Rapidly contracting subdivision yields finite, effectively C^2 surfaces

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Vilnius University

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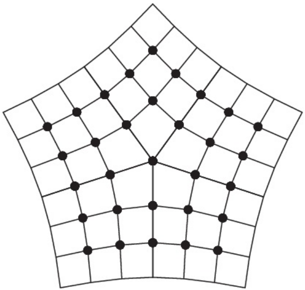
Outline

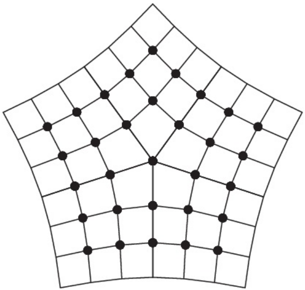
1. Multi-sided surfaces as contracting rings
2. Toolkit for guided surfacing
3. Examples
4. Conclusion

Input – Catmull-Clark (CC) nets



B-spline (CC) net

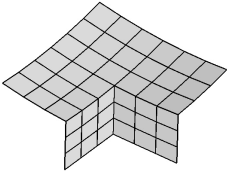
Input – Catmull-Clark (CC) nets



B-spline (CC) net

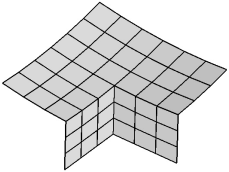
bicubic ring + tensor-border of degree 3

Guided subdivision



CC-net, $n = 5$

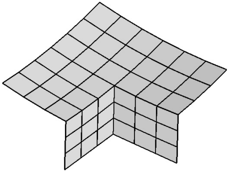
Guided subdivision



CC-net, $n = 5$

bicubic ring

Guided subdivision

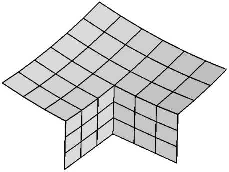


CC-net, $n = 5$

bicubic ring

guide

Guided subdivision



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bicubic ring

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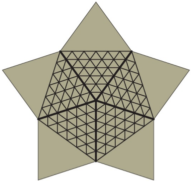
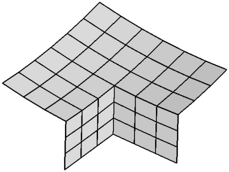


of degree 6

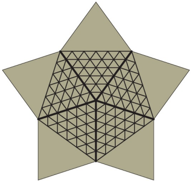
Guided subdivision



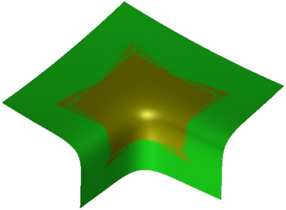
CC-net, $n = 5$

bicubic ring

guide

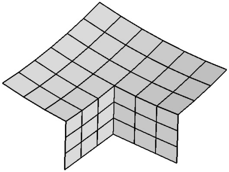


of degree 6



bicubic ring + guide

Guided subdivision



CC-net, $n = 5$

bicubic ring

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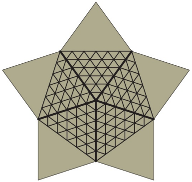
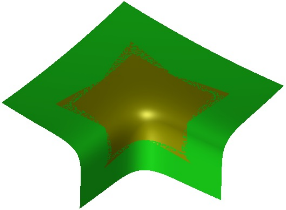
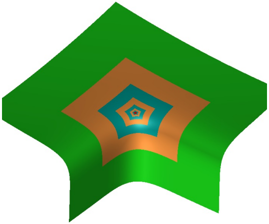


of degree 6

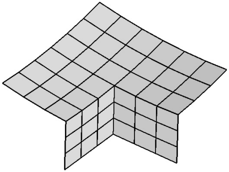


bicubic ring + guide



guided rings

Guided subdivision



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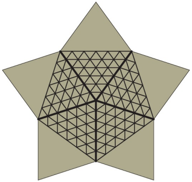
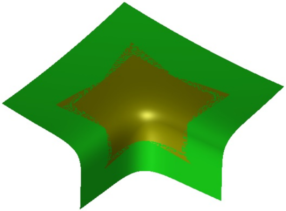
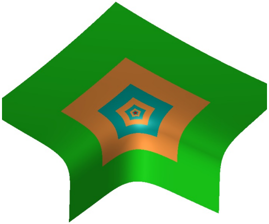
guide



of degree 6



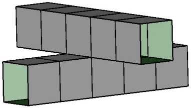
bicubic ring + guide



guided rings

highlight lines

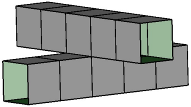
Two crossing beams



mesh
(constructed by
kids under 5)

(four $n = 6$ eons
are separated)

Two crossing beams



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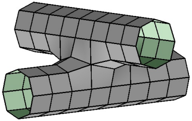

CC refinement
(four $n = 6$ eons
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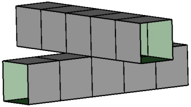
Two crossing beams



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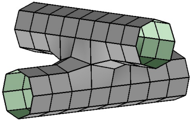

