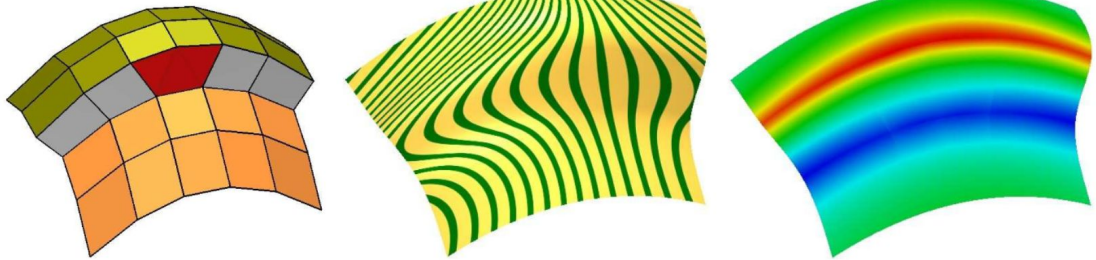
T-junctions in spline surfaces



(a) $\dot{T}$-net (b) highlight lines (c) mean curvature

Kęstutis Karčiauskas, Daniele Panozzo, Jörg Peters

Funded by NSF-CCF and DARPA TRADES

Siggraph 2018, Vancouver

T-junctions in spline surfaces

(a) $\dot{T}$-net (b) highlight lines (c) mean curvature

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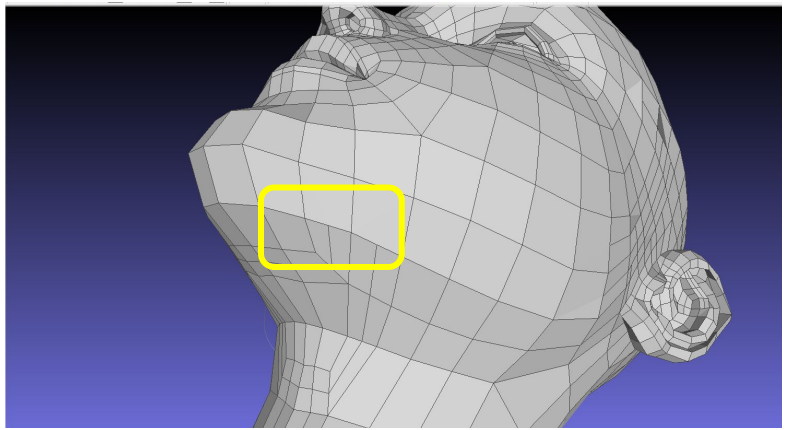
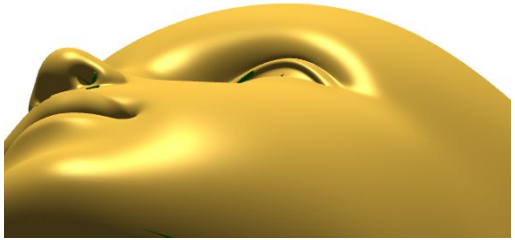
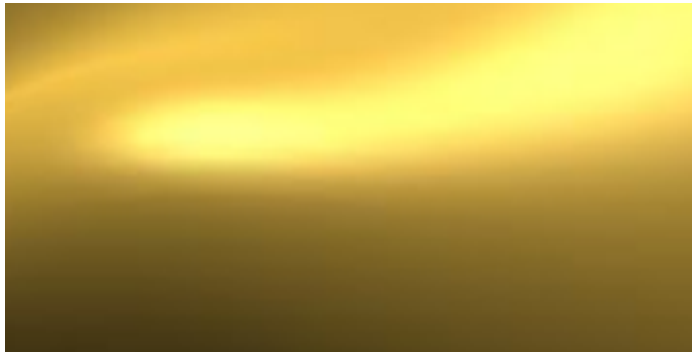
Siggraph 2018, Vancouver

T-junctions = where features start or terminate

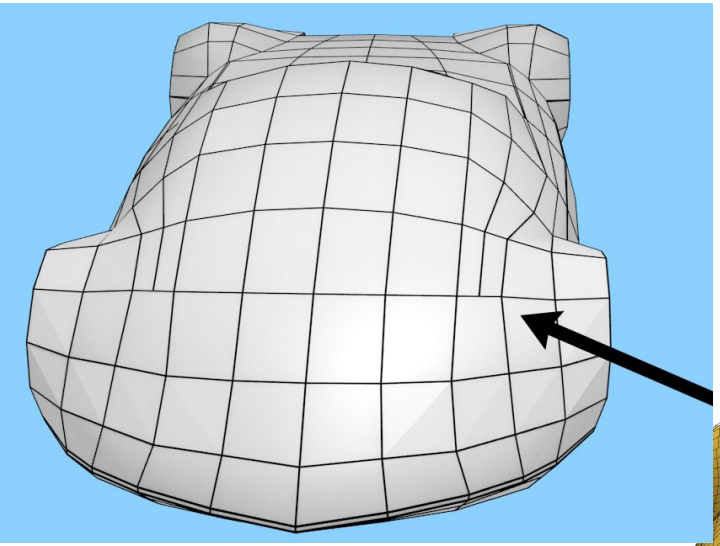


“Make irregularities (T-junctions) disappear”

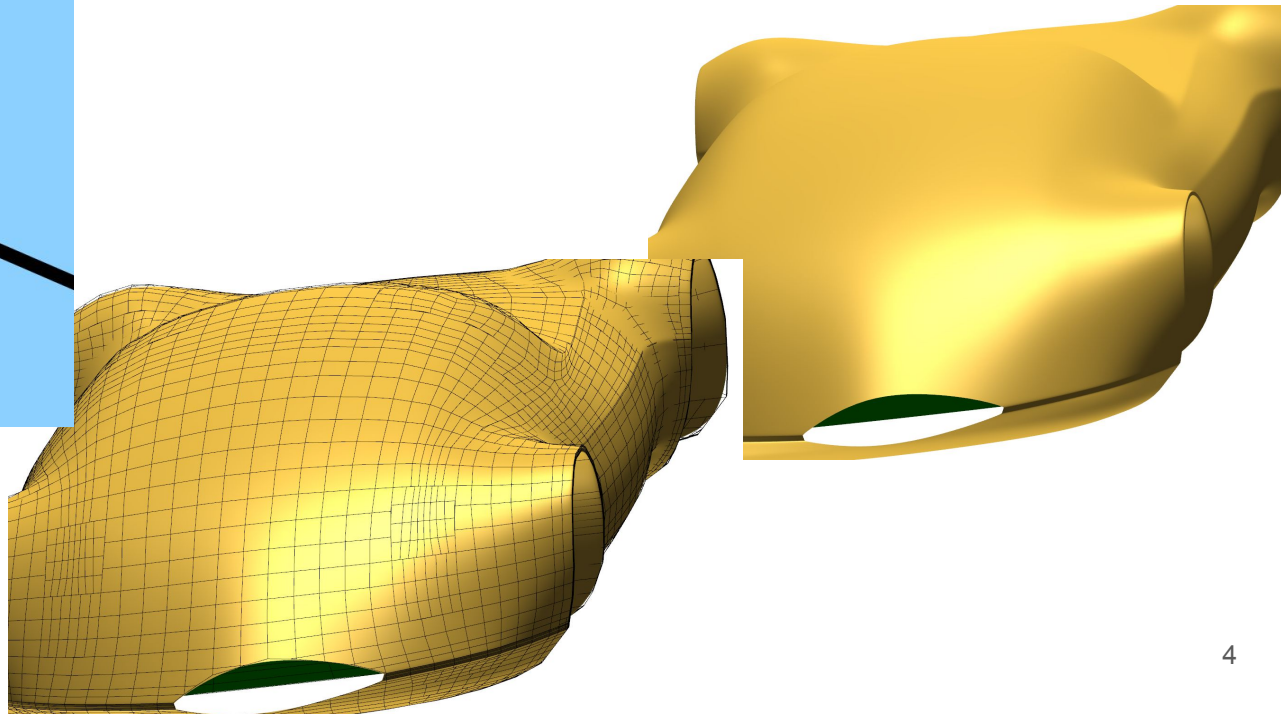




T-junctions = where features start or terminate

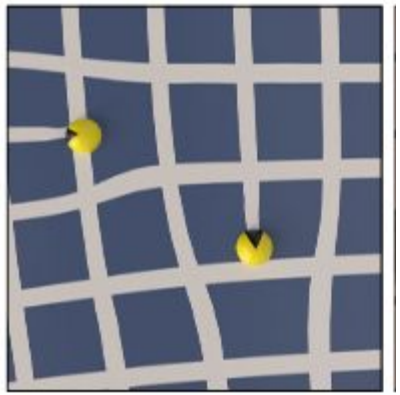


“Make T-junction disappear”



T-junctions = where features start or terminate

➢ Automatic quad meshing



strict quad-meshing [Bommes et al. 2012; Vaxman et al. 2016] complex and global

→ T-meshes [Li et al. 2006; Lai et al. 2008, Alliez et al. 2003; Marinov Kobbelt 2004, Myles et al. 2010; 2014a; Pietroni et. 2016, Zadravec et al. 2010; PengWonka 2013, Ray et al. 2006; Jakob et al. 2015] Instant field-aligned meshes

T-junctions = where features start or terminate

- Automatic quad meshing
- Merge separately-developed spline surfaces

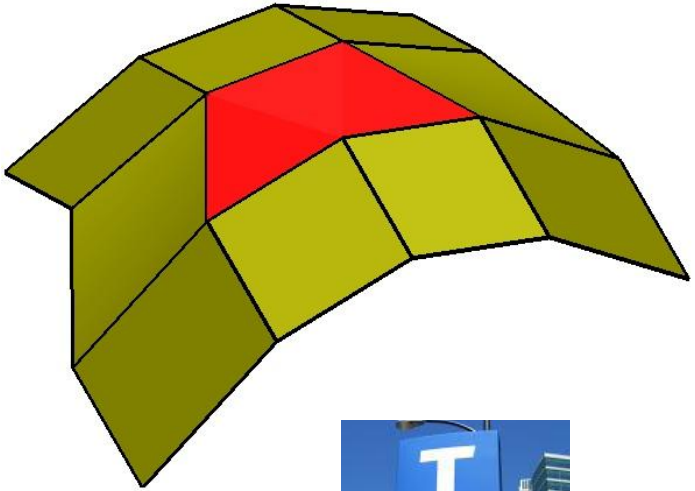
Overview

- Configurations
- Alternatives
  - T-junctions → T-splines ?
  - T-junctions → Catmull-Clark subdivision ?
  - T-junctions → Geometric continuity ?
- Construction
- T- G-splines = merging meshes T1 T2 T3

