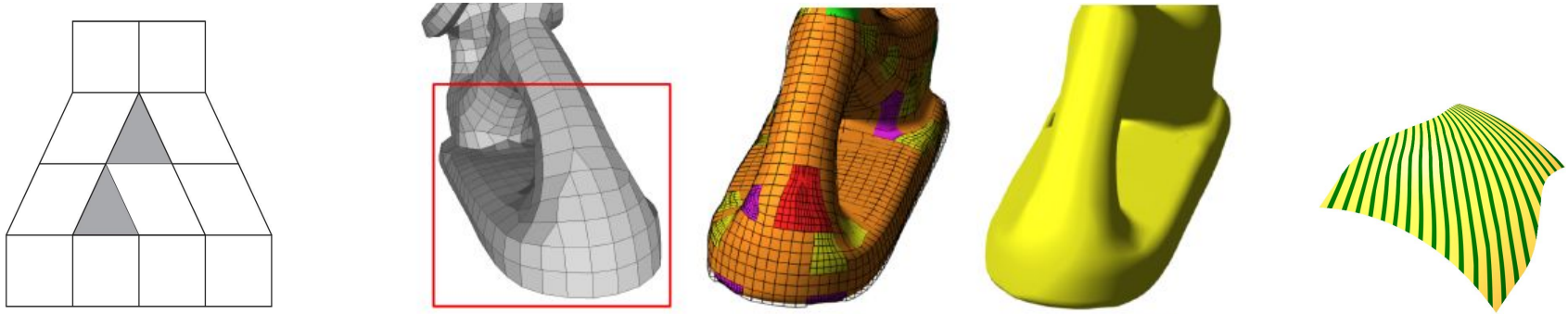
Splines for **fast-contracting** polyhedral control nets



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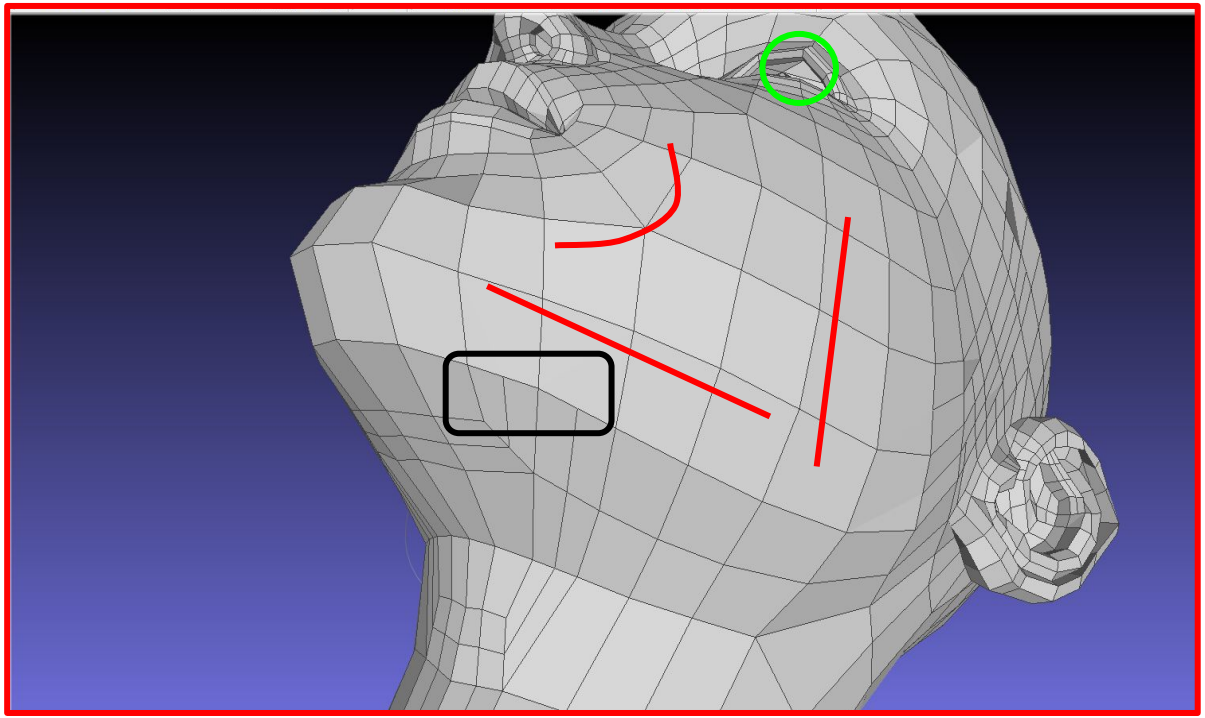
Istanbul Technical University, Vilnius University, University of Florida