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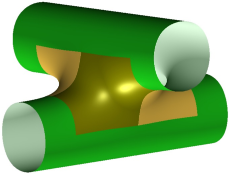
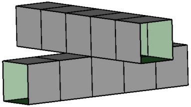
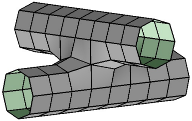


layout

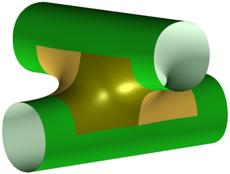
Two crossing beams



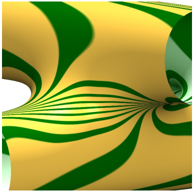
mesh (constructed by kids under 5)



CC refinement (four $n = 6$ eons are separated)

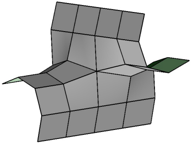


layout



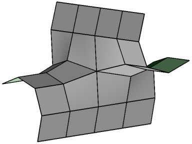
highlight lines

Two crossing beams: six-sided part

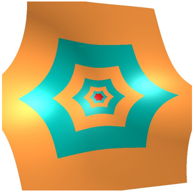


CC-net

Two crossing beams: six-sided part

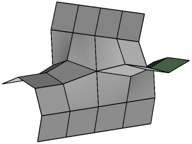


CC-net



6 rings + cap

Two crossing beams: six-sided part



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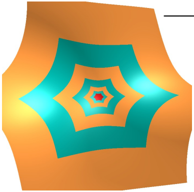
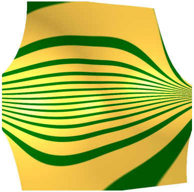


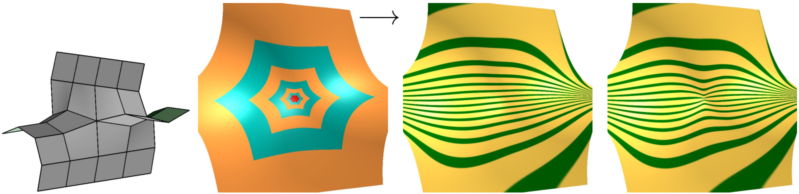
6 rings + cap

→



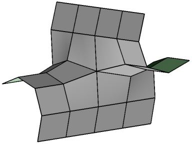
guided

Two crossing beams: six-sided part



CC-net | 6 rings + cap | guided | Catmull-Clark

Two crossing beams: six-sided part



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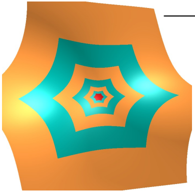

6 rings + cap

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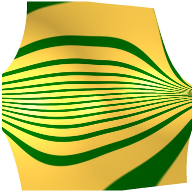

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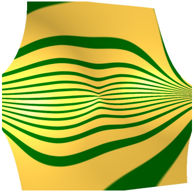
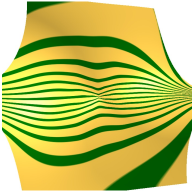
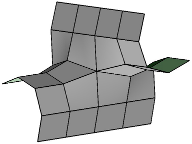


Catmull-Clark



KP2009: double-speed
bicubic subdivision

Two crossing beams: six-sided part



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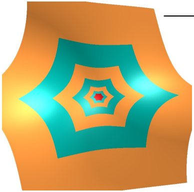
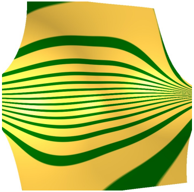
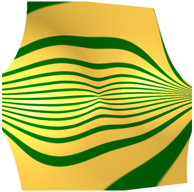


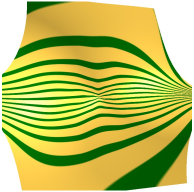
6 rings + cap



guided



Catmull-Clark



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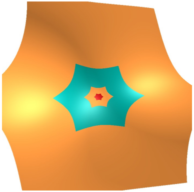
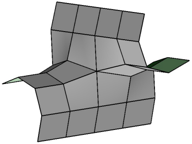
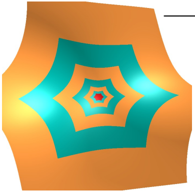
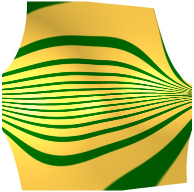


3 double speed
guided rings + cap

Two crossing beams: six-sided part


CC-net


6 rings + cap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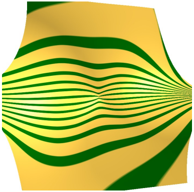
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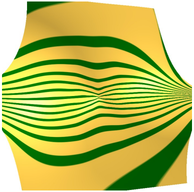
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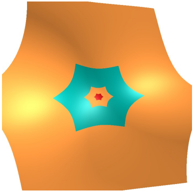
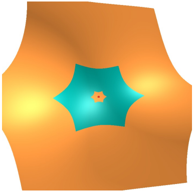
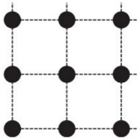

3 double speed
guided rings + cap


3 yet faster guided
rings + cap

Assembling Bézier patches from corner jets

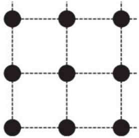
$$\begin{pmatrix} \partial_v^2 f & \partial_u \partial_v^2 f & \partial_u^2 \partial_v^2 f \\ \partial_v f & \partial_u \partial_v f & \partial_u^2 \partial_v f \\ f & \partial_u f & \partial_u^2 f \end{pmatrix} \rightarrow$$



