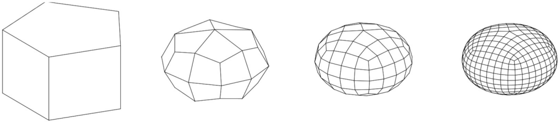
Quadratic-Attraction Subdivision with Contraction-Ratio $\lambda = \frac{1}{2}$

Kęstutis Karčiauskas
Vilnius University

Jörg Peters
University of Florida

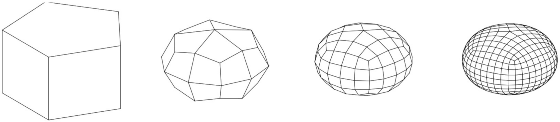
Quadratic-Attraction Subdivision with Contraction-Ratio $\lambda = \frac{1}{2}$

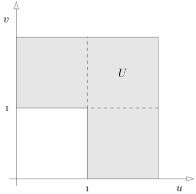
Kęstutis Karčiauskas   Jörg Peters

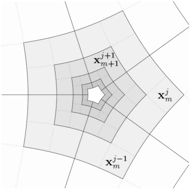


Quadratic-Attraction Subdivision with Contraction-Ratio $\lambda = \frac{1}{2}$

Kęstutis Karčiauskas Jörg Peters







Quadratic-Attraction Subdivision with Contraction-Ratio $\lambda = \frac{1}{2}$

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QAS $\lambda_{CC} > \frac{1}{2}$

Quadratic-Attraction Subdivision with Contraction-Ratio $\lambda = \frac{1}{2}$

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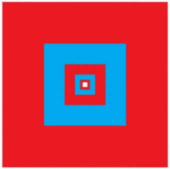

regular spline: $\lambda = \frac{1}{2}$

Quadratic-Attraction Subdivision with Contraction-Ratio $\lambda = \frac{1}{2}$

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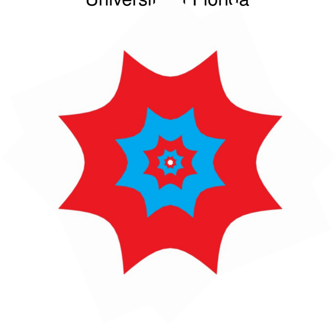
QAS $\lambda_{CC} > \frac{1}{2}$



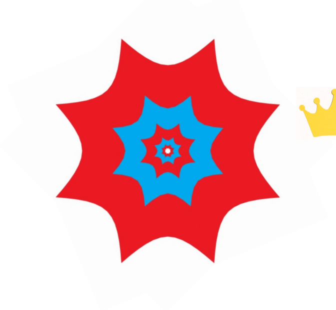
SMI'24 $\lambda = \frac{1}{2}$

Quadratic-Attraction Subdivision with Contraction-Ratio $\lambda = \frac{1}{2}$

Kęstutis Karčiauskas
Vilnius University

Jörg Peters
University of Florida

QAS $\lambda_{CC} > \frac{1}{2}$



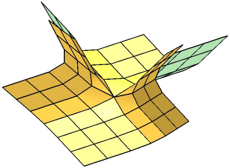
SMI'24 $\lambda = \frac{1}{2}$

Outline

1. Conventional vs quadratic-attraction subdivision (in pictures)
2. Quadratic-attraction subdivision (QAS)
   - Structure
   - Derivation
   - $\lambda = \frac{1}{2}$
   - Analysis
3. Comparison
4. Summary

Wavy $n = 8$ input

Conventional vs QA subdivision



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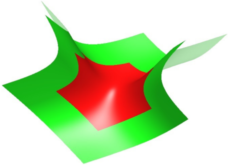
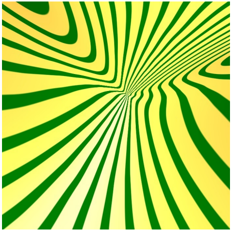


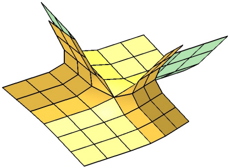
bi-3 ring + surface



CC CATMULL, CLARK: Recursively generated B-spline surfaces on arbitrary topological meshes. CAD 10,1978)

Wavy $n = 8$ input

Conventional vs QA subdivision



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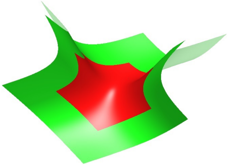
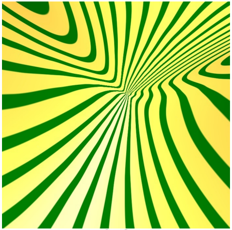


bi-3 ring + surface



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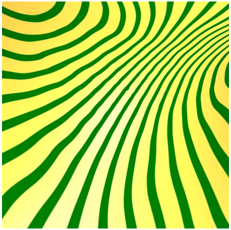
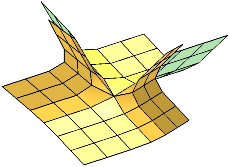