SPM 2024

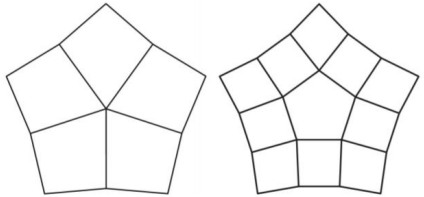
Polyhedral net patterns

Fast-contracting polyhedral-net Splines E. Gunpinar, K. Karčiauskas, J. Peters

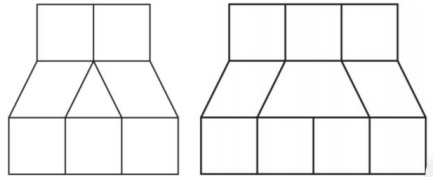


Missing fast-contracting net!

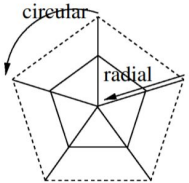
merge parameter directions



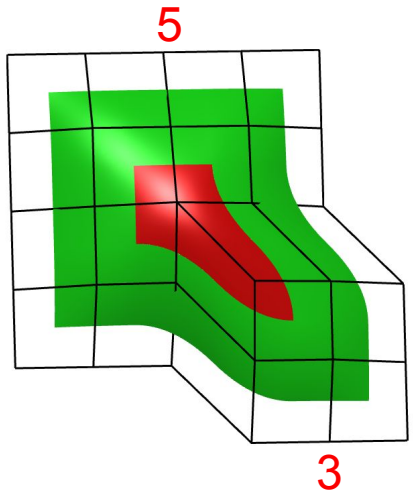
transition between coarse and fine meshes