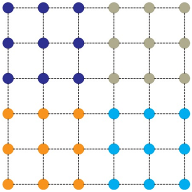
Hermite data in Bernstein-Bézier form

Assembling Bézier patches from corner jets

$$\begin{pmatrix} \partial_v^2 f & \partial_u \partial_v^2 f & \partial_u^2 \partial_v^2 f \\ \partial_v f & \partial_u \partial_v f & \partial_u^2 \partial_v f \\ f & \partial_u f & \partial_u^2 f \end{pmatrix} \rightarrow$$



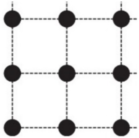
Hermite data   in Bernstein-Bézier form



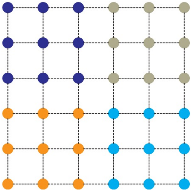
bi-5

Assembling Bézier patches from corner jets

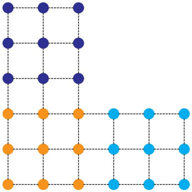
$$\begin{pmatrix} \partial_v^2 f & \partial_u \partial_v^2 f & \partial_u^2 \partial_v^2 f \\ \partial_v f & \partial_u \partial_v f & \partial_u^2 \partial_v f \\ f & \partial_u f & \partial_u^2 f \end{pmatrix} \rightarrow$$



Hermite data — in Bernstein-Bézier form

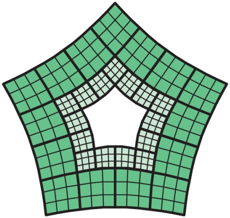


bi-5

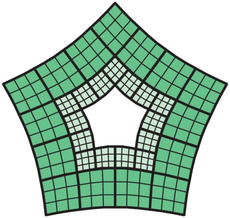


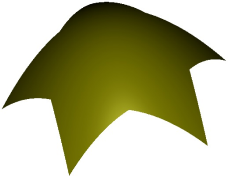
assembled tensor-border

Characteristic map of Catmull-Clark subdivision

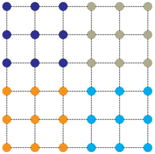


Characteristic map of Catmull-Clark subdivision as sampling tool

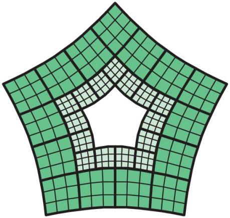


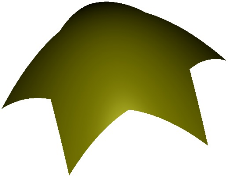


guide



Characteristic map of Catmull-Clark subdivision as sampling tool





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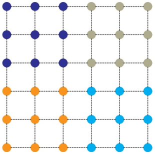


sampled rings

Characteristic parameterizations

KP, Adjustable speed surface subdivision, CAGD 2009.

$$\lambda := \frac{\tilde\sigma}{2}((1+c)\sigma^2 + 2\tilde\sigma + \sigma\sqrt{(1+c)((1+c)\sigma^2+4\tilde\sigma)}) \,,$$

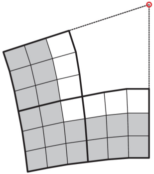
$$c := \cos\frac{2\pi}{n}, \quad \tilde\sigma := 1-\sigma, \quad 0<\sigma<1.$$

Characteristic parameterizations

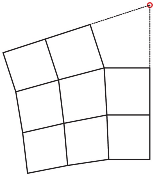
KP, Adjustable speed surface subdivision, CAGD 2009.

$$\lambda := \frac{\tilde\sigma}{2}((1+c)\sigma^2 + 2\tilde\sigma + \sigma\sqrt{(1+c)((1+c)\sigma^2+4\tilde\sigma)}) \,,$$

$$c := \cos\frac{2\pi}{n}, \quad \tilde\sigma := 1-\sigma, \quad 0<\sigma<1.$$



$\sigma = \frac{1}{2}$

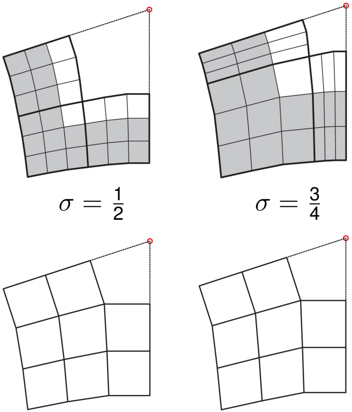


Characteristic parameterizations

KP, Adjustable speed surface subdivision, CAGD 2009.

$$\lambda := \frac{\tilde\sigma}{2}((1+\mathrm{c})\sigma^2 + 2\tilde\sigma + \sigma\sqrt{(1+\mathrm{c})((1+\mathrm{c})\sigma^2+4\tilde\sigma)})\ ,$$

$$\mathrm{c} := \cos\frac{2\pi}{n}, \quad \tilde\sigma := 1-\sigma, \quad 0<\sigma<1.$$