MM18

MA Y., MA W.: Subdivision schemes with optimal bounded curvature near extraordinary vertices. CGF 2018

Wavy $n = 8$ input

Conventional vs QA subdivision



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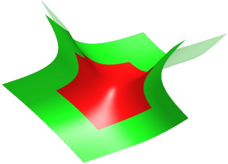
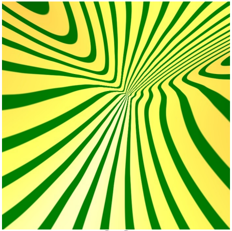


bi-3 ring + surface



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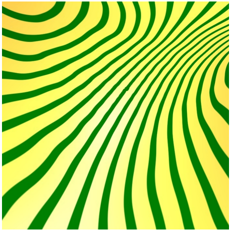
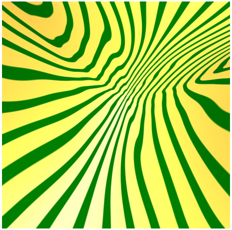


MM18

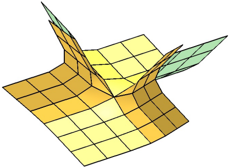


WM23

WANG MA: An extended tuned subdivision scheme with optimal convergence for isogeometric analysis. CAD 2023

Wavy $n = 8$ input

Conventional vs QA subdivision



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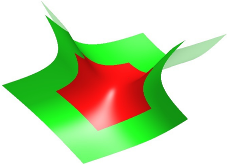
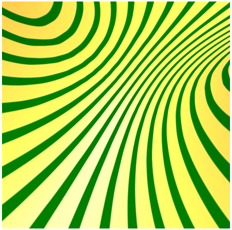
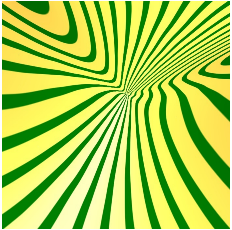


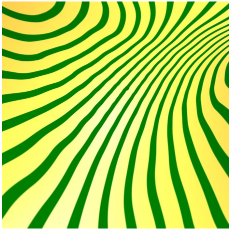
bi-3 ring + surface



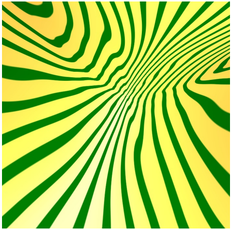
KP23 QAS



CC



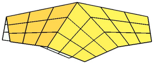
MM18



WM23

Convex $n = 6$ input

Conventional vs QA subdivision



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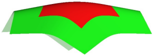
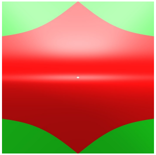
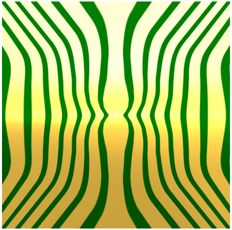




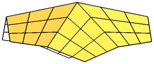
bi-3 ring + surface



CC

Convex $n = 6$ input

Conventional vs QA subdivision



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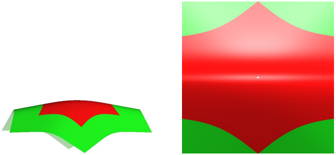
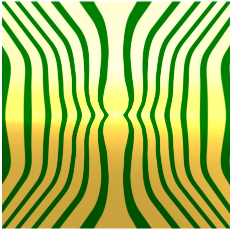


bi-3 ring + surface



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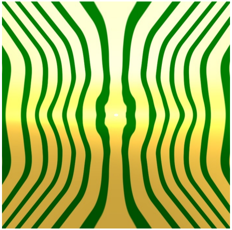
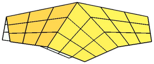


MM18

Convex $n = 6$ input

Conventional vs QA subdivision



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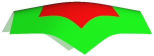
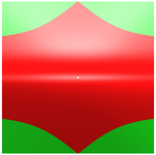
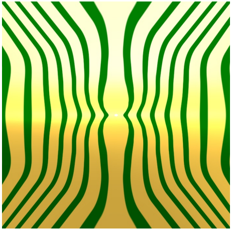




bi-3 ring + surface



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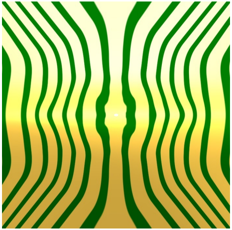
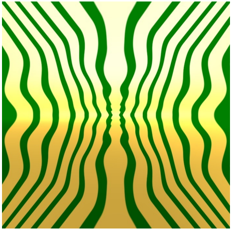


