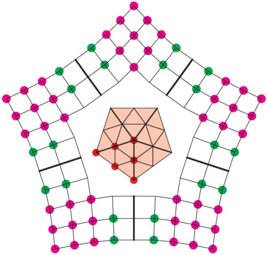
C^2
macropatches
internally C^3

quality of C^1 and C^2 surfaces is alike;
C^1: more analysis functions, more sparse analysis matrix.
By contrast, in regular bi-3 case:
more C^1 functions, more dense analysis matrix.

Reformulation towards traditional subdivision

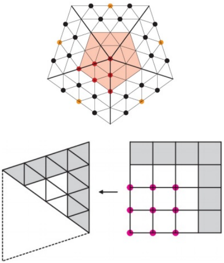


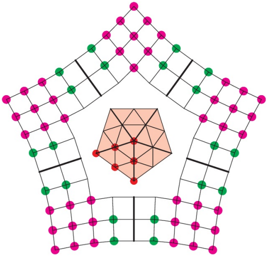
Original dof

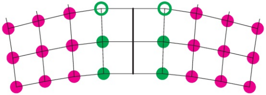


New structure

Reformulation towards traditional subdivision



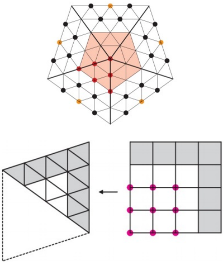




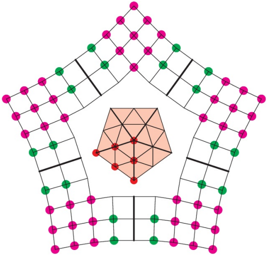
Original dof New structure

contains almost all data for assembling bi-4 rings.

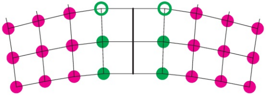
Reformulation towards traditional subdivision



Original dof



New structure

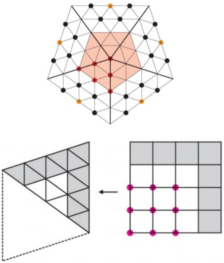


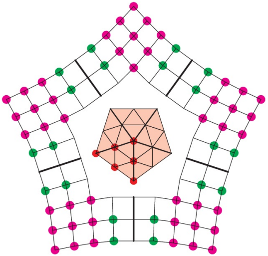
Completion

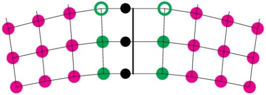
contains almost all data for assembling bi-4 rings.

of tensor-border: o are defined by • and •;

Reformulation towards traditional subdivision







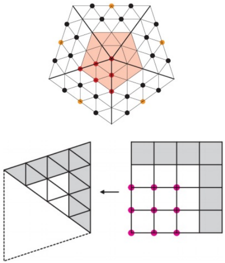
Original dof New structure Completion

contains almost all data for assembling bi-4 rings.

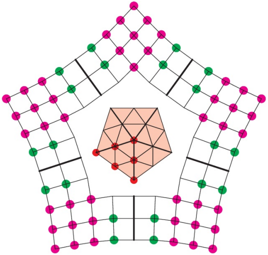
of tensor-border: o are defined by • and •;

averaging;

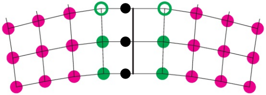
Reformulation towards traditional subdivision



Original dof



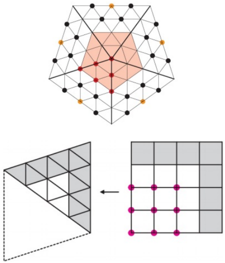
New structure



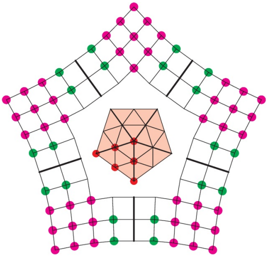
Completion

contains almost all data for assembling bi-4 rings.
of tensor-border: o are defined by • and •;
averaging; correction to C^2.