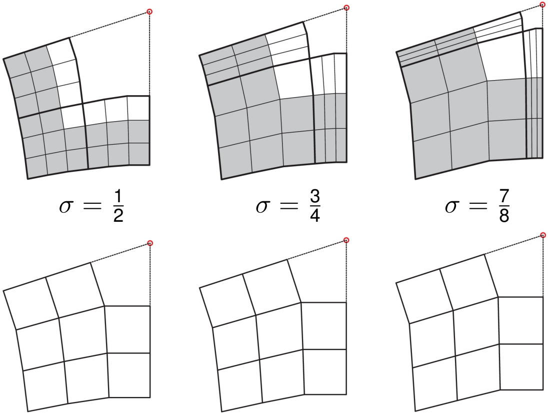
$\sigma = \frac{1}{2}$ $\sigma = \frac{3}{4}$

Characteristic parameterizations

KP, Adjustable speed surface subdivision, CAGD 2009.

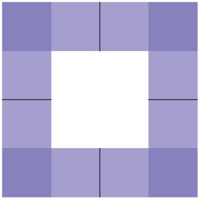
$$\lambda := \frac{\tilde\sigma}{2}((1+c)\sigma^2 + 2\tilde\sigma + \sigma\sqrt{(1+c)((1+c)\sigma^2+4\tilde\sigma)})\ ,$$