The extremely short (use it) presentation

T-G-spline surface construction: executive version

Turn into smooth surface



Highlight lines

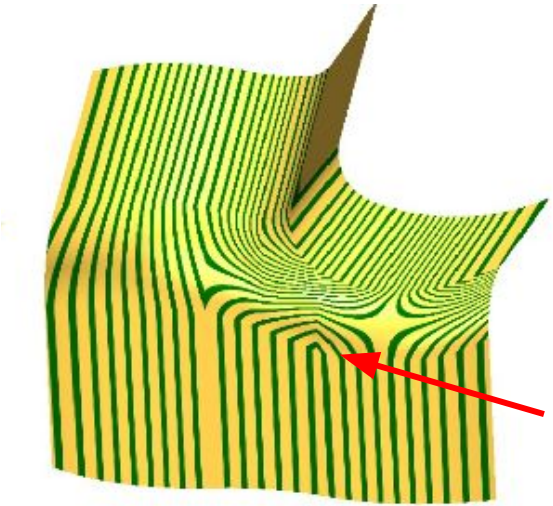
Farin

(a) reflection lines

uniform, parallel = good (unless feature)

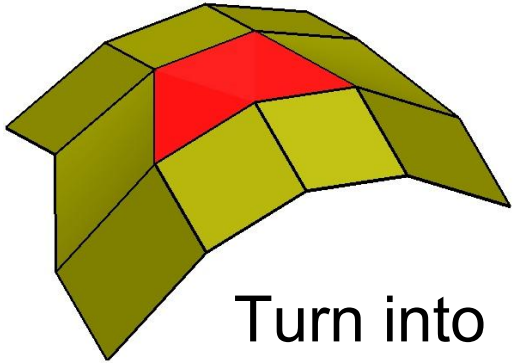
Highlight lines

reflection lines



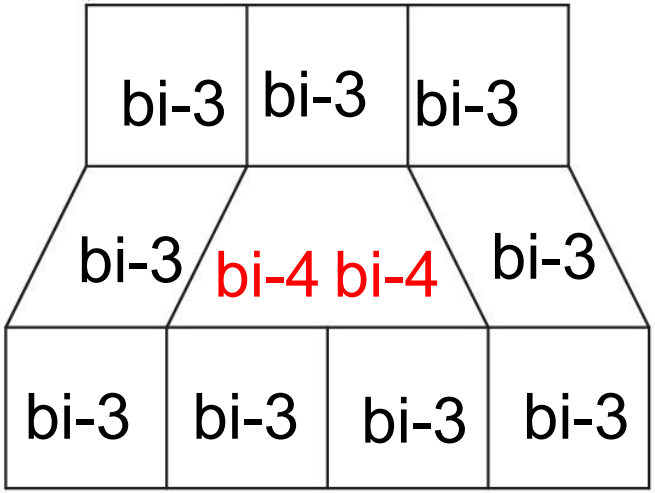
highlight lines

T-G-spline surface construction: executive version



Turn into smooth surface

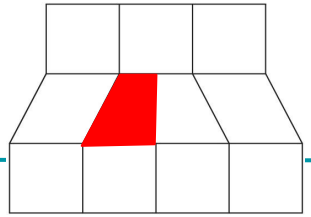
Outer can be irregular



bi-3 = bi-cubic = polynomial patch of coordinate degree 4

assume isolated T-junction

T-G-spline bi-4 = 5 x 5 Bezier coefficients



$$\begin{bmatrix} 4 & 16 & 16 & 4 & 0 \\ 16 & 64 & 64 & 16 & 0 \\ 4 & 16 & 16 & 4 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}
\begin{bmatrix} 5 & 16 & \frac{11}{2} & 0 \\ 10 & 64 & 22 & 0 \\ \frac{5}{2} & 16 & \frac{11}{2} & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}
\begin{bmatrix} \frac{3}{2} & 15 & \frac{15}{2} & 0 \\ 6 & 60 & 30 & 0 \\ \frac{11}{8} & 13 & \frac{55}{4} & 1 \\ 0 & 0 & 0 & \frac{-1}{8} \end{bmatrix}
\begin{bmatrix} \frac{7}{8} & \frac{107}{8} & \frac{77}{8} & \frac{1}{8} \\ \frac{7}{2} & \frac{107}{2} & \frac{77}{2} & \frac{1}{2} \\ \frac{5}{8} & 9 & \frac{33}{2} & 3 \\ 0 & 0 & 0 & \frac{-1}{8} \end{bmatrix}
\begin{bmatrix} \frac{1}{2} & \frac{23}{2} & \frac{23}{2} & \frac{1}{2} \\ 6 & 46 & 46 & 6 \\ \frac{1}{4} & \frac{33}{2} & \frac{33}{2} & \frac{1}{4} \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 4 & 4 & 1 & 0 \\ 16 & 64 & 64 & 16 & 0 \\ 7 & 28 & 28 & 7 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}
\begin{bmatrix} \frac{1}{4} & 4 & \frac{7}{4} & 0 \\ \frac{17}{2} & 64 & \frac{47}{2} & 0 \\ 4 & 28 & 10 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}
\begin{bmatrix} \frac{1}{8} & \frac{13}{4} & \frac{21}{8} & 0 \\ 5 & 58 & 33 & 0 \\ \frac{67}{32} & \frac{85}{4} & \frac{267}{16} & \frac{9}{4} \\ 0 & 0 & 0 & \frac{-9}{32} \end{bmatrix}
\begin{bmatrix} \frac{1}{8} & \frac{23}{8} & \frac{23}{8} & \frac{1}{8} \\ \frac{23}{8} & \frac{413}{8} & \frac{323}{8} & \frac{7}{8} \\ 1 & 15 & \frac{161}{8} & 6 \\ 0 & 0 & 0 & \frac{-1}{8} \end{bmatrix}
\begin{bmatrix} \frac{1}{8} & \frac{23}{8} & \frac{23}{8} & \frac{1}{8} \\ 2 & 46 & 46 & 2 \\ \frac{7}{16} & \frac{21}{2} & \frac{161}{16} & \frac{21}{2} \\ 0 & 0 & 0 & \frac{7}{16} \end{bmatrix}$$

$$\begin{bmatrix} 0 & 0 & 0 & 0 & 0 \\ 12 & 48 & 48 & 12 & 0 \\ 12 & 48 & 48 & 12 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}
\begin{bmatrix} 0 & 0 & 0 & 0 \\ \frac{19}{8} & 48 & \frac{39}{4} & 0 \\ 6 & 48 & 18 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}
\begin{bmatrix} \frac{-1}{6} & \frac{-1}{3} & \frac{1}{2} & 0 \\ \frac{19}{8} & 43 & 25 & \frac{9}{8} \\ \frac{7}{6} & \frac{116}{3} & 31 & \frac{4}{3} \\ 0 & \frac{-8}{3} & \frac{-8}{3} & 0 \end{bmatrix}
\begin{bmatrix} \frac{-1}{16} & \frac{-5}{16} & \frac{5}{16} & \frac{1}{16} \\ \frac{7}{2} & \frac{207}{8} & \frac{231}{8} & \frac{9}{2} \\ \frac{9}{16} & \frac{47}{2} & \frac{87}{2} & \frac{9}{2} \\ 0 & \frac{-1}{4} & \frac{-1}{4} & 0 \end{bmatrix}
\begin{bmatrix} 0 & 0 & 0 & 0 & 0 \\ 3 & 33 & 33 & 3 & 0 \\ \frac{1}{4} & 14 & \frac{87}{2} & 14 & \frac{1}{4} \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

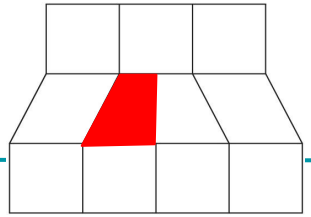
$$\begin{bmatrix} 0 & 0 & 0 & 0 & 0 \\ 7 & 28 & 28 & 7 & 0 \\ 16 & 64 & 64 & 16 & 0 \\ 1 & 4 & 4 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}
\begin{bmatrix} 0 & 0 & 0 & 0 \\ \frac{17}{8} & 28 & \frac{25}{8} & 0 \\ \frac{11}{2} & 64 & \frac{53}{2} & 0 \\ \frac{5}{8} & 4 & \frac{11}{8} & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}
\begin{bmatrix} 0 & 0 & 0 & 0 \\ \frac{3}{2} & 25 & 14 & \frac{3}{2} \\ 0 & 52 & 44 & 0 \\ 0 & 4 & 2 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}
\begin{bmatrix} 0 & 0 & 0 & 0 \\ \frac{7}{2} & \frac{125}{16} & \frac{235}{16} & \frac{7}{2} \\ 0 & \frac{59}{2} & 64 & \frac{5}{2} \\ 0 & \frac{17}{8} & 4 & \frac{-1}{8} \\ 0 & 0 & 0 & 0 \end{bmatrix}
\begin{bmatrix} 0 & 0 & 0 & 0 \\ \frac{7}{2} & \frac{85}{2} & \frac{85}{2} & \frac{7}{2} \\ 0 & 16 & 64 & 16 & 0 \\ 0 & 1 & 4 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