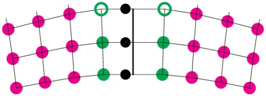
Reformulation towards traditional subdivision



Original dof



New structure



Completion

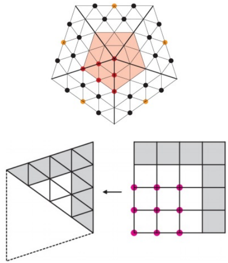
contains almost all data for assembling bi-4 rings.

of tensor-border: o are defined by • and •;

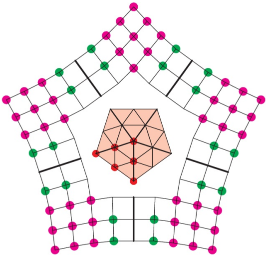
averaging; correction to C^2.

▷ Fewer arithmetic operations ⇒ faster evaluation;

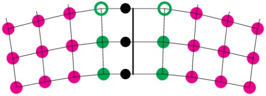
Reformulation towards traditional subdivision



Original dof



New structure

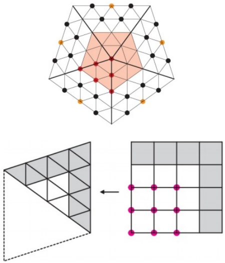


Completion

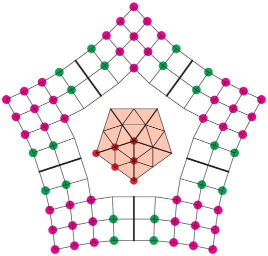
contains almost all data for assembling bi-4 rings.
of tensor-border: o are defined by • and •;
averaging; correction to C^2.

- ▷ Fewer arithmetic operations ⇒ faster evaluation;
- ▷ new refinement is akin to traditional subdivision;

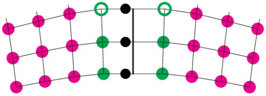
Reformulation towards traditional subdivision



Original dof



New structure



Completion

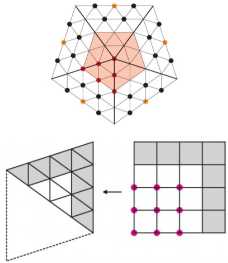
contains almost all data for assembling bi-4 rings.

of tensor-border: o are defined by • and •;

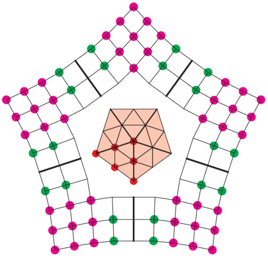
averaging; correction to C^2.

- ▷ Fewer arithmetic operations ⇒ faster evaluation;
- ▷ new refinement is akin to traditional subdivision;
- ▷ considerably larger precalculated stencils;

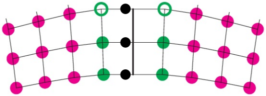
Reformulation towards traditional subdivision



Original dof



New structure

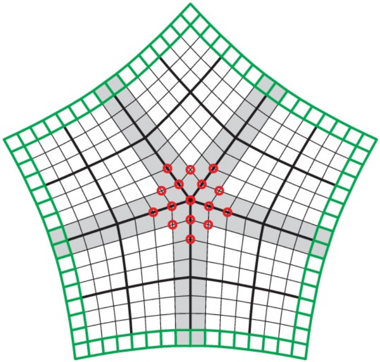


Completion

contains almost all data for assembling bi-4 rings.
of tensor-border: o are defined by • and •;
averaging; correction to C^2.

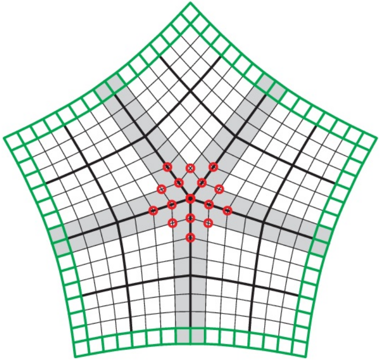
- ▷ Fewer arithmetic operations ⇒ faster evaluation;
- ▷ new refinement is akin to traditional subdivision;
- ▷ considerably larger precalculated stencils;
- ▷ considerably better quality.