$$c := \cos\frac{2\pi}{n}, \quad \tilde\sigma := 1-\sigma, \quad 0<\sigma<1.$$



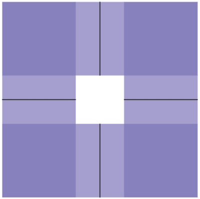
Adjustable speed splitting

$\sigma : \tilde{\sigma}$



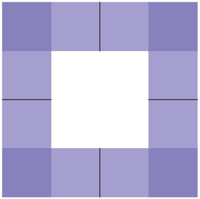
$\sigma := \frac{1}{2}$

$\sigma : \tilde{\sigma}$

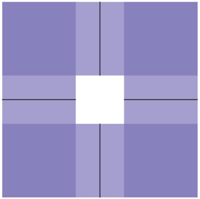


$\sigma := \frac{3}{4}$

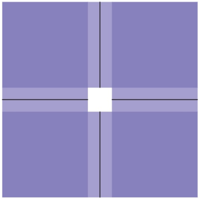
Adjustable speed splitting



$\sigma := \frac{1}{2}$



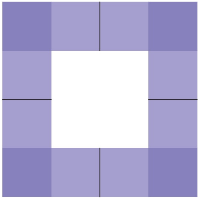
$\sigma := \frac{3}{4}$



$\sigma := \frac{7}{8}$

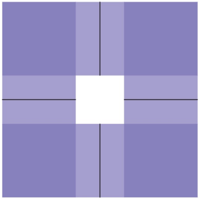
Adjustable speed splitting

$\sigma : \tilde{\sigma}$

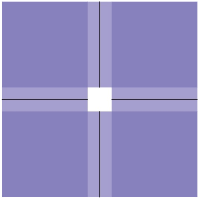


$\sigma := \frac{1}{2}$

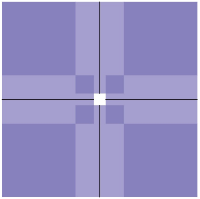
$\sigma : \tilde{\sigma}$



$\sigma := \frac{3}{4}$



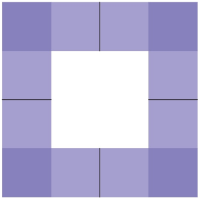
$\sigma := \frac{7}{8}$



$2\times \{\frac{3}{4}\}$

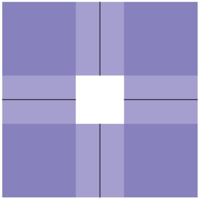
Adjustable speed splitting

$\sigma : \tilde{\sigma}$

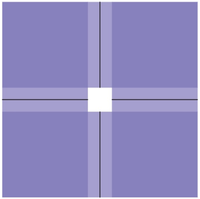


$\sigma := \frac{1}{2}$

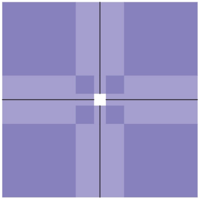
$\sigma : \tilde{\sigma}$



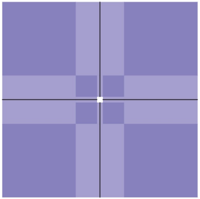
$\sigma := \frac{3}{4}$



$\sigma := \frac{7}{8}$

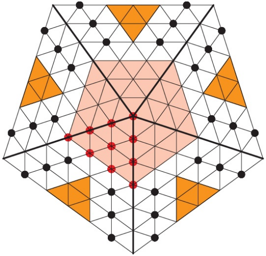


$2\times \{\frac{3}{4}\}$



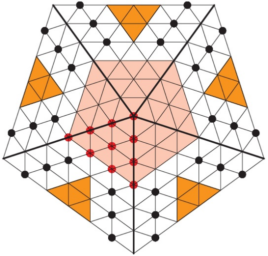
$\{\frac{3}{4}\} + \{\frac{7}{8}\}$

Guide of total degree 6 and its sampling

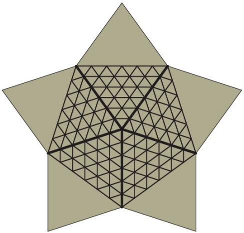


structure:
piecewise C^1;
C^3 at central point

Guide of total degree 6 and its sampling

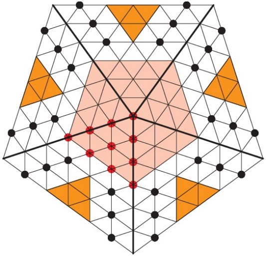


structure:
piecewise C^1;
C^3 at central point



guide to sample
with characteristic
tensor-border

Guide of total degree 6 and its sampling



structure:
piecewise C^1;
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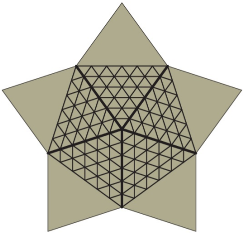
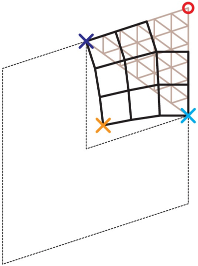
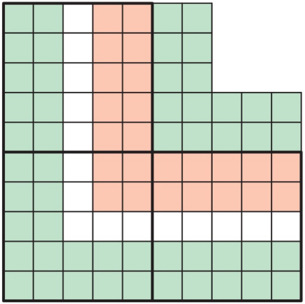


guide to sample
with characteristic
tensor-border



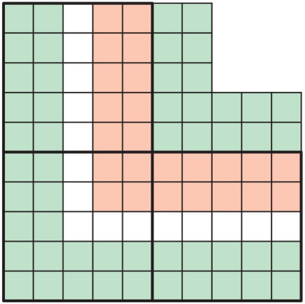
scaled sampling

Assembling bi-5 rings

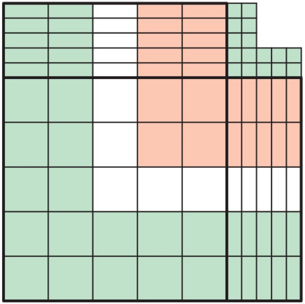


$$\sigma := \frac{1}{2}$$

Assembling bi-5 rings

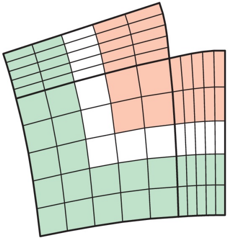


$\sigma := \frac{1}{2}$



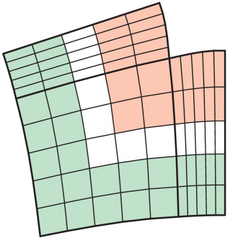
$\sigma := \frac{3}{4}$

Assembling bi-5 rings: combining various speeds

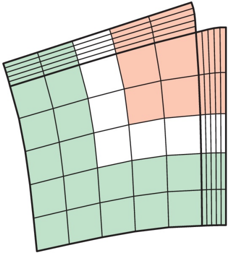


$\sigma : \frac{1}{2} \rightarrow \frac{3}{4}$

Assembling bi-5 rings: combining various speeds

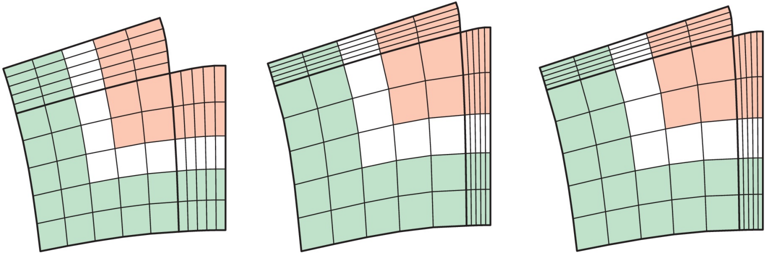


$\sigma : \frac{1}{2} \rightarrow \frac{3}{4}$



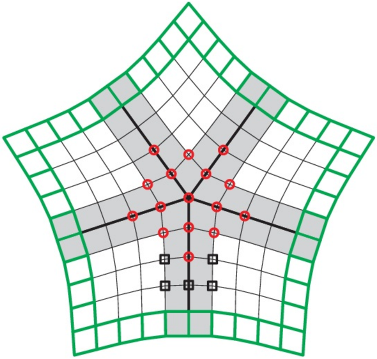
$\sigma : \frac{3}{4} \rightarrow \frac{7}{8}$

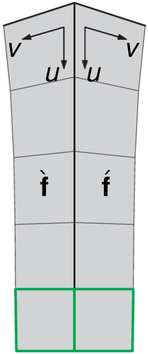
Assembling bi-5 rings: combining various speeds



$\sigma : \frac{1}{2} \to \frac{3}{4}$ $\sigma : \frac{3}{4} \to \frac{7}{8}$ $\sigma : \frac{1}{2} \to \frac{7}{8}$

Tiny G^1 bi-5 cap



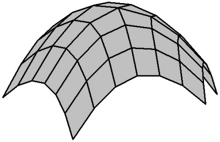


structure: $\partial \grave{\mathbf{f}}_v + \partial \acute{\mathbf{f}}_v - 2c(1-u)^2 \partial \grave{\mathbf{f}}_u = 0$

well-defined curvature at eop;