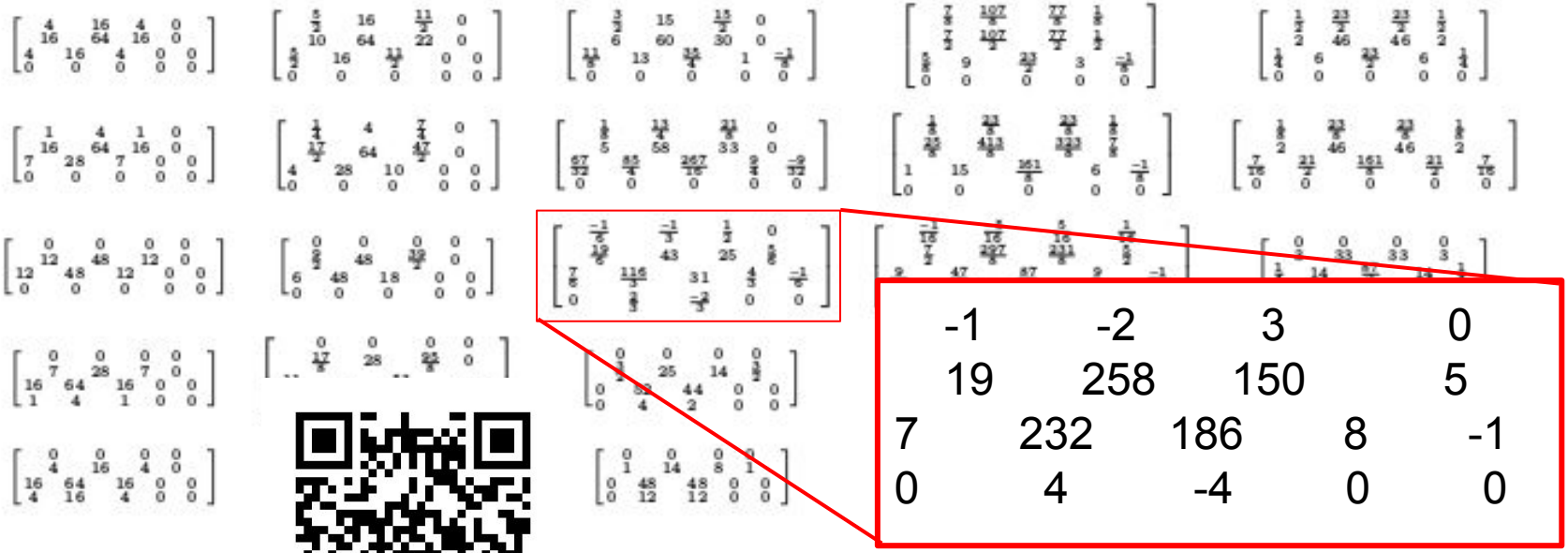
$$\begin{bmatrix} 0 & 0 & 0 & 0 \\ 4 & 16 & 16 & 4 \\ 16 & 64 & 64 & 16 \\ 4 & 16 & 16 & 4 \\ 0 & 0 & 0 & 0 \end{bmatrix}
\begin{bmatrix} 0 & 0 & 0 & 0 \\ 1 & 16 & 7 & 0 \\ 4 & 64 & 28 & 0 \\ 1 & 16 & 7 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}
\begin{bmatrix} 0 & 0 & 0 & 0 \\ 1 & 14 & 8 & 1 \\ 4 & 48 & 48 & 4 \\ 0 & 12 & 12 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}
\begin{bmatrix} 0 & 0 & 0 & 0 \\ 2 & \frac{23}{2} & \frac{17}{2} & 2 \\ 0 & 28 & 64 & 4 \\ 0 & 7 & 16 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix}
\begin{bmatrix} 0 & 0 & 0 & 0 \\ 2 & 10 & 10 & 2 \\ 0 & 16 & 64 & 16 \\ 0 & 4 & 16 & 4 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

/144

Short explicit formulas (stencils)

T-G-spline bi-4 = 5x5 coefficient stencils





-1	-2	3		0
19	258	150		5
7	232	186	8	-1
0	4	-4	0	0

F# Code:

Short explicit formulas /864

Long version: Overview

- **Some T-configurations**
- Alternatives
  - T-junctions → T-splines ?
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  - T-junctions → Geometric continuity ?
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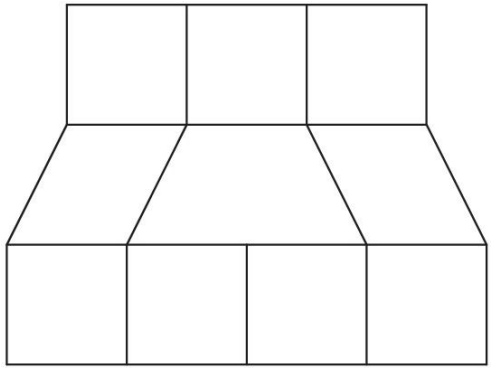
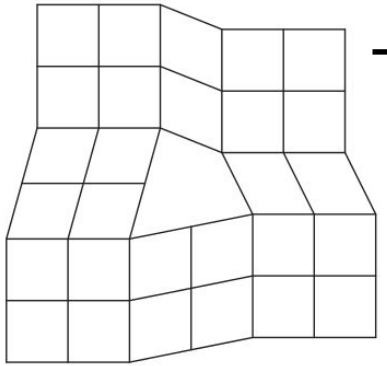
T-junctions (Extended) Configurations T1 T2 T3



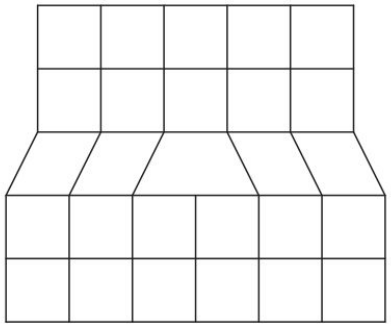
Extended to regular bi-3 neighborhood



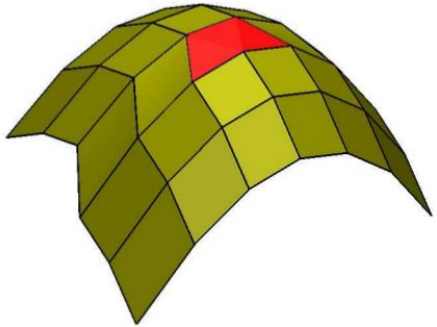
T2

(a) $\ddot{T}$-net

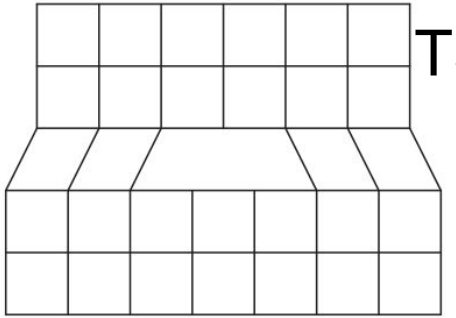
T1



(a) $\dot{T}$-net layout



(b) convex $\dot{T}$-net



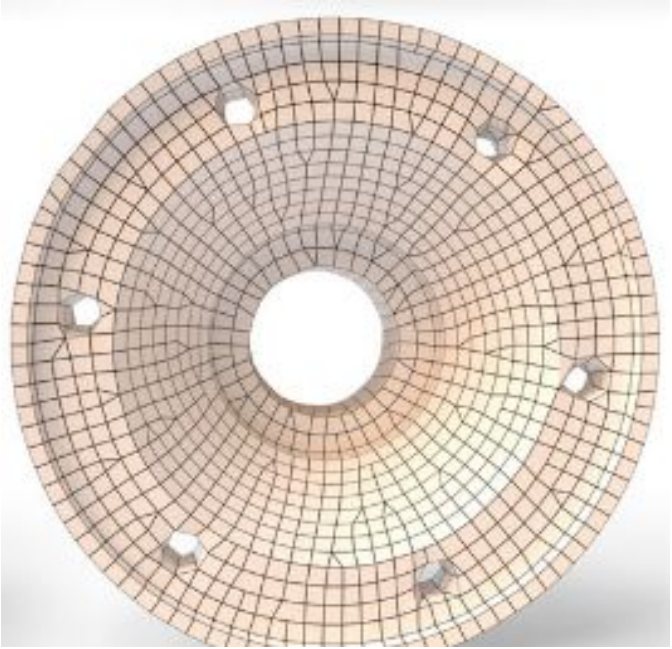
T3

(b) $\dddot{T}$-net

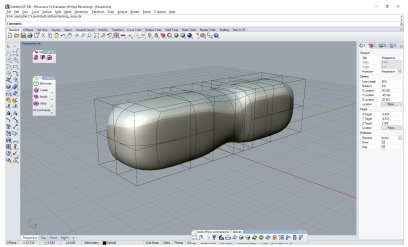
Configurations: Meshing & Surface quality

Trade off between meshing work and surface construction:

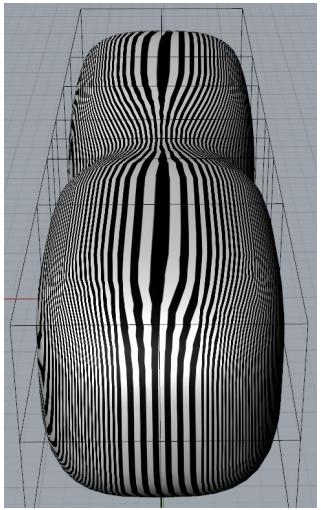
Bad mesh → bad surface

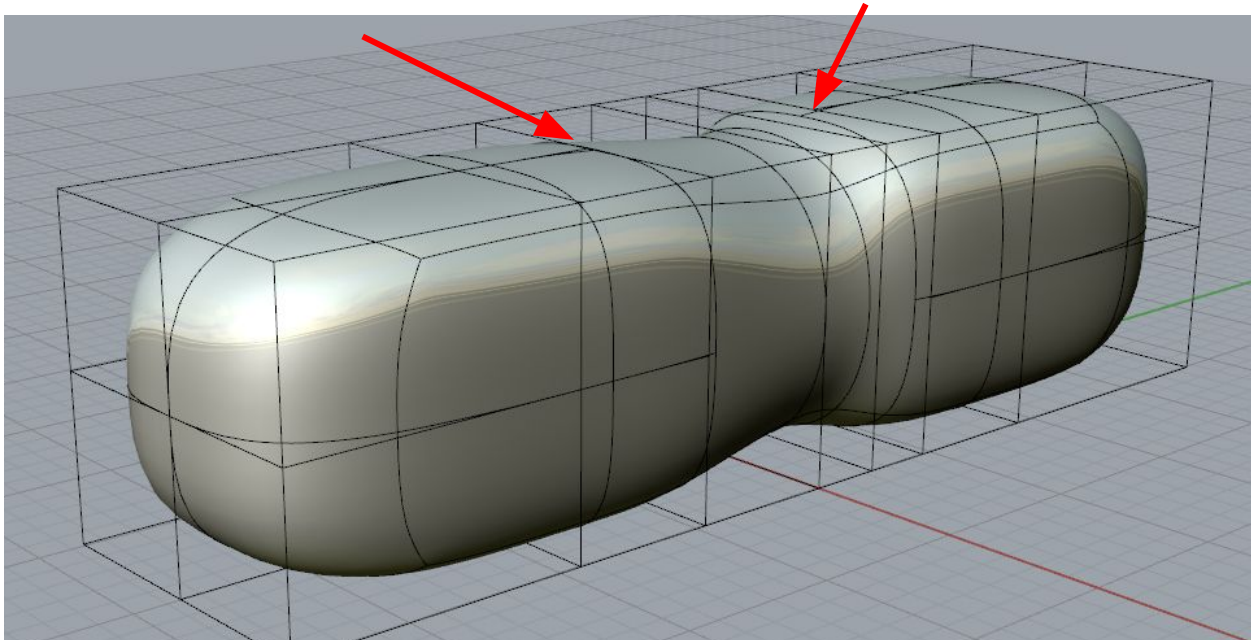


T-junctions -- use with care !



bad design or intended?