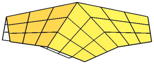
MM18



WM23

Convex $n = 6$ input

Conventional vs QA subdivision



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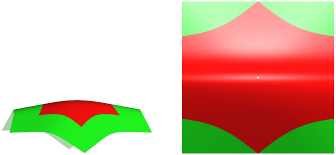
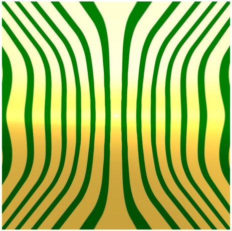
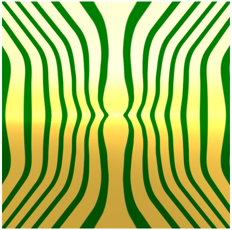


bi-3 ring + surface



KP23



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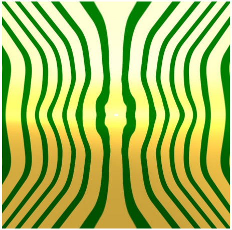
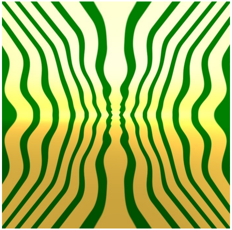


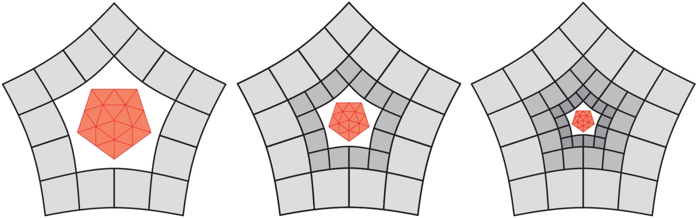
MM18



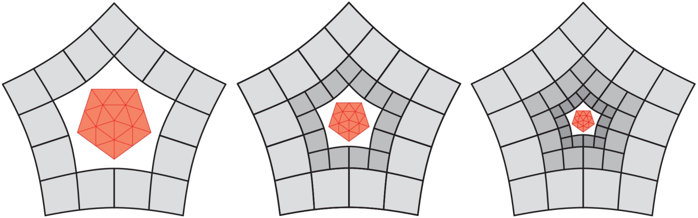
WM23

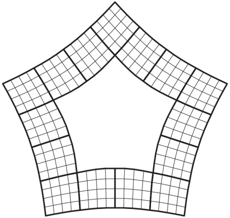
1. Conventional vs quadratic-attraction subdivision (in pictures)
2. Quadratic-attraction subdivision (QAS)
   - Structure
   - Derivation
   - $\lambda = \frac{1}{2}$
   - Analysis
3. Comparison
4. Summary

Layout of quadratic-attraction subdivision (QAS)



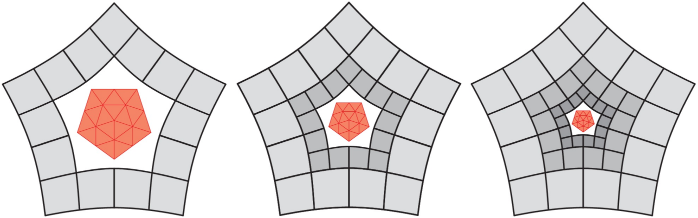
Layout of quadratic-attraction subdivision (QAS)

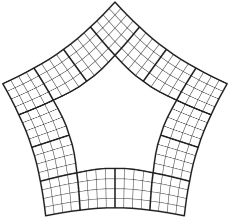




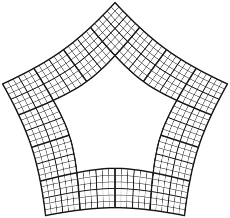
bi-4

Layout of quadratic-attraction subdivision (QAS)