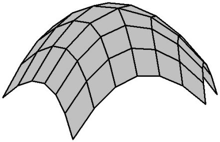
C^1 connection to last guided ring

Convex shape (Fig. 14)

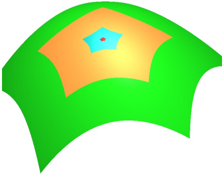


CC-net, $n = 5$

Convex shape (Fig. 14)

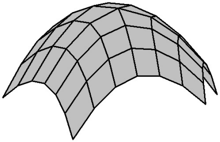


CC-net, $n = 5$

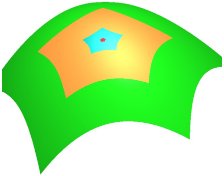


2 rings, $\sigma : \frac{3}{4}, \frac{7}{8}$

Convex shape (Fig. 14)



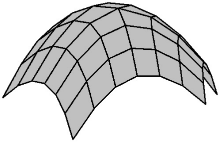
CC-net, $n = 5$



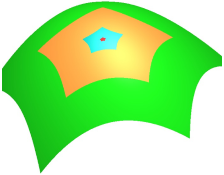
2 rings, $\sigma : \frac{3}{4}, \frac{7}{8}$

highlight lines

Convex shape (Fig. 14)

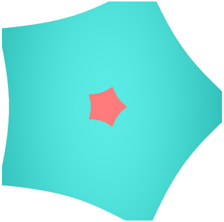


CC-net, $n = 5$

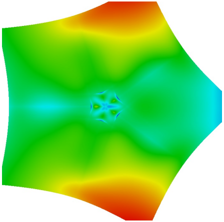


2 rings, $\sigma : \frac{3}{4}, \frac{7}{8}$

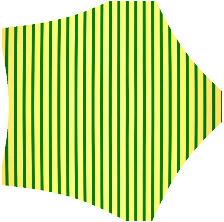
highlight lines



zoom to 2nd ring:

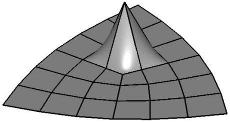


Gauss curvature



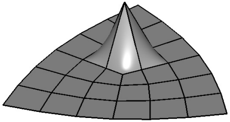
highlight lines

$n = 3$ (Fig. 12)

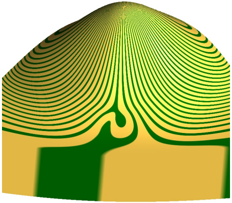


CC-net, $n = 3$

$n = 3$ (Fig. 12)

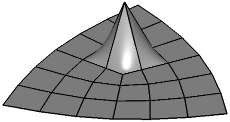


CC-net, $n = 3$

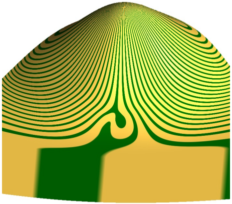


Catmull-Clark

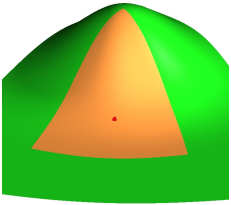
$n = 3$ (Fig. 12)



CC-net, $n = 3$

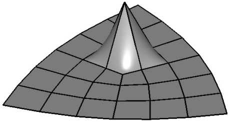


Catmull-Clark

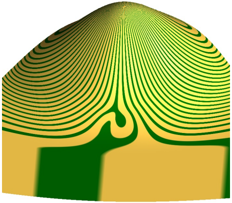


1 ring, $\sigma := \frac{15}{16}$

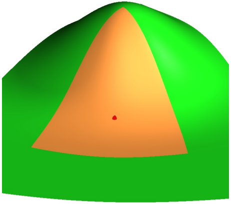
$n = 3$ (Fig. 12)



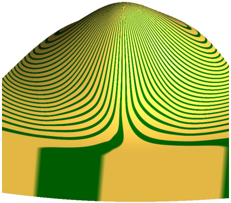
CC-net, $n = 3$



Catmull-Clark

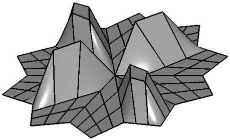


1 ring, $\sigma := \frac{15}{16}$



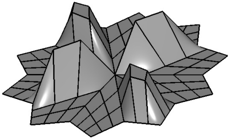
highlight lines

Exotic shape (Fig. 16)

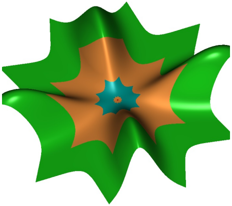


CC-net, $n = 10$

Exotic shape (Fig. 16)

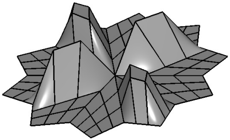


CC-net, $n = 10$

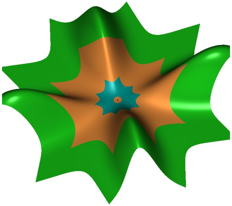


3 rings, $\sigma : \frac{3}{4}, \frac{7}{8}, \frac{7}{8}$

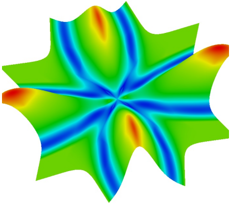
Exotic shape (Fig. 16)