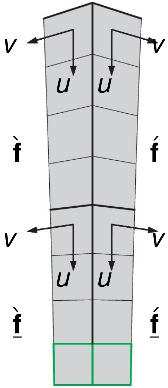
Central G^1 bi-4 cap



well-defined curvature at eop;
C^1 connection to last guided ring

Central G^1 bi-4 cap





well-defined curvature at eop;

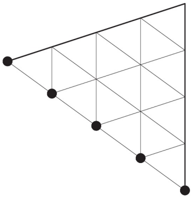
C^1 connection to last guided ring

$$\partial\grave{\mathbf{f}}_v + \partial\acute{\mathbf{f}}_v - (2c(1-u) + \tfrac{2}{3}cu)\partial\grave{\mathbf{f}}_u = 0$$

$$\partial\underline{\grave{\mathbf{f}}}_v + \partial\underline{\acute{\mathbf{f}}}_v - \tfrac{2}{3}c(1-u)^2\partial\underline{\grave{\mathbf{f}}}_u = 0$$

Homogeneous functions I

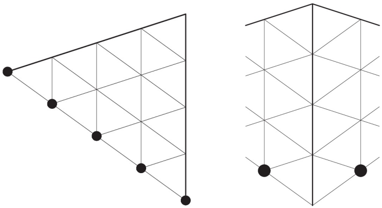
Homogeneous function of degree d: $F(\lambda x) = \lambda^d F(x)$



$d = 4$

Homogeneous functions I

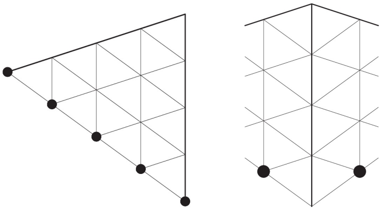
Homogeneous function of degree d: $F(\lambda x) = \lambda^d F(x)$



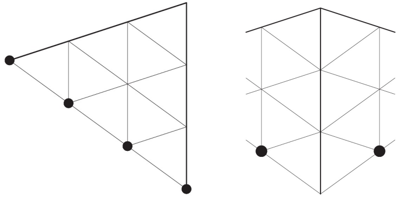
$d = 4$

Homogeneous functions I

Homogeneous function of degree d: $F(\lambda x) = \lambda^d F(x)$



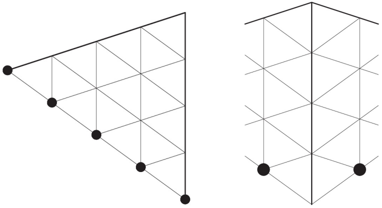
$d = 4$



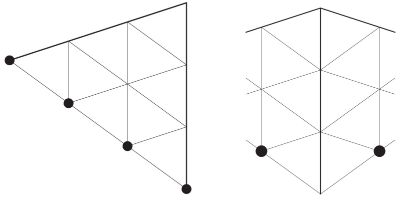
$d = 3$

Homogeneous functions I

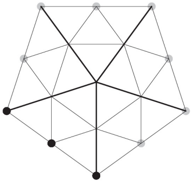
Homogeneous function of degree d: $F(\lambda x) = \lambda^d F(x)$



