T-junctions in spline surfaces

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T-junctions in spline surfaces

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T-junctions = where features start or terminate

“Make irregularities (T-junctions) disappear”
T-junctions = where features start or terminate

“Make T-junction disappear”
T-junctions = where features start or terminate

➢ Automatic quad meshing

strict quad-meshing [Bommes et al. 2012; Vaxman et al. 2016]  complex and global

T-junctions = where features start or terminate

➢ Automatic quad meshing

➢ Merge separately-developed spline surfaces
Overview

➢ Configurations
➢ Alternatives
   ○ T-junctions → T-splines ?
   ○ T-junctions → Catmull-Clark subdivision ?
   ○ T-junctions → Geometric continuity ?
➢ Construction
➢ T- G-splines = merging meshes T1 T2 T3

The extremely short (use it) presentation
T-G-spline surface construction: executive version

Turn into smooth surface
Highlight lines

T-junctions in Spline Surfaces

uniform, parallel = good (unless feature)

(a) reflection lines
Highlight lines

T-junctions in Spline Surfaces

reflection lines

highlight lines
**T-G-spline surface construction: executive version**

Outer can be irregular

Turn into smooth surface

**bi-3** = bi-cubic = polynomial patch of coordinate degree 4

assume isolated T-junction
T-G-spline $bi-4 = 5 \times 5$ Bezier coefficients

\[
\begin{bmatrix}
4 & 16 & 64 & 32 & 0 \\
4 & 16 & 64 & 32 & 0 \\
0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 \\
\end{bmatrix}
\]

\[
\begin{bmatrix}
\frac{1}{2} & 1 & 4 & 0 & 0 \\
\frac{1}{2} & 1 & 4 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 \\
\end{bmatrix}
\]

/144 Short explicit formulas (stencils)
T-G-spline bi-4 = 5x5 coefficient

F# Code:

```
-1  -2   3   0
19  258  150  5
 7  232  186  8  -1
 0   4  -4   0  0
```

Short explicit formulas

/864
Some T-configurations

Alternatives
- T-junctions $\rightarrow$ T-splines ?
- T-junctions $\rightarrow$ Catmull-Clark subdivision ?
- T-junctions $\rightarrow$ Geometric continuity ?

Construction

T-G-splines = merging meshes T1 T2 T3
T-junctions (Extended) Configurations  T1 T2 T3

Extended to regular bi-3 neighborhood

(a) $\hat{T}$-net layout
(b) convex $\hat{T}$-net

(a) $\hat{T}$-net
(b) $\bar{T}$-net
Configurations: **Meshing & Surface quality**

**Trade off between meshing work and surface construction:**

Bad mesh $\rightarrow$ bad surface
T-junctions -- use with care!

bad design or intended?
Long version: Overview

➢ Some T-configurations

➢ Alternatives
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  ○ T-junctions → Geometric continuity ?

➢ Construction

➢ T-G-splines = merging meshes T1 T2 T3
T-junctions → T-splines ?

T Sederberg, J Zheng, A Bakinov, A. Nasri 03

global

local
T-junctions → T-splines (hierarchical splines)?

Approach 1: T-splines

“cast” global

“band-aid” local
where T-splines fail

**T-splines**: “Rule 1”
sum of knot intervals on opposing edges of any face must be equal

→ horizontal knot intervals of the grey helical strip have 0 knot intervals
→ no smooth T-spline parameterization!

Denis Zorin et al: two different knots sets on either side of an edge
where T-splines (hierarchical splines) fail

no smooth T-spline parameterization!
When (not) to use Hierarchical Splines

Hierarchical splines
[Kraft1998;Seder2003;Giannelli12;Dokken13,Kang15]

➢ well-suited for *introducing* T-junctions in quad meshes (refinement)
➢ **not** naturally suited for creating smooth surfaces from *given* quad meshes with T-junctions.
T-junctions $\rightarrow$ Catmull-Clark subdivision?

(a) $\hat{T}$-net layout

(b) convex $\hat{T}$-net
How Catmull-Clark subdivision fails

T-junctions in Spline Surfaces
T-junctions → Geometric Continuity?

(a) $T$-net layout

(b) convex $T$-net
T1-G-spline surface construction

(a) $\tilde{T}$-net layout

(b) convex $\tilde{T}$-net
(T)-spline vs T-G-spline
Summary: T-junctions in spline surfaces

T-splines (global parameterization)

Catmull-Clark (local, shape?)

T-G-splines (local) 😊
Long version: Overview

- Some T-configurations
- Alternatives
  - T-junctions → T-splines?
  - T-junctions → Catmull-Clark subdivision?
  - T-junctions → Geometric continuity?
- Construction
  - T-G-splines = merging meshes T1 T2 T3
**T1-G-spline surface construction**

(a) $T$-net layout

(a) $\hat{T}$-net layout
**T1-G-spline** surface construction

(a) $T$-net layout

(b) $\dot{T}$-net layout
T1-G-spline surface construction

T-junctions in Spline Surfaces

Short explicit formulas
T1-G-spline surface construction

Short explicit formulas
T1-G-spline surface construction

Short explicit formulas
T1-G-spline surface construction
T2-G-spline surface construction

T-junctions in Spline Surfaces
T3-G-spline surface construction recommended

(b) T-net

T3-junctions in Spline Surfaces
Long version: Overview

➢ Some T-configurations
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  ○ T-junctions → Catmull-Clark subdivision ?
  ○ T-junctions → Geometric continuity ?
➢ Construction T1 T2 T3
➢ T-G-splines
Long version: Overview

➢ Some T-configurations
➢ Alternatives
  ○ T-junctions → T-splines ?
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➢ Construction T1 T2 T3
➢ T-G-splines

valence n=3
Combining T-junctions with other irregularities

bi-3 (gold) + bi-4 T-G-spline

mean curvature
T3-G-spline surface construction

A truly watertight tea pot!
**T3-G-spline** surface construction of the *truly watertight tea pot*

T-junctions in Spline Surfaces

Cut open
T3-G-spline surface construction of the truly watertight tea pot

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T-junctions in Spline Surfaces

Thank you