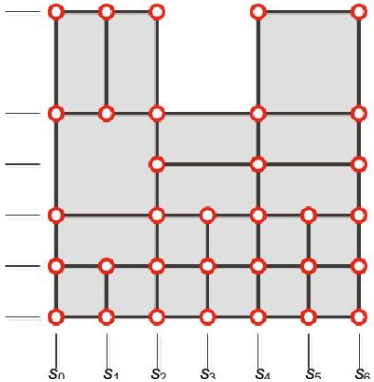


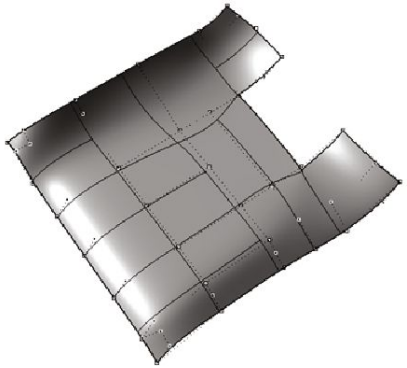


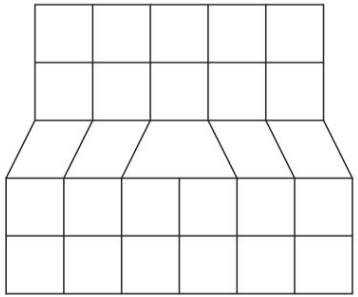
Long version: Overview

- Some T-configurations
- **Alternatives**
  - T-junctions → T-splines ?
  - T-junctions → Catmull-Clark subdivision ?
  - T-junctions → Geometric continuity ?
- Construction
- T-G-splines = merging meshes T1 T2 T3

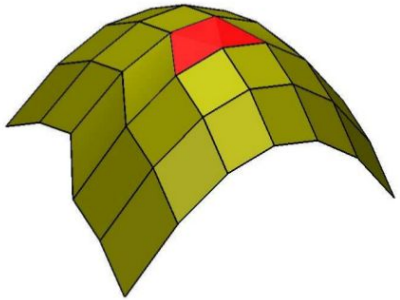
T-junctions → T-splines ?







(a) $\dot{T}$-net layout



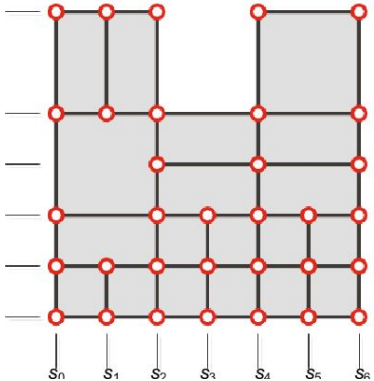
(b) convex $\dot{T}$-net

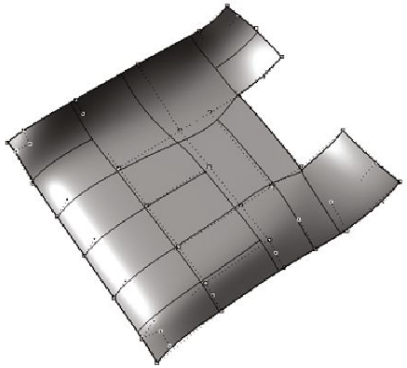
T Sederberg, J Zheng, A Bakinov, A. Nasri 03

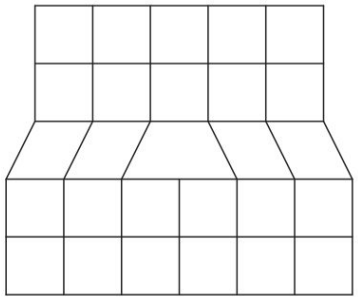
global

local

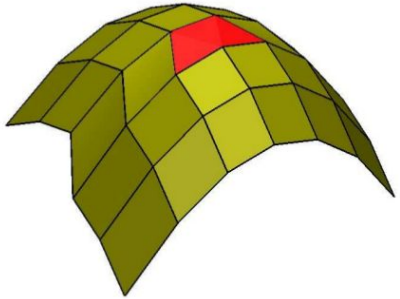
T-junctions → T-splines (hierarchical splines)?







(a) $\dot{T}$-net layout



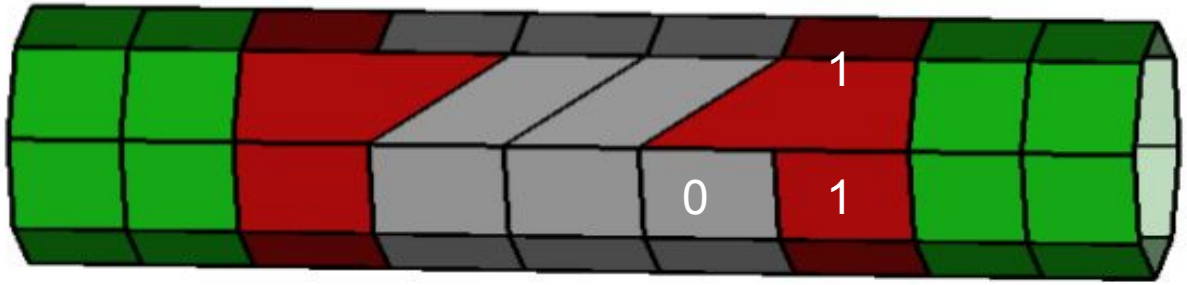
(b) convex $\dot{T}$-net

"cast"
global

"band-aid"
local

where T-splines fail

T-splines: “Rule 1”
sum of knot intervals on opposing edges of any face must be equal

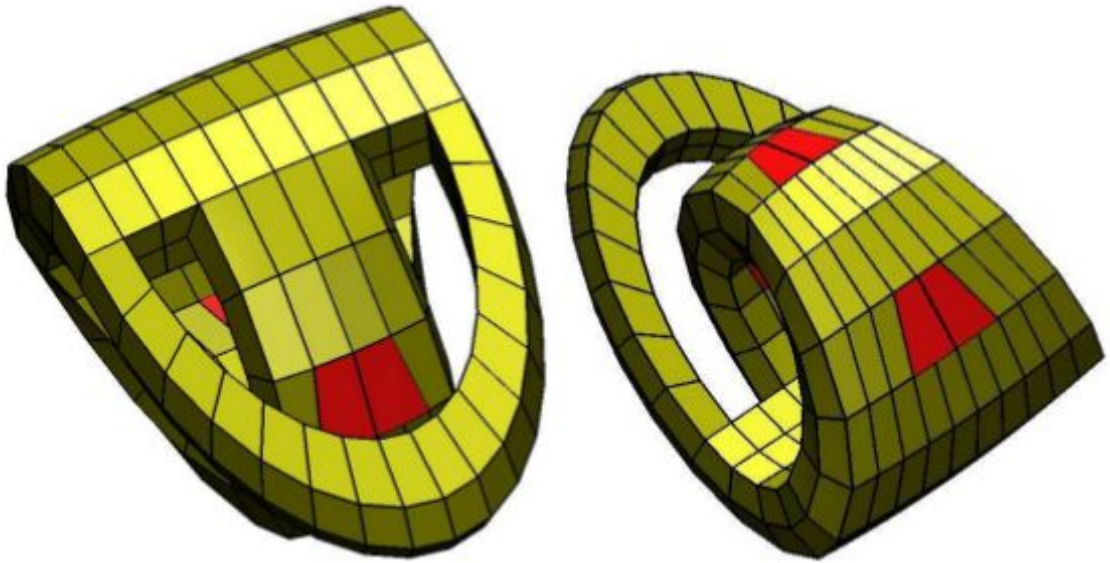


Dagstuhl 2016

→ horizontal knot intervals of the grey helical strip have 0 knot intervals
→ no smooth T-spline parameterization!

Denis Zorin et al: two different knots sets on either side of an edge

where T-splines (hierarchical splines) fail



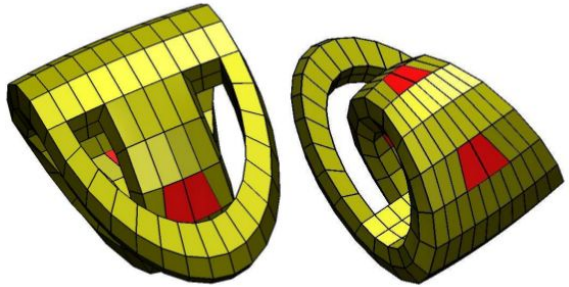
no smooth T-spline parameterization!

When (not) to use Hierarchical Splines

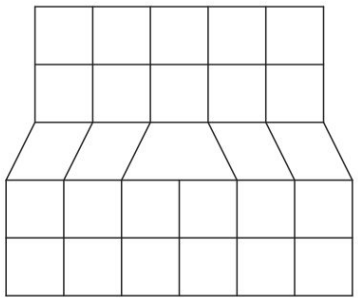
Hierarchical splines

[Kraft1998;Seder2003;Giannelli12;Dokken13,Kang15]

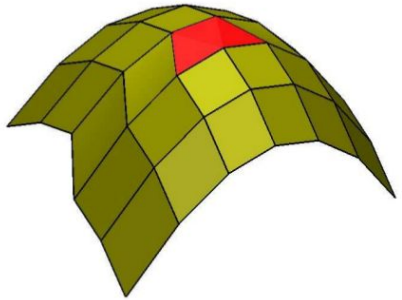
- well-suited for *introducing* T-junctions in quad meshes (refinement)
- **not** naturally suited for creating smooth surfaces from *given* quad meshes with T-junctions.