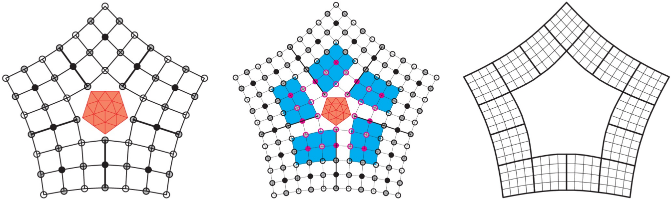
bi-4



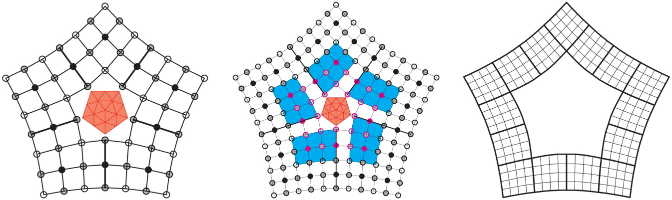
bi-3

Choice of bi-4 or bi-3 QAS

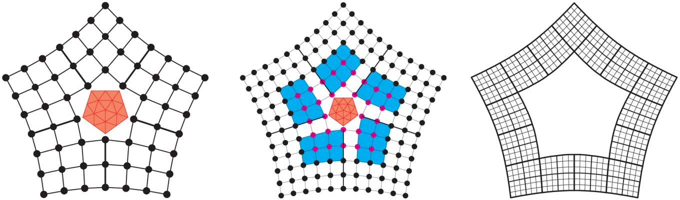
bi-4 **d**-net



Choice of bi-4 or bi-3 QAS

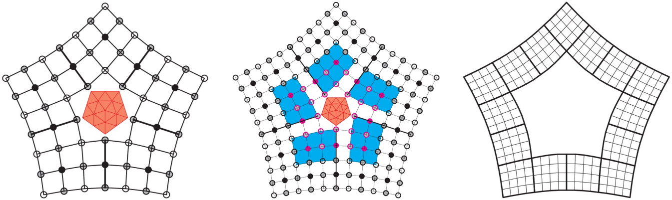


bi-4 **d**-net

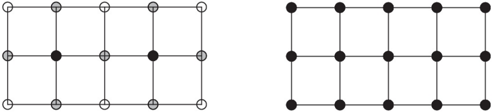


bi-3 **d**-net

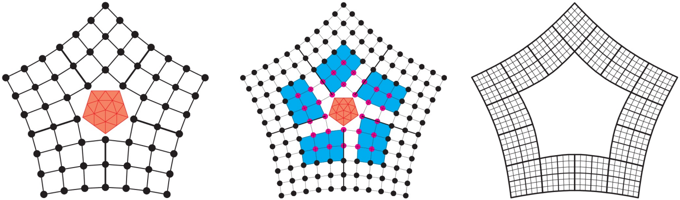
Choice of bi-4 or bi-3 QAS



bi-4 **d**-net



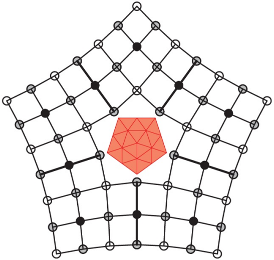
bi-4 $\leftrightarrow$ bi-3



bi-3 **d**-net

Earlier work with $\lambda \neq \frac{1}{2}$ using χcc

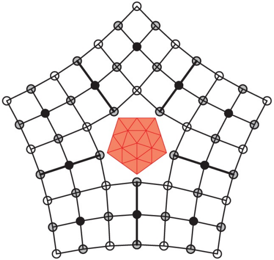
All data and calculations are symbolic!



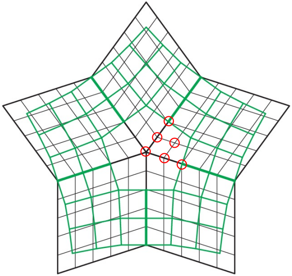
bi-4 **d**-net
+ quadratic expansion **q**

Earlier work with $\lambda \neq \frac{1}{2}$ using χcc

All data and calculations are symbolic!



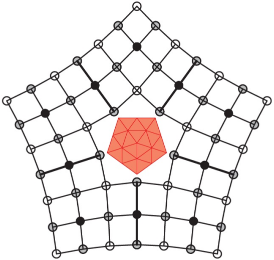
bi-4 **d**-net
+ quadratic expansion **q**



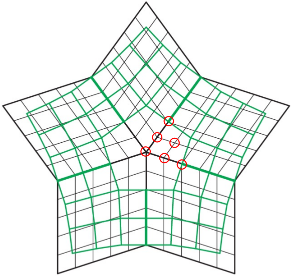
sampling bi-5 guide
with χcc

Earlier work with $\lambda \neq \frac{1}{2}$ using χ_{cc}

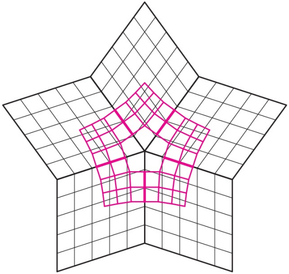
All data and calculations are symbolic!



bi-4 **d**-net
+ quadratic expansion **q**



sampling bi-5 guide
with χ_{cc}

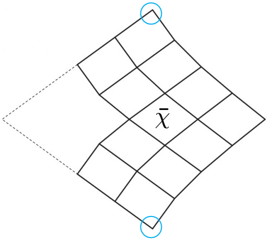


sampling with
$\chi_{cc} := \lambda_{cc}\chi_{cc}$

Constructing $\chi_{0.5}$ that scales by $\lambda = \frac{1}{2}$

$\bar{\chi}$ (one sector; ◯ lies on unit circle).

Symmetry and C^2 join of adjacent sectors leave 11 free parameters.

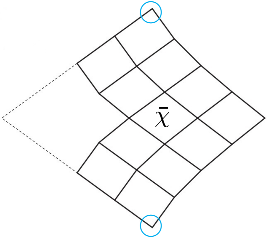


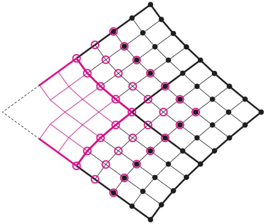
Constructing $\chi_{0.5}$ that scales by $\lambda = \frac{1}{2}$

$\bar{\chi}$ (one sector; ◯ lies on unit circle).