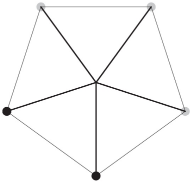
$d = 4$



$d = 3$

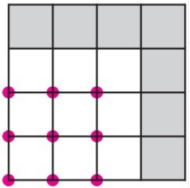


$d = 2$

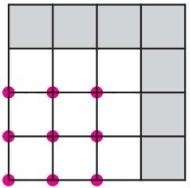


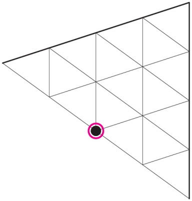
$d = 1$

Homogeneous functions II



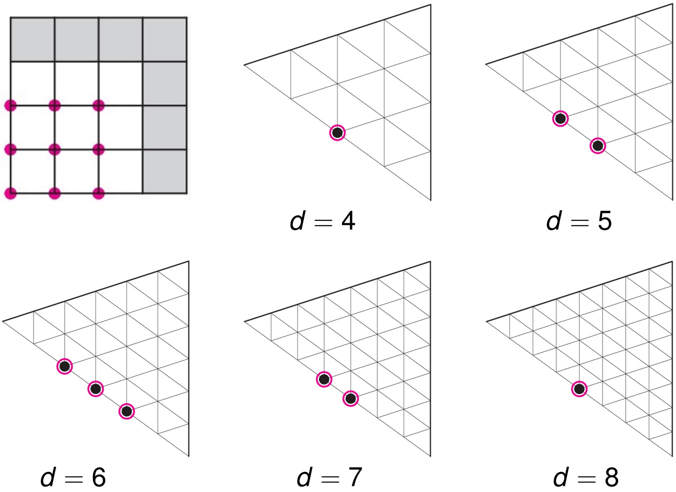
Homogeneous functions II





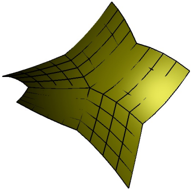
$d = 4$

Homogeneous functions II

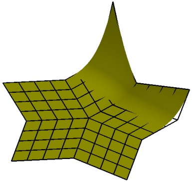


Eigenfunctions

Top row: $d = 2$ (hyperbolic shape); bottom row: $d = 3$

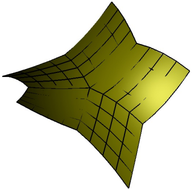


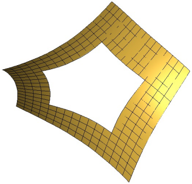
eigen-guide



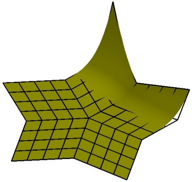
Eigenfunctions

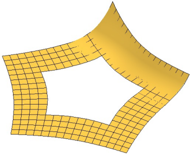
Top row: $d = 2$ (hyperbolic shape); bottom row: $d = 3$





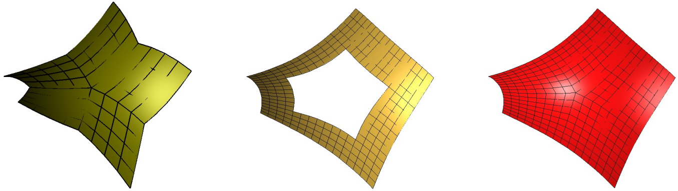
eigen-guide   eigen-ring



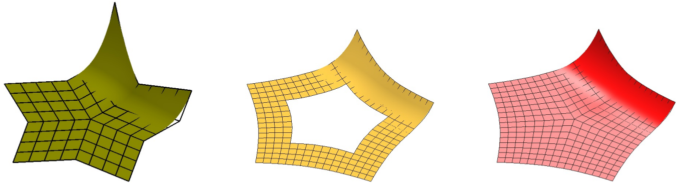


Eigenfunctions

Top row: $d = 2$ (hyperbolic shape); bottom row: $d = 3$

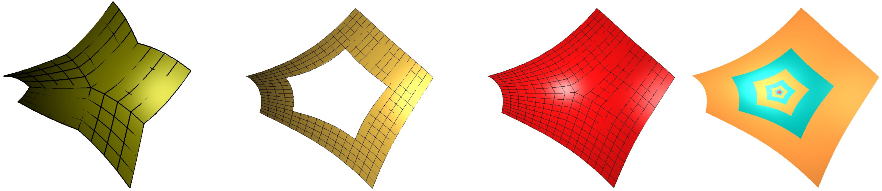


eigen-guide   eigen-ring   eigen-cap

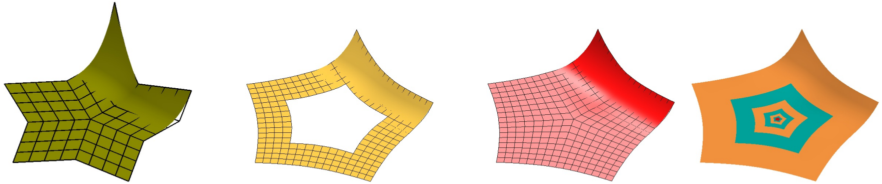


Eigenfunctions

Top row: $d = 2$ (hyperbolic shape); bottom row: $d = 3$

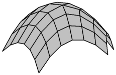


eigen-guide eigen-ring eigen-cap surface:



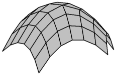
eigen-ring scaled by λ^s, $s = 0, 1, \dots, m-1$ and eigen-cap scaled by λ^m;
λ is subdominant eigenvalue of Catmull-Clark subdivision.

Convex shape



CC-net, $n = 5$

Convex shape



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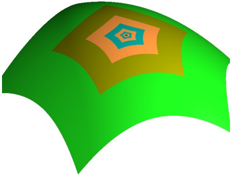
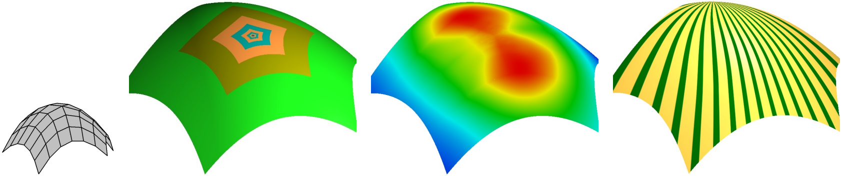


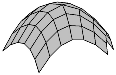
layout

Convex shape

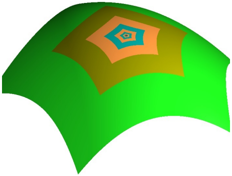


CC-net, $n = 5$ layout Gauss curvature highlight lines

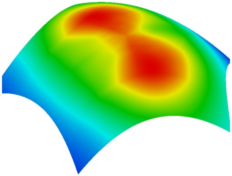
Convex shape



CC-net, $n = 5$

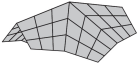


layout

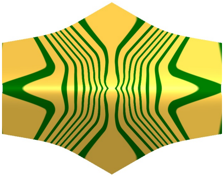


Gauss curvature

highlight lines

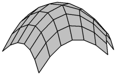


CC-net, $n = 6$

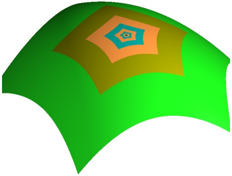


Catmull-Clark

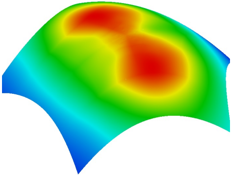
Convex shape



CC-net, $n = 5$

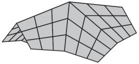


layout

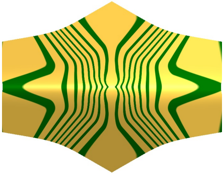


Gauss curvature

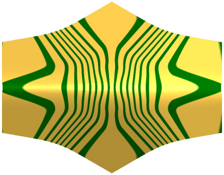
highlight lines



CC-net, $n = 6$

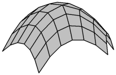


Catmull-Clark

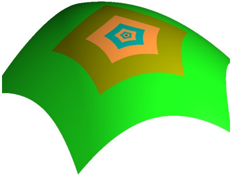


guided after one
CC refinement

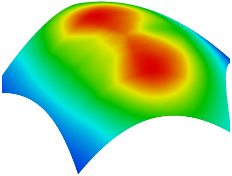
Convex shape



CC-net, $n = 5$

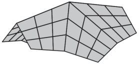


layout

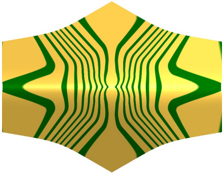


Gauss curvature

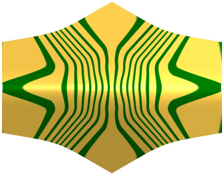
highlight lines



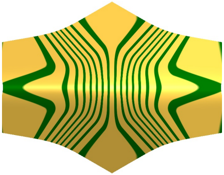
CC-net, $n = 6$



Catmull-Clark

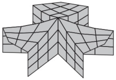


guided after one CC refinement



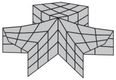
default

Exotic shape (Mitsubishi logo)