CC-net, $n = 10$

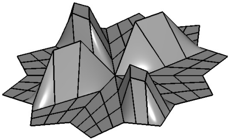


3 rings, $\sigma : \frac{3}{4}, \frac{7}{8}, \frac{7}{8}$

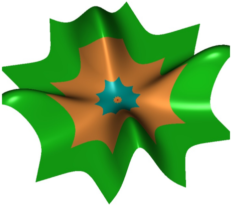


max curvature

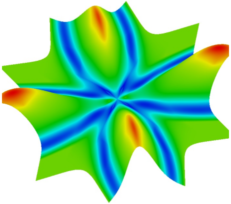
Exotic shape (Fig. 16)



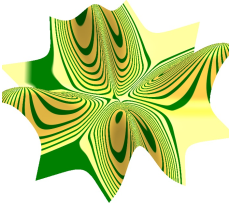
CC-net, $n = 10$



3 rings, $\sigma : \frac{3}{4}, \frac{7}{8}, \frac{7}{8}$

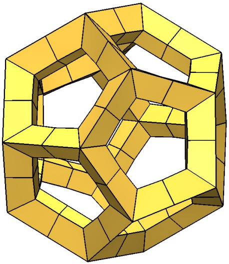


max curvature



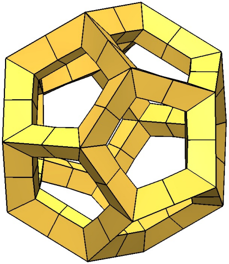
highlight lines

Dodecahedron (Fig. 1)

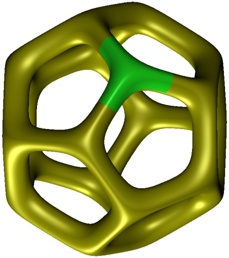


polyhedron with $n = 6$ eons

Dodecahedron (Fig. 1)

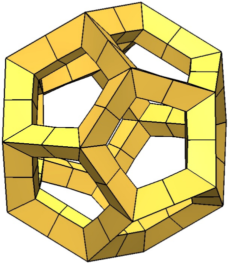


polyhedron with $n = 6$ eons

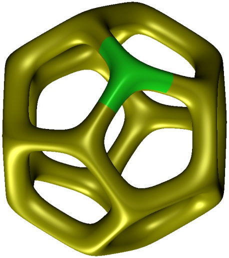


surface

Dodecahedron (Fig. 1)

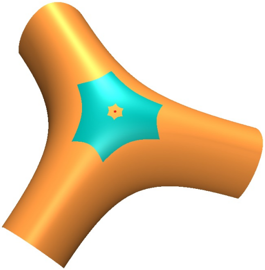


polyhedron with $n = 6$ eons



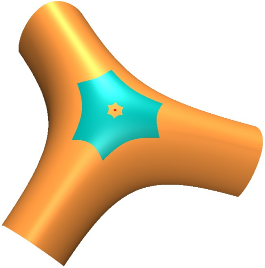
surface

Default sequence of speeds

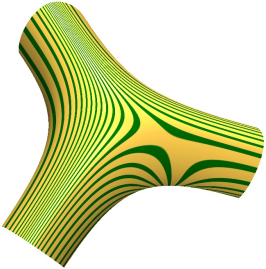


3 rings, $\sigma : \frac{3}{4}, \frac{7}{8}, \frac{7}{8}$

Default sequence of speeds

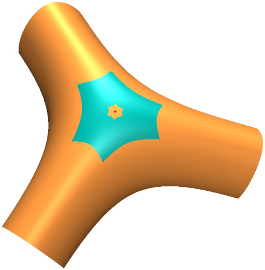


3 rings, $\sigma : \frac{3}{4}, \frac{7}{8}, \frac{7}{8}$



highlight lines

Default sequence of speeds



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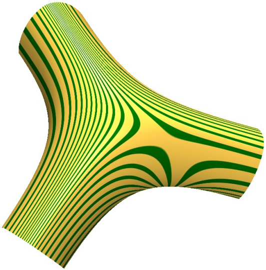
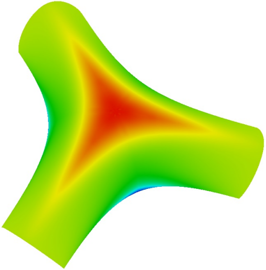
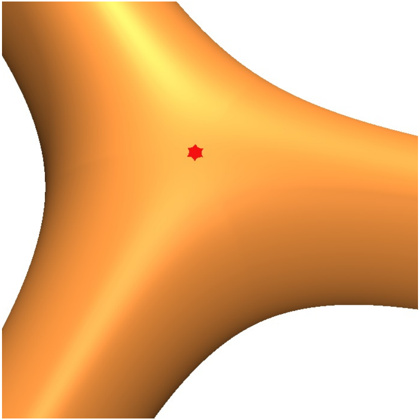