Symmetry and C^2 join of adjacent sectors leave 11 free parameters.

$\frac{1}{2}\bar{\chi}$: • uniformly split $\bar{\chi}$ ○: C^2 extension backwards

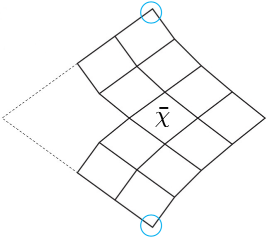


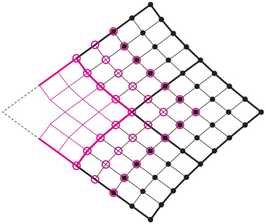


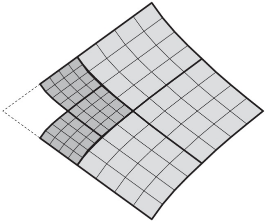
Constructing $\chi_{0.5}$ that scales by $\lambda = \frac{1}{2}$

$\bar{\chi}$ (one sector; ◯ lies on unit circle).
Symmetry and C^2 join of adjacent sectors leave 11 free parameters.

$\frac{1}{2}\bar{\chi}$: • uniformly split $\bar{\chi}$ ∘: C^2 extension backwards
Matching ∘ to • leaves 5 free parameters
set by 'matheneering': heuristic and derivative functional

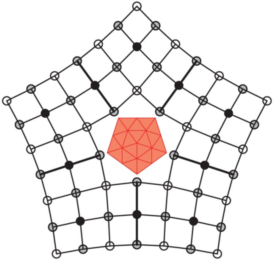




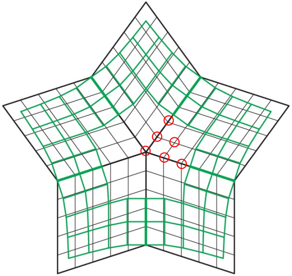


$\frac{1}{2}\chi_{0.5}$ $\chi_{0.5}$

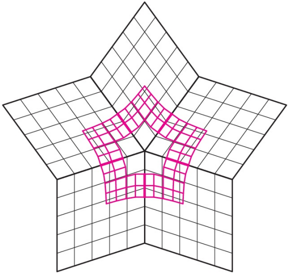
$\lambda = \frac{1}{2}$: uses newly constructed $\chi_{0.5}$



bi-4 **d**-net
+ quadratic expansion **q**



sampling bi-5 guide
with $\chi_{0.5}$



sampling with
$\chi_{0.5} := \frac{1}{2}\chi_{0.5}$

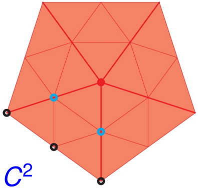
1 Conventional vs quadratic-attraction subdivision (in pictures)

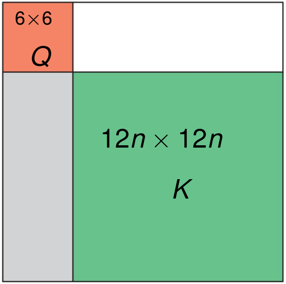
2 Quadratic-attraction subdivision (QAS)
- Structure
- Derivation
- $\lambda = \frac{1}{2}$
- Analysis

3 Comparison

4 Summary

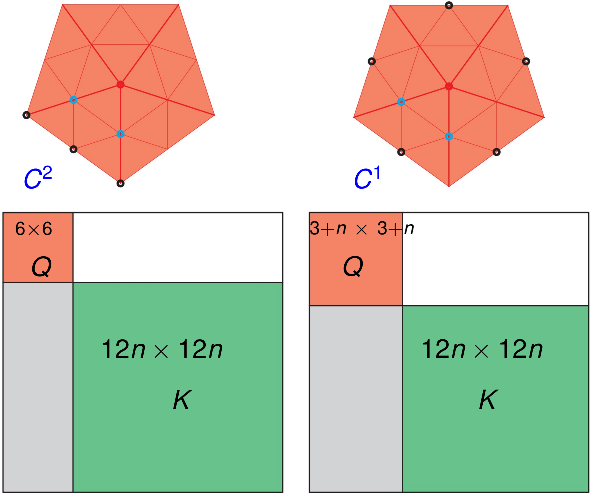
Analysis choice of ever freer attracting quadratics





$1, \lambda, \lambda, \lambda^2, \lambda^2, \lambda^2$

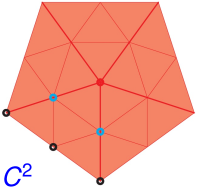
Analysis choice of ever freer attracting quadratics