CC-net,
$n = 9$

Exotic shape (Mitsubishi logo)



CC-net,
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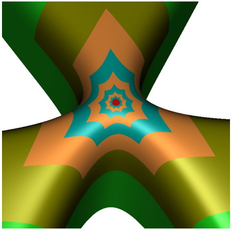
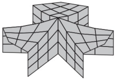


6 guided rings
+ cap

Exotic shape (Mitsubishi logo)



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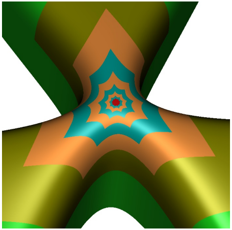

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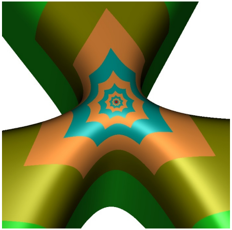
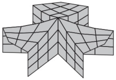
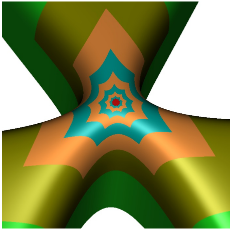


8 guided rings
+ cap

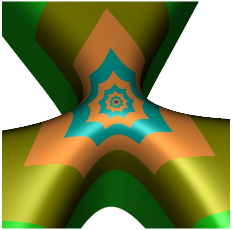
Exotic shape (Mitsubishi logo)



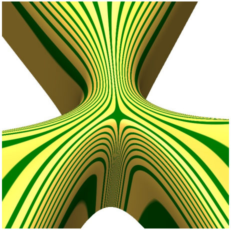
CC-net, $n = 9$



6 guided rings + cap

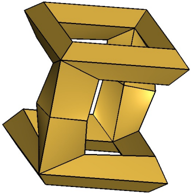


8 guided rings + cap



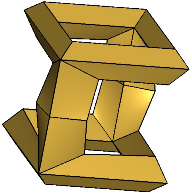
highlight lines

Dominant multi-sided surfaces

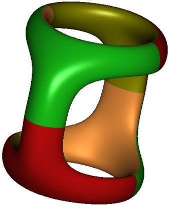


mesh, $n = 6$

Dominant multi-sided surfaces

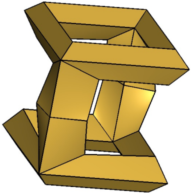


mesh, $n = 6$



layout

Dominant multi-sided surfaces



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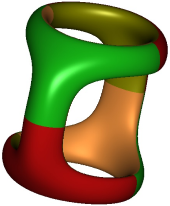

layout

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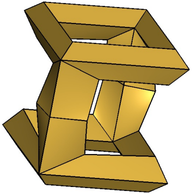

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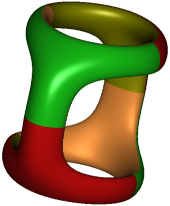

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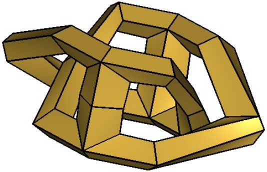

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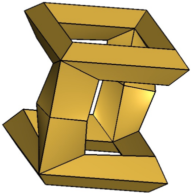

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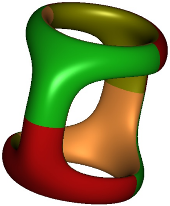
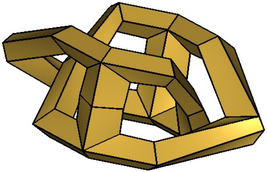
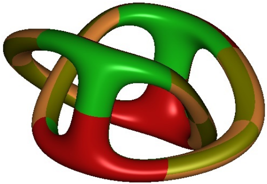