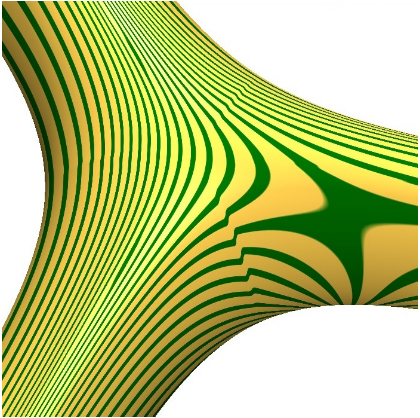
highlight lines



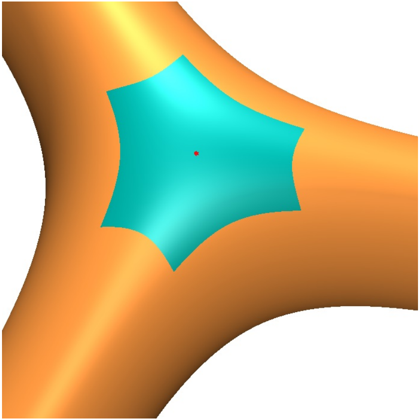
Gauss curvature

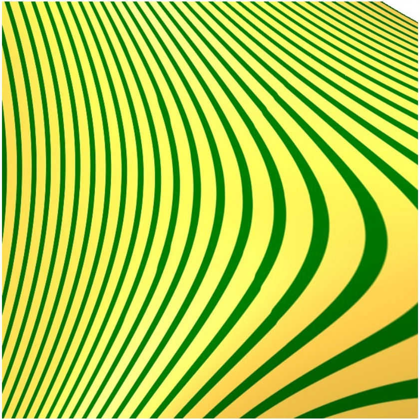
Supersonic speed I: $\sigma : \frac{63}{64}$



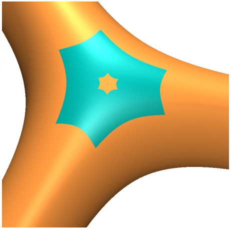


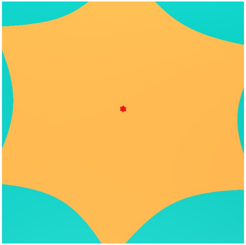
Supersonic speed II: $\sigma : \frac{3}{4}, \frac{63}{64}$

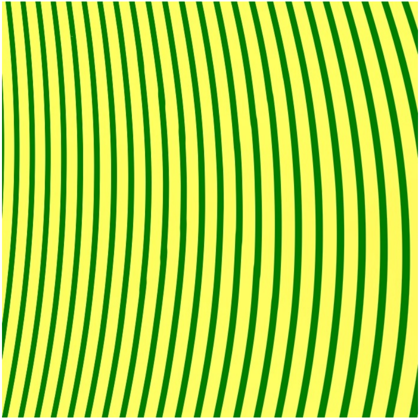




Supersonic speed III: $\sigma : \frac{3}{4}, \frac{7}{8}, \frac{63}{64}$







1. Multi-sided surfaces as contracting rings
2. Toolkit for guided surfacing
3. Examples
4. Conclusion

Conclusion

New class of smooth high quality bi-5 surfaces using

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- subdivision ⇒ refinable C^2 surfaces;

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New class of smooth high quality bi-5 surfaces using

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Conclusion

New class of smooth high quality bi-5 surfaces using

- subdivision ⇒ refinable C^2 surfaces;
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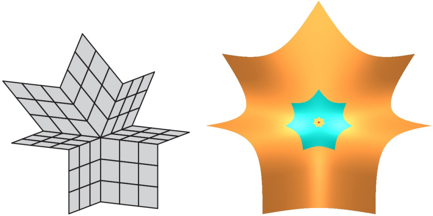
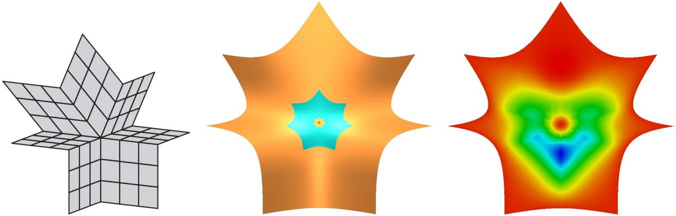
- accelerated guided subdivision ⇒ finite patchwork with G^1 central cap.



Conclusion

New class of smooth high quality bi-5 surfaces using

- subdivision $\Rightarrow$ refinable C^2 surfaces;
- guided subdivision $\Rightarrow$ good highlight line distribution;
- accelerated guided subdivision $\Rightarrow$ finite patchwork with G^1 central cap.



Gaussian smile

Conclusion

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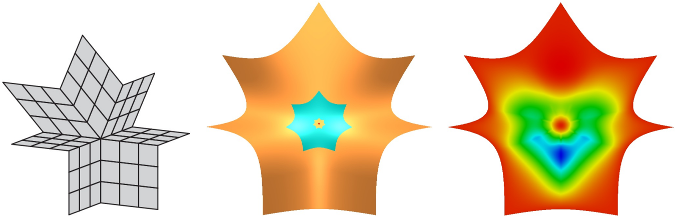

Gaussian smile

Thank you!