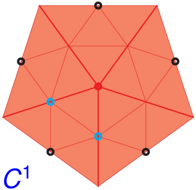
$$1, \lambda, \lambda, \lambda^2, \lambda^2, \lambda^2$$

$$1, \lambda, \lambda, \underbrace{\lambda^2, \ldots, \lambda^2}_{n-fold}$$

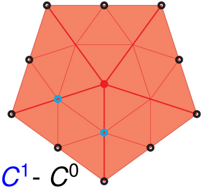
Analysis
choice of ever freer attracting quadratics



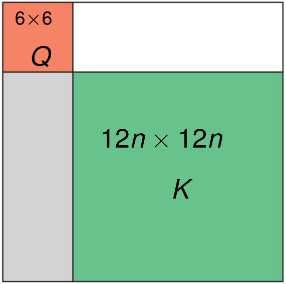
C^2



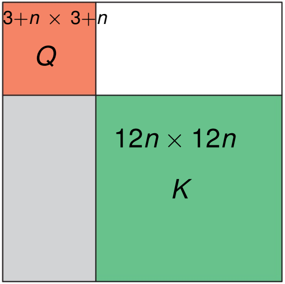
C^1



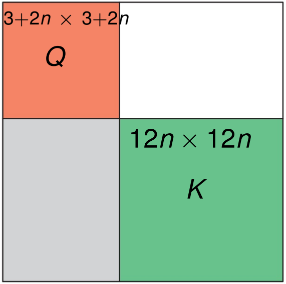
C^1 - C^0



$$1, \lambda, \lambda, \lambda^2, \lambda^2, \lambda^2$$

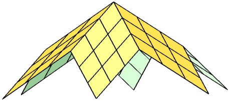


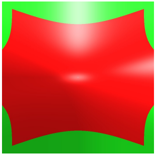
$$1, \lambda, \lambda, \underbrace{\lambda^2, \ldots, \lambda^2}_{n-fold}$$



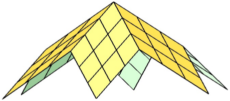
$$1, \lambda, \lambda, \underbrace{\lambda^2, \ldots, \lambda^2}_{2n-fold}$$

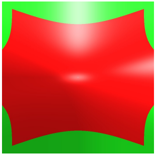
The need for a scalable $\chi_{0.5}$ map

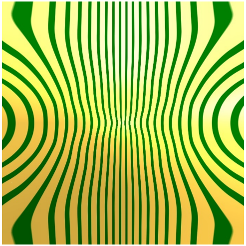




The need for a scalable $\chi_{0.5}$ map

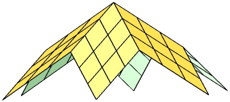


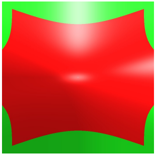


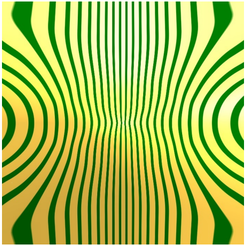


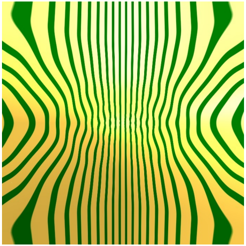
KP23 but
q scaled $\frac{1}{2}$

The need for a scalable $\chi_{0.5}$ map





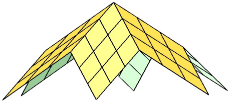


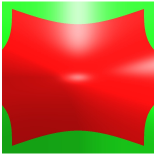


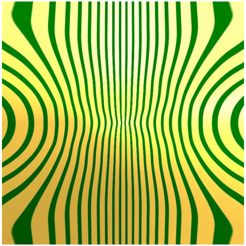
KP23 but
q scaled $\frac{1}{2}$

q scaled $\frac{1}{2}$
in derivation

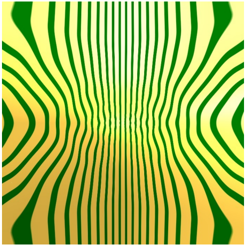
The need for a scalable $\chi_{0.5}$ map



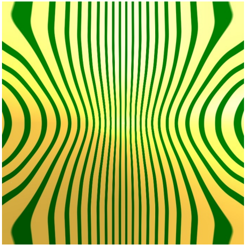




KP23 but
q scaled $\frac{1}{2}$

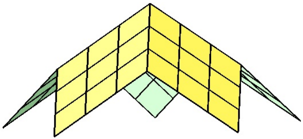


q scaled $\frac{1}{2}$
in derivation

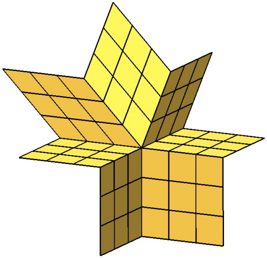


SMI24

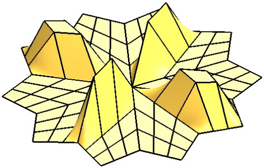
Three input nets of the ‘obstacle course’