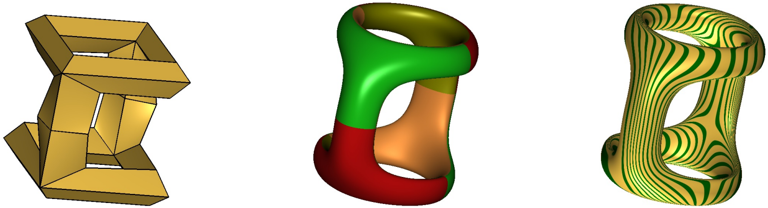
layout

highlight lines

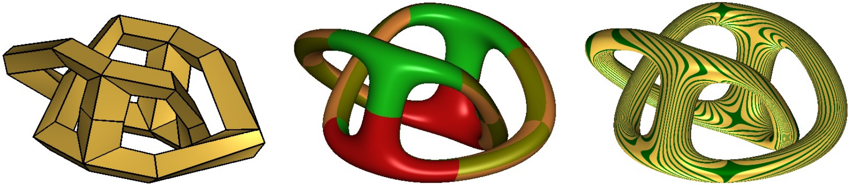




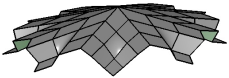
Dominant multi-sided surfaces



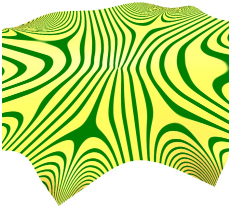
mesh, $n = 6$ layout highlight lines



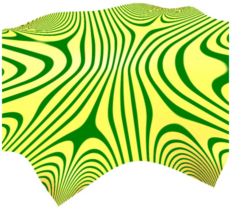
Refinability: embossing the details



CC-net, $n = 8$



Catmull-Clark



default

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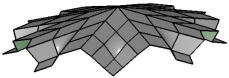

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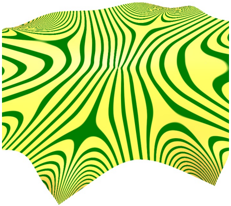
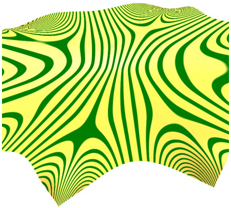
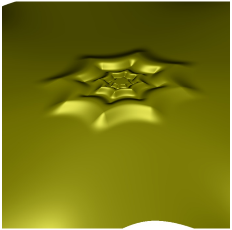
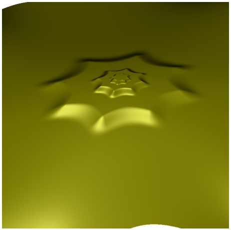


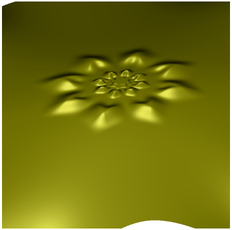
Catmull-Clark



default







Summary

New class of smooth high quality bi-4 surfaces using

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- subdivision $\Rightarrow$ refinable C^1 (C^2) surfaces;
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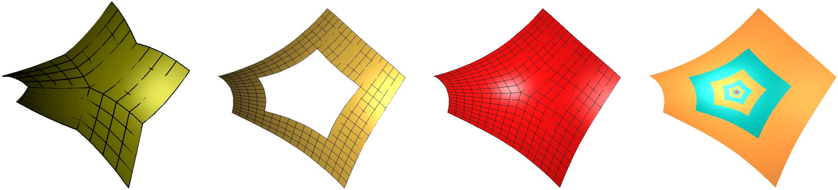
- Built-in eigen-structure characterized and determined by the guide.



Summary

New class of smooth high quality bi-4 surfaces using

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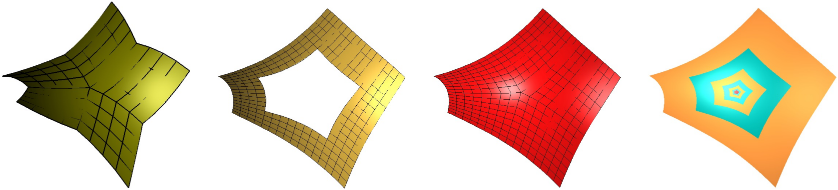

Thank you!