T-junctions → **Catmull-Clark** subdivision?

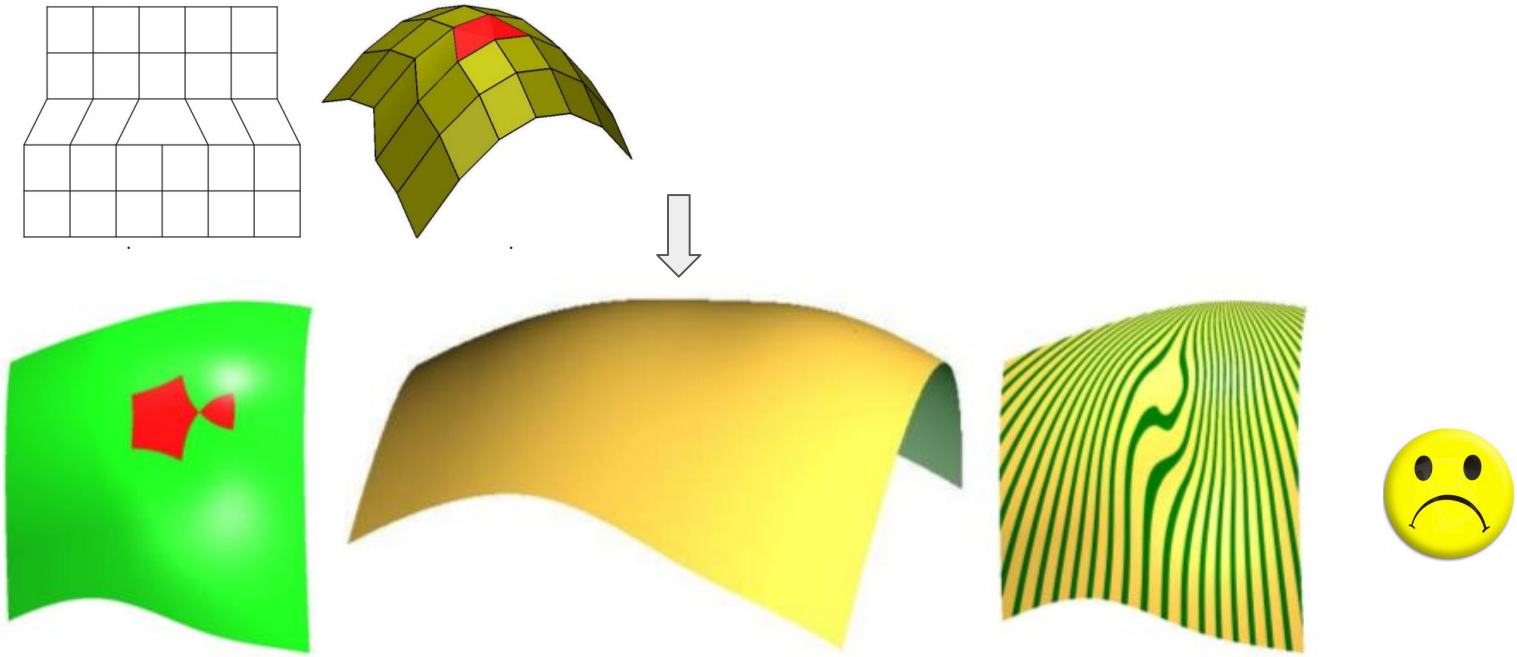


(a) $\dot{T}$-net layout

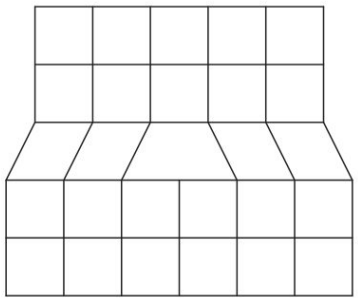


(b) convex $\dot{T}$-net

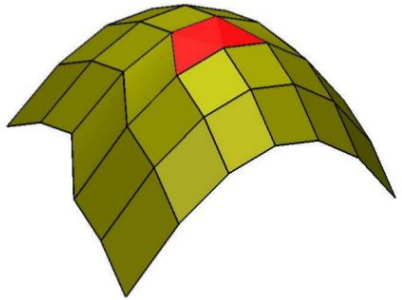
How Catmull-Clark subdivision fails



T-junctions → Geometric Continuity?

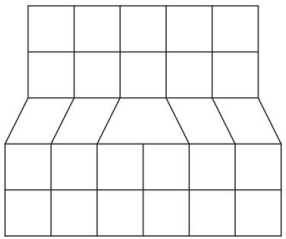


(a) $\dot{T}$-net layout

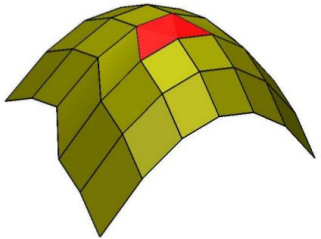


(b) convex $\dot{T}$-net

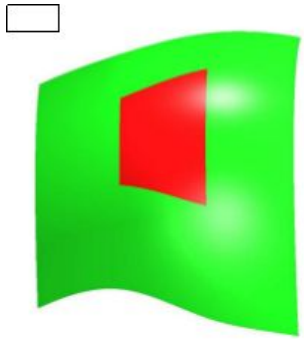
T1-G-spline surface construction

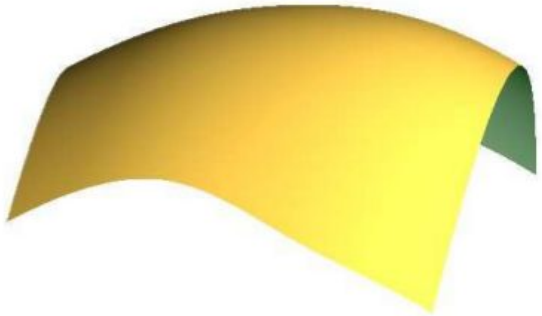


(a) $\dot{T}$-net layout

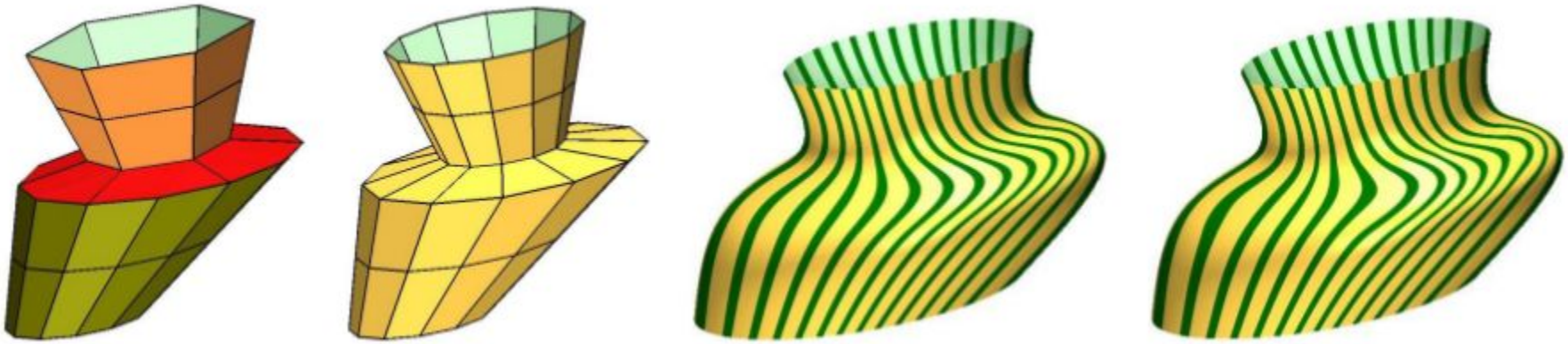


(b) convex $\dot{T}$-net





(T)-spline vs T-G-spline



Summary: T-junctions in spline surfaces

T-splines (global parameterization)

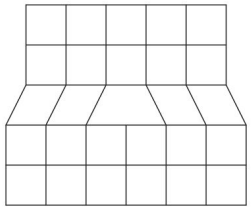
Catmull-Clark (local, shape?)

T-G-splines (local)

Long version: Overview

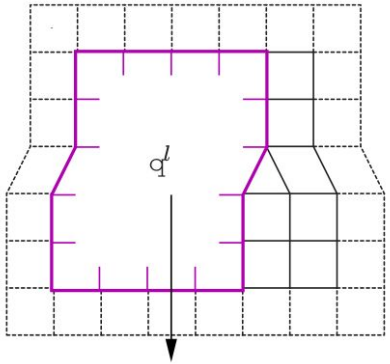
- Some T-configurations
- Alternatives
  - T-junctions → T-splines ?
  - T-junctions → Catmull-Clark subdivision ?
  - T-junctions → Geometric continuity ?
- **Construction**
- T-G-splines = merging meshes T1 T2 T3