$n = 5$ (Fig 18)

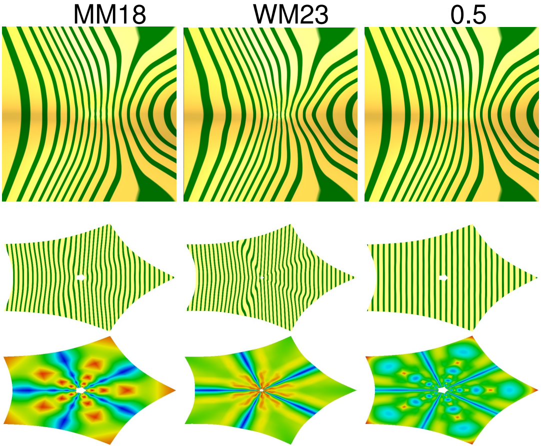


$n = 7$ (Fig 22)

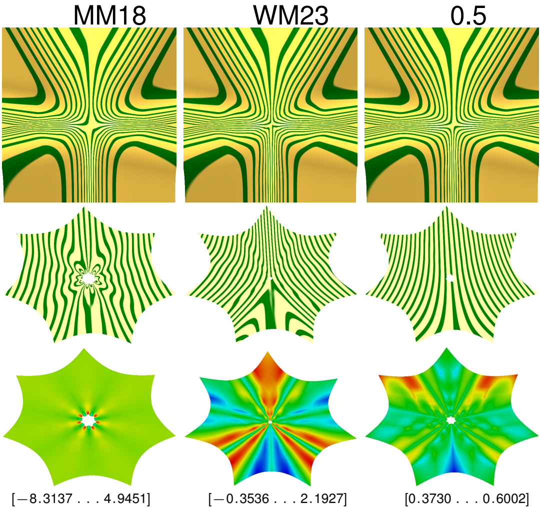


$n = 10$ (Fig 24)

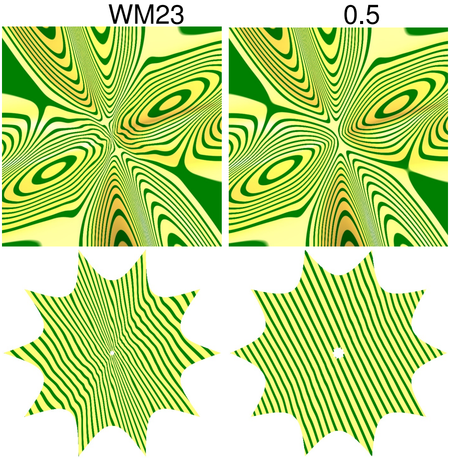
$n = 5$ 



$n = 7$ 



$n = 10$ 



QAS($\frac{1}{2}$) Properties

1. Baked-in evolving guides $\rightarrow$ high surface quality
2. Explicit formulas hide complexity of derivation
3. Explicit formulas are *local*
4. bi-4 QAS $\rightarrow$ bi-3 QAS
5. Case $n = 3$ is not considered, since for $n = 3$ the $\lambda_{CC} \approx 0.41 < \frac{1}{2}$
6. Graphics Replicability Stamp 
7. Quadratic expansion adds flexibility

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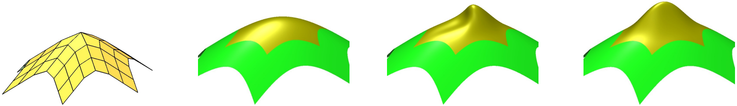

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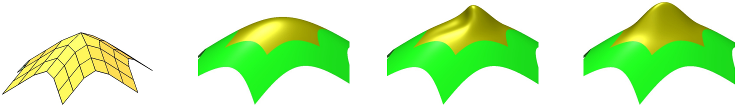
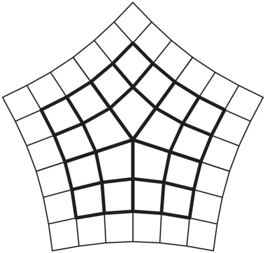
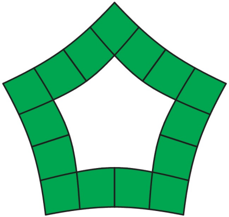


Thank You!