T1-G-spline surface construction

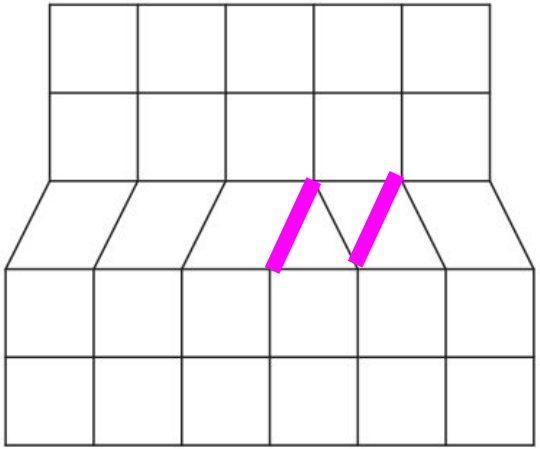


(a) $\dot{T}$-net layout

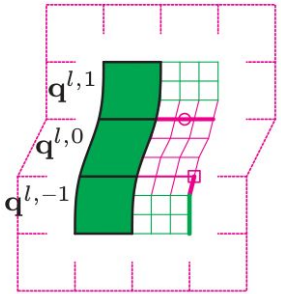
T1



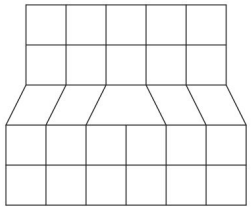
(a) $\mathfrak{q}^l$



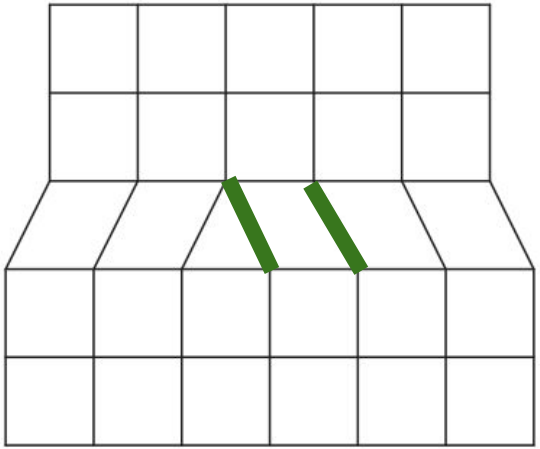
(a) $\dot{T}$-net layout



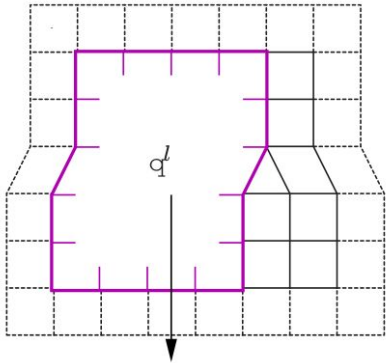
T1-G-spline surface construction



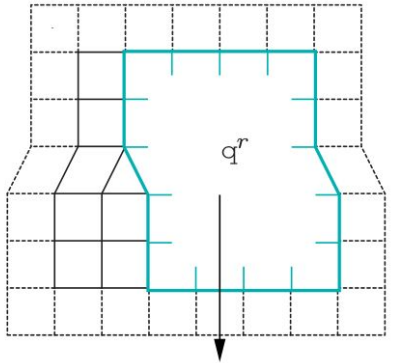
(a) $\dot{T}$-net layout



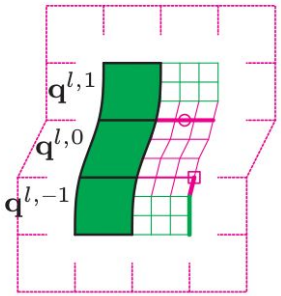
(a) $\dot{T}$-net layout

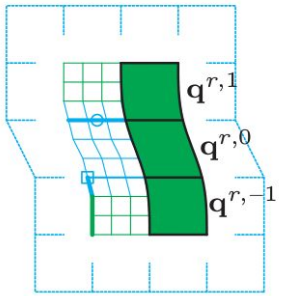


(a) $\mathfrak{q}^l$

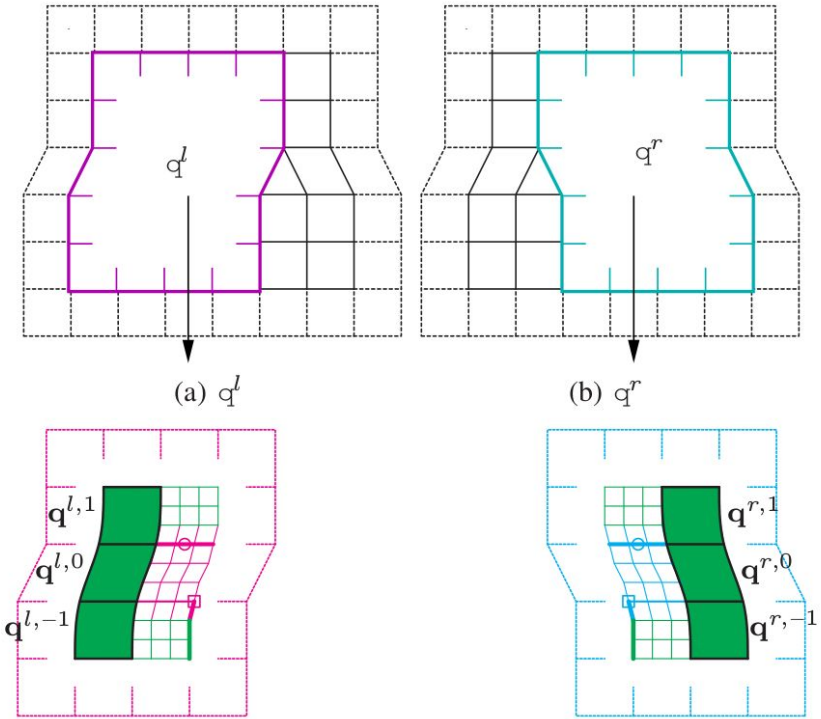


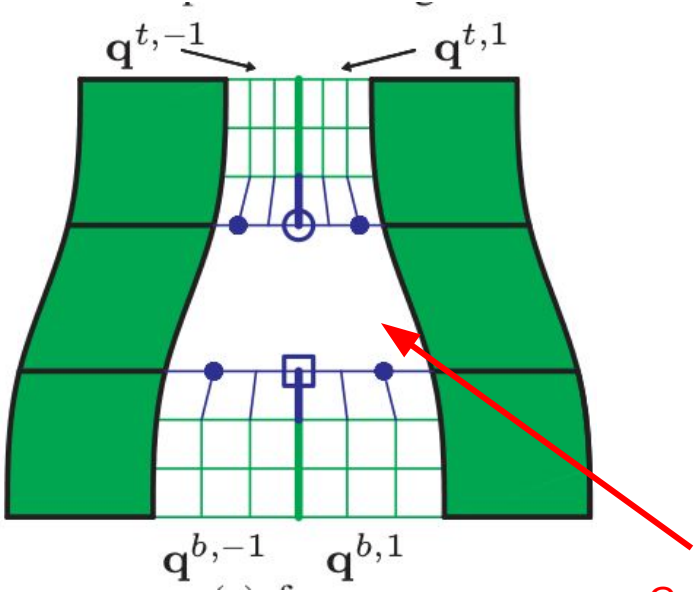
(b) $\mathfrak{q}^r$





T1-G-spline surface construction

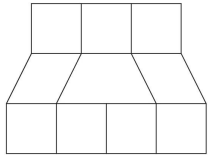


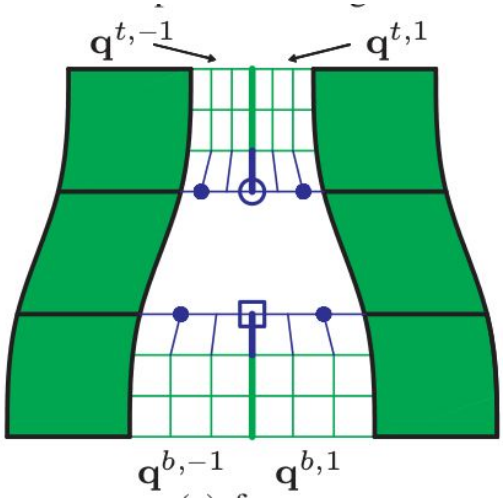


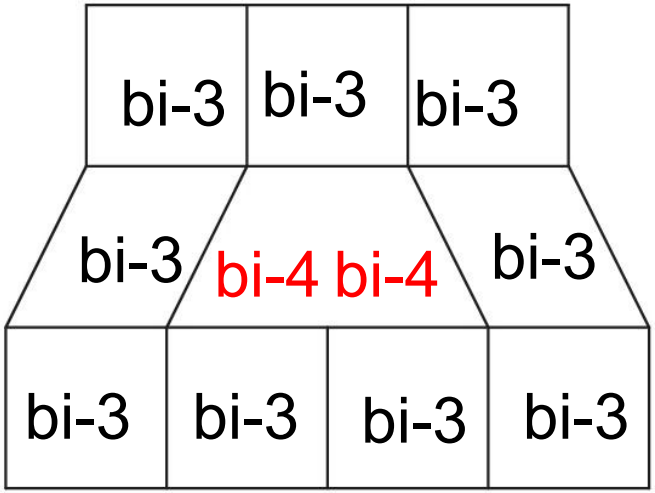
Short explicit formulas

Geometric continuity

T1-G-spline surface construction

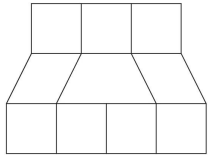


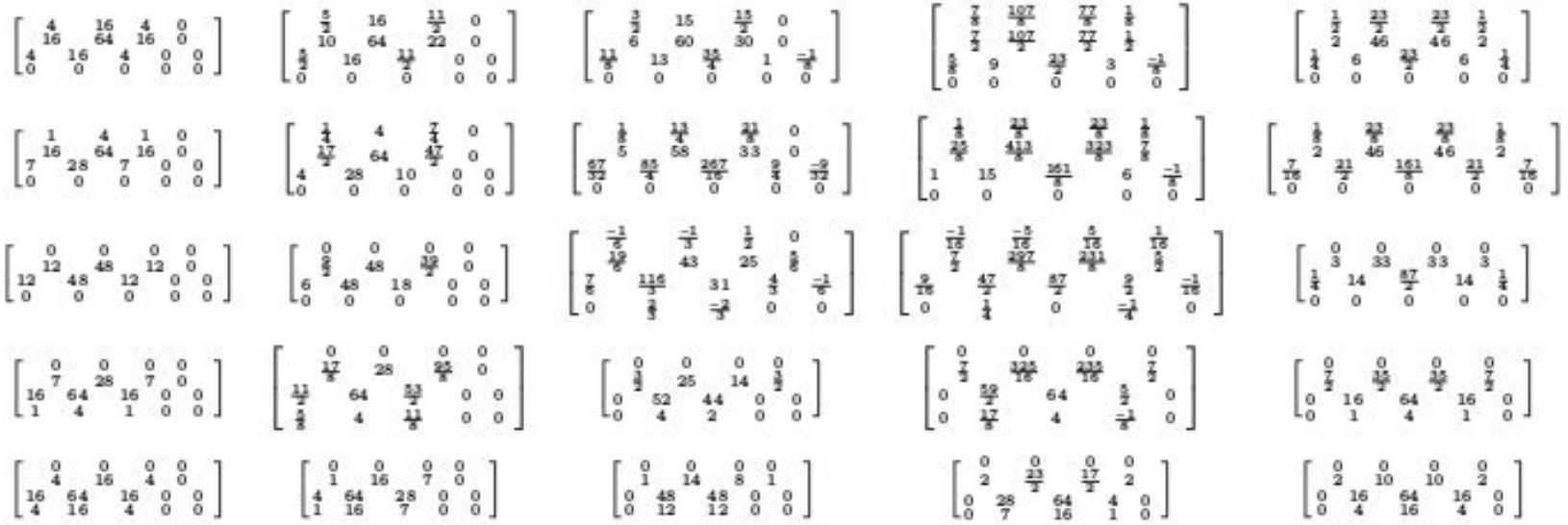




Short explicit formulas

T1-G-spline surface construction

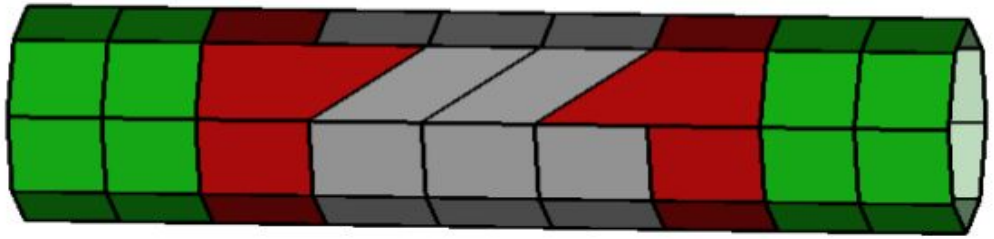


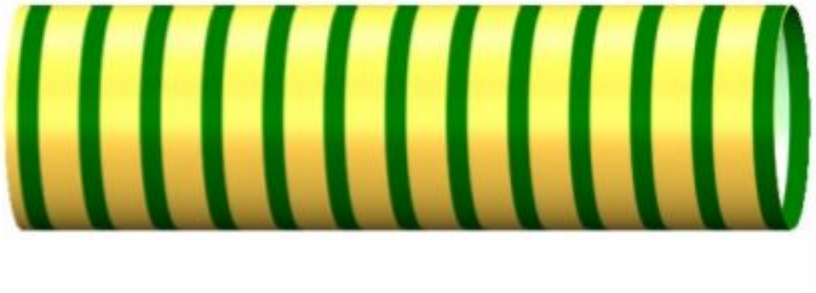


/144

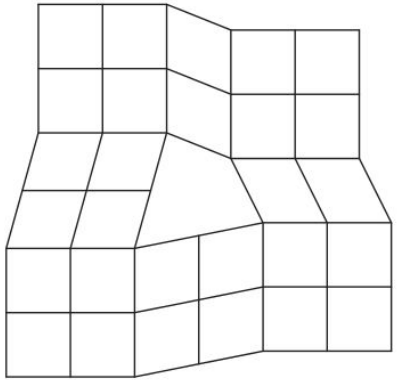
Short explicit formulas

T1-G-spline surface construction

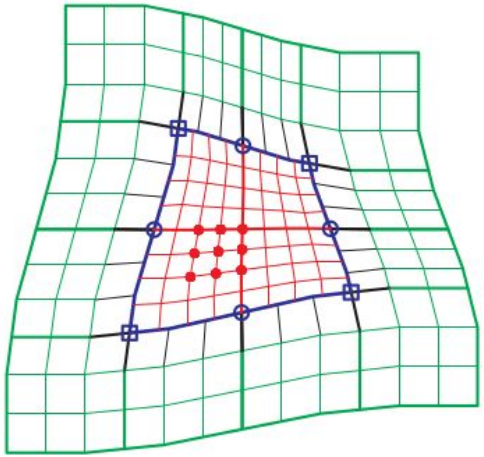


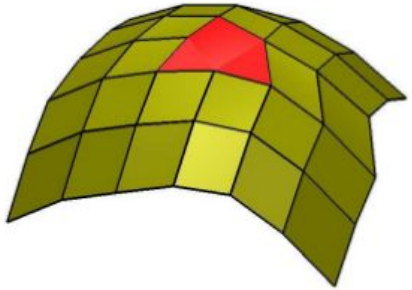


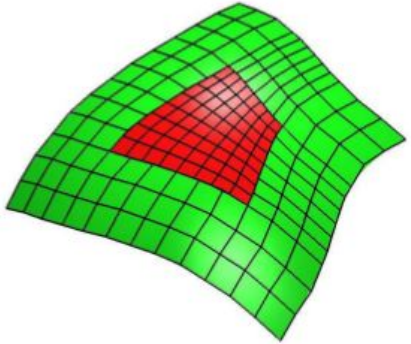
T2-G-spline surface construction



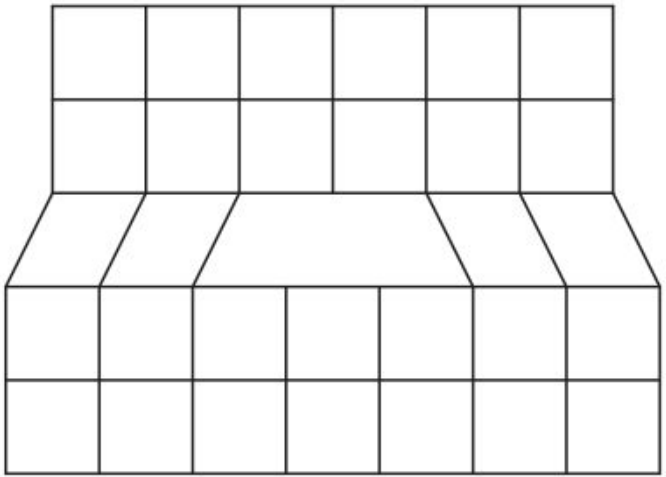
(a) $\ddot{T}$-net



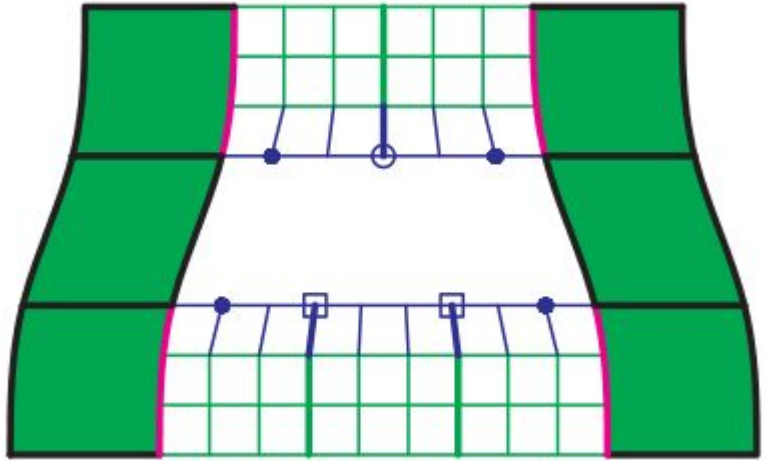




T3-G-spline surface construction **recommended**



(b) $\dddot{T}$-net



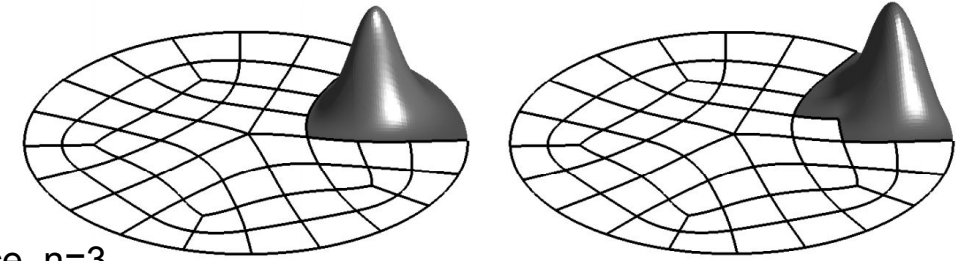
T3

Long version: Overview

- Some T-configurations
- Alternatives
  - T-junctions → T-splines ?
  - T-junctions → Catmull-Clark subdivision ?
  - T-junctions → Geometric continuity ?
- Construction T1 T2 T3
- **T-G-splines**

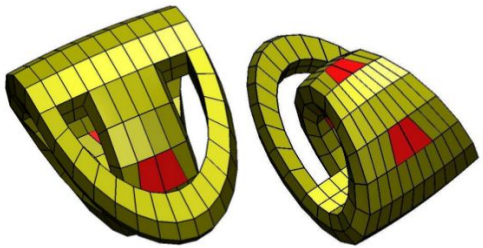
Long version: Overview

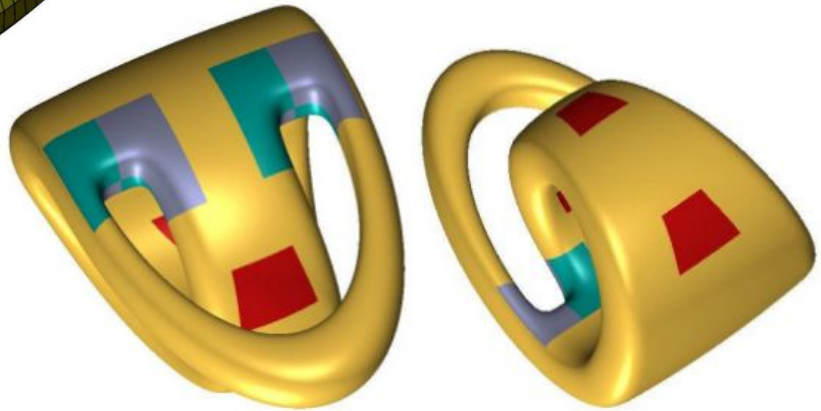
- Some T-configurations
- Alternatives
  - T-junctions → T-splines ?
  - T-junctions → Catmull-Clark subdivision ?
  - T-junctions → Geometric continuity ?
- Construction T1 T2 T3
- **T-G-splines**