Initialization of **d**-nets

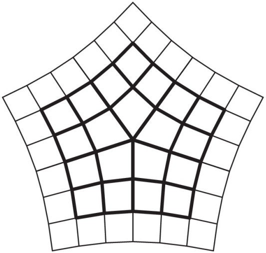


c-net

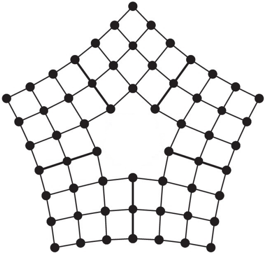


bi-3 ring

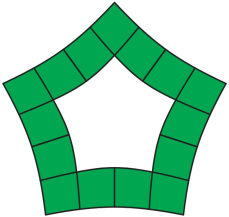
Initialization of **d**-nets



c-net

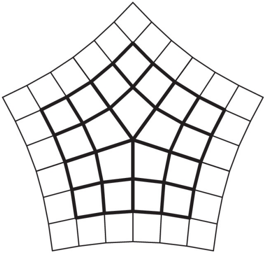


bi-3 **d**-net
regular refinement

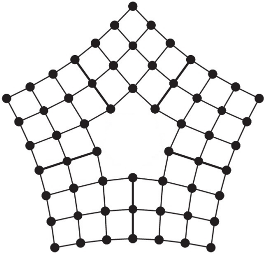


bi-3 ring

Initialization of **d**-nets



c-net



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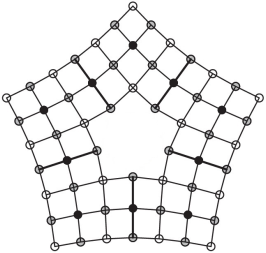
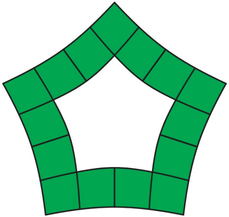
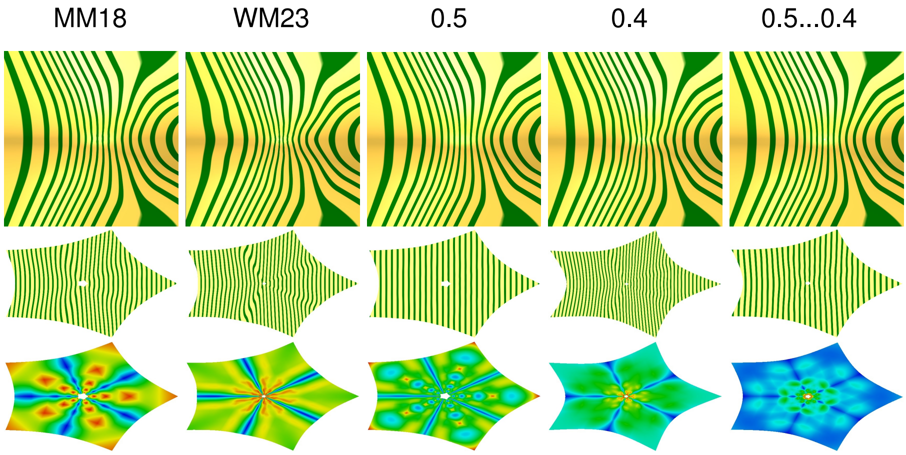


bi-4 **d**-net
degree-raising



bi-3 ring

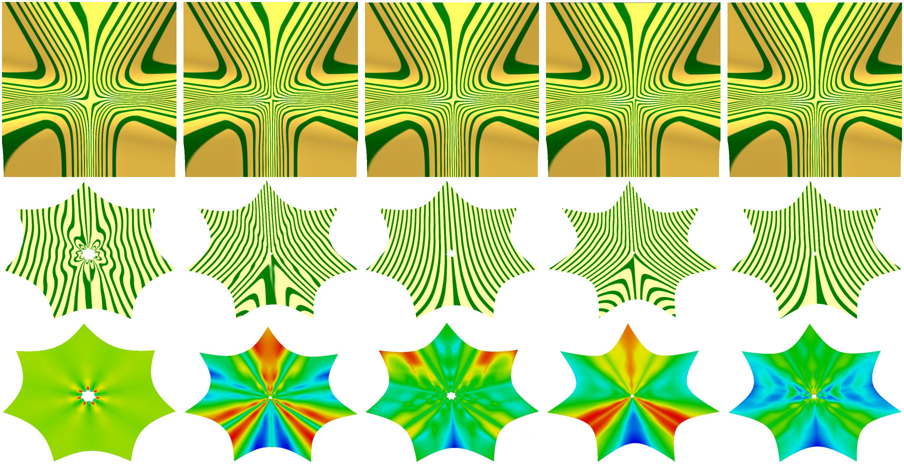
$n = 5$ 



[20.4557 . . . 38.4520] [29.4124 . . . 117.7662] [25.6783 . . . 29.3760] [47.1327 . . . 143.7433] [24.8400 . . . 54.0622]

$n = 7$ 

MM18	WM23	0.5	0.4	0.5...0.4



$[-8.3137 \ldots 4.9451]$	$[-0.3536 \ldots 2.1927]$	$[0.3730 \ldots 0.6002]$	$[-0.5784 \ldots 1.9864]$	$[0.3037 \ldots 0.8788]$

$n = 10$ 

WM23 | 0.5 | 0.4 | 0.5...0.4