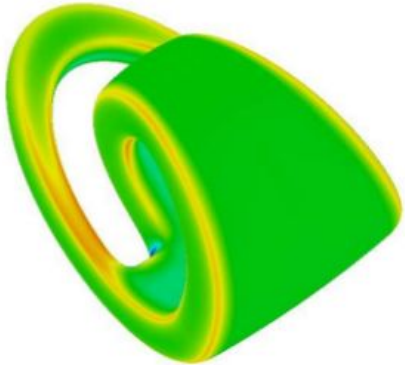


valence n=3

Combining T-junctions with other irregularities



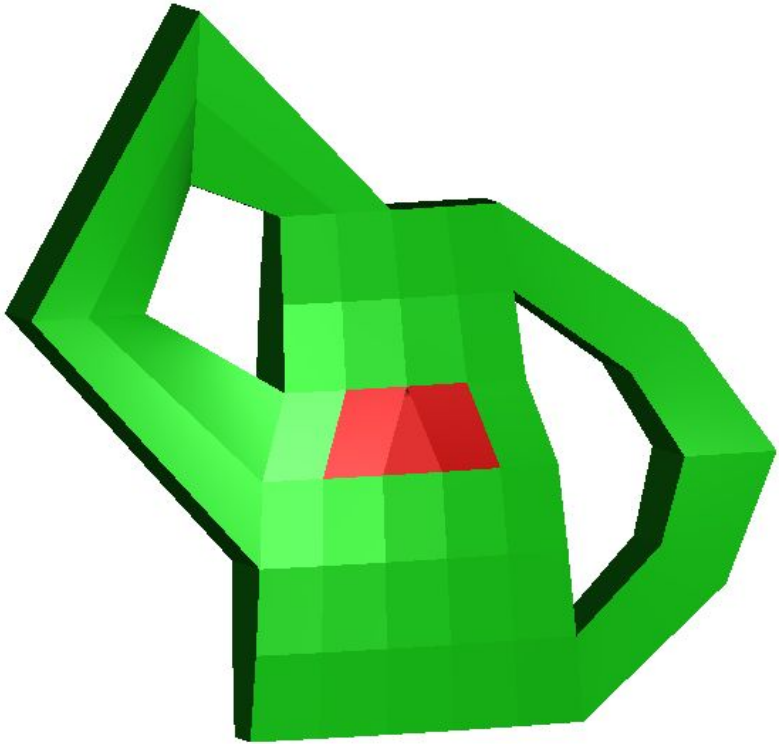




bi-3 (gold) + bi-4 T-G-spline

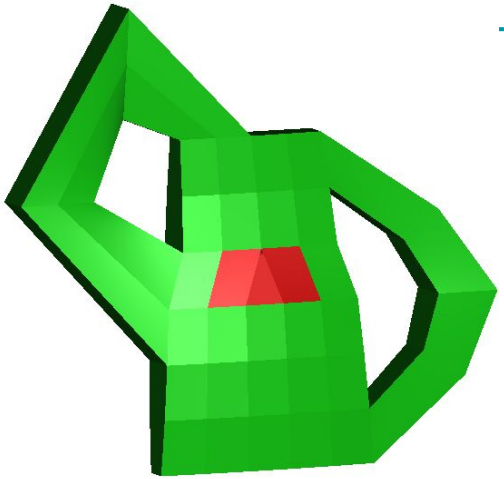
mean curvature

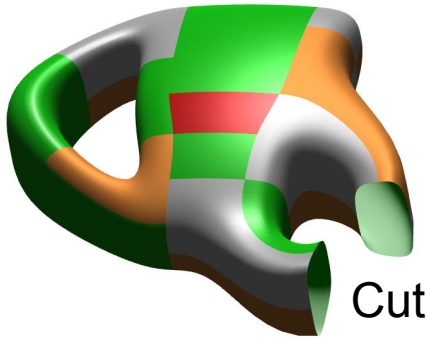
T3-G-spline surface construction



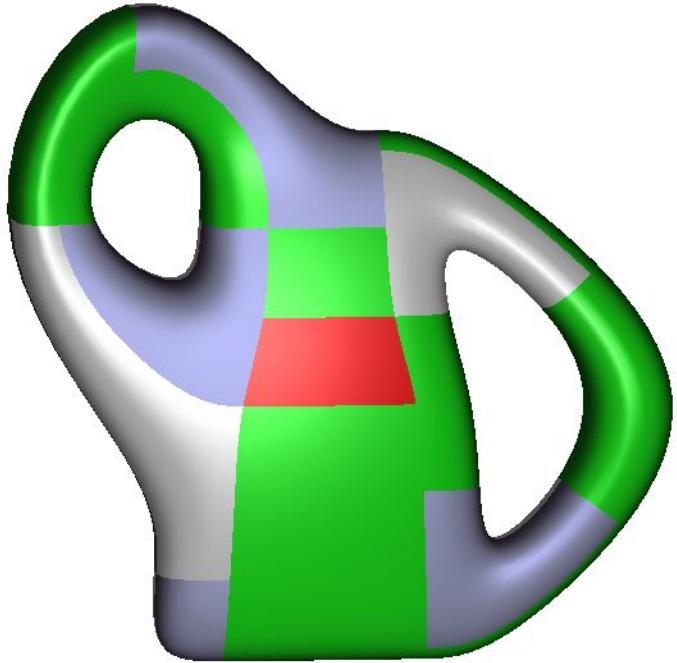
A truly watertight tea pot!

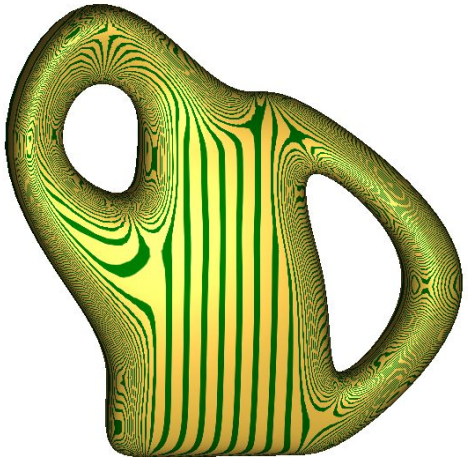
T3-G-spline surface construction of the *truly watertight tea pot*



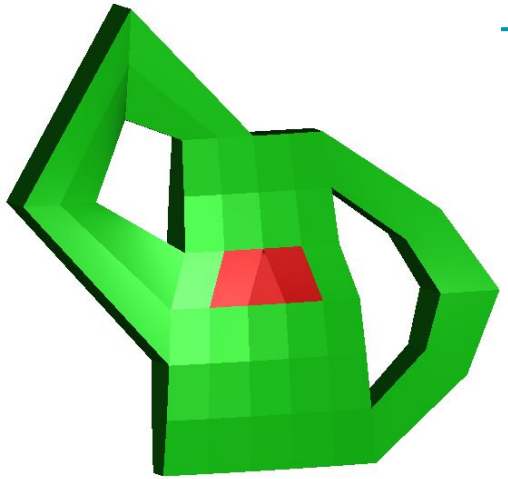


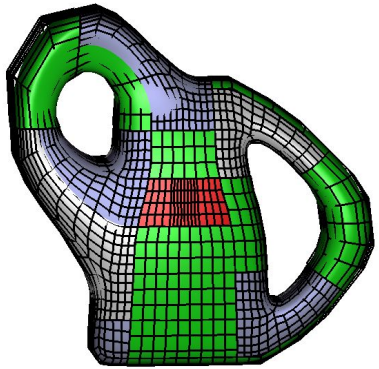
Cut open

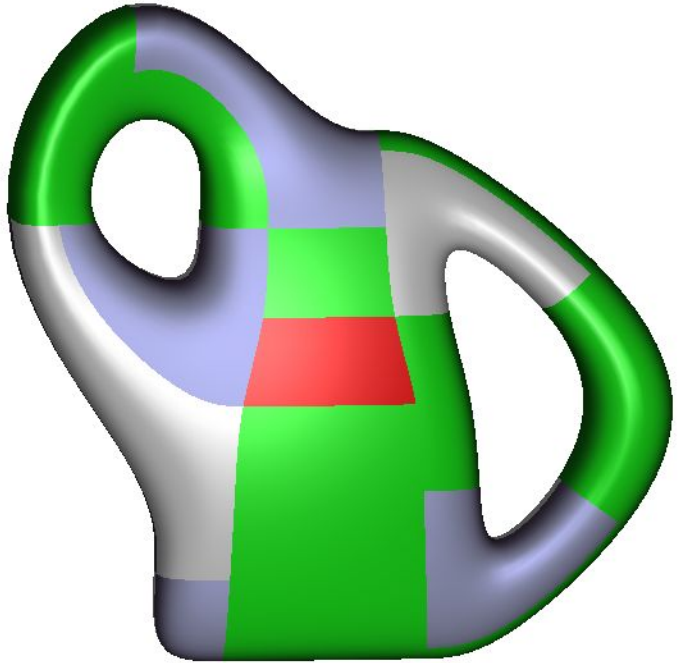


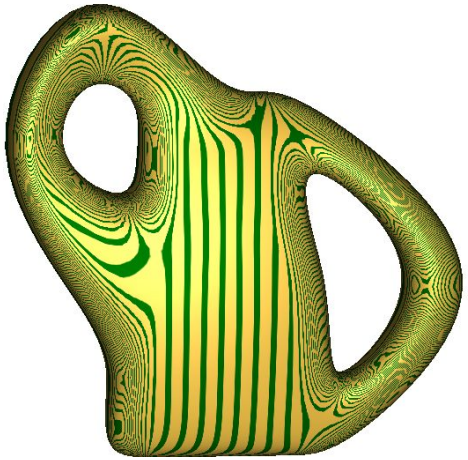


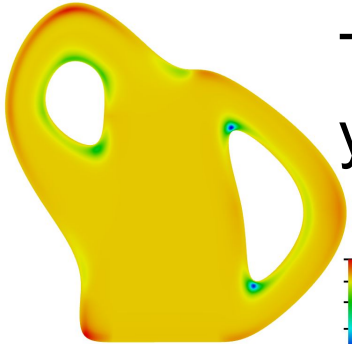
T3-G-spline surface construction of the *truly watertight tea pot*











Thank you