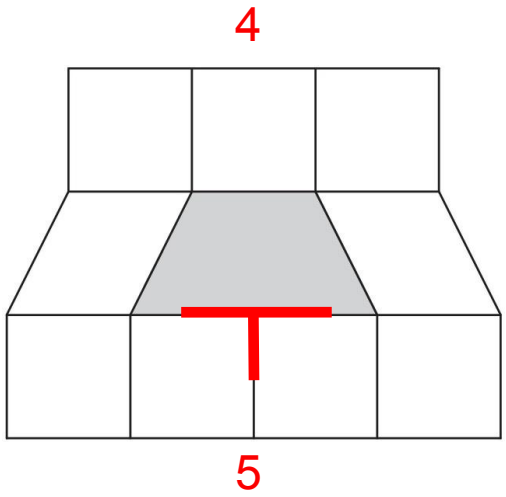
polar

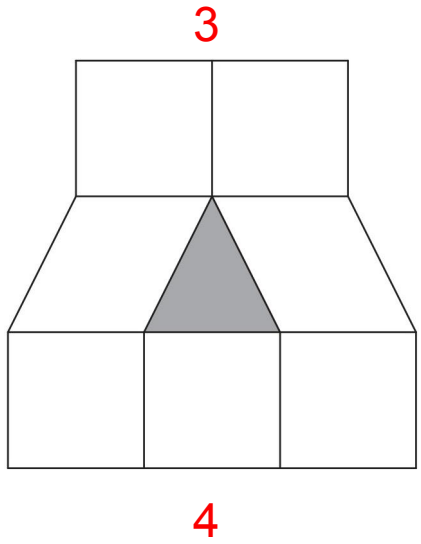


Contraction in quad mesh design



diagonal 5-3 contraction

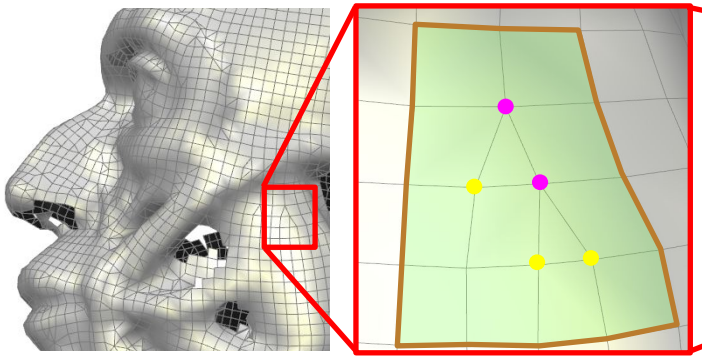




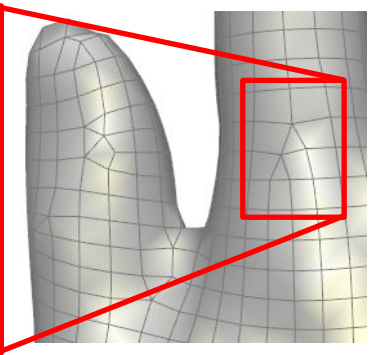
single, direction-aligned contraction

Cascading triangles in quad-dom. meshes

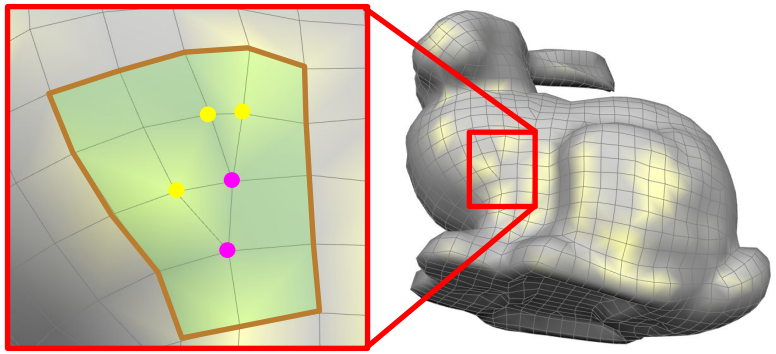
- Is this just academia making this cases?



Schertler et al. (2017)



Ray et al. (2006)

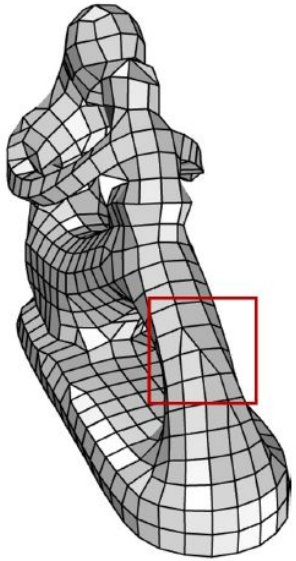


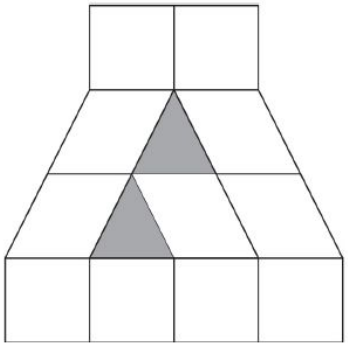
Jakob et al. (2015)

valence 5 and valence 4 vertices

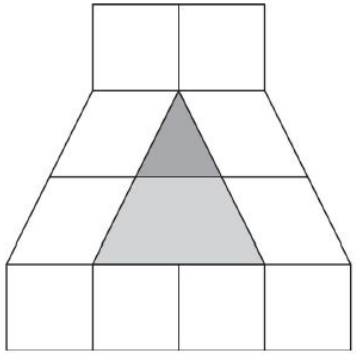
Fast-contracting polyhedral control nets

Fast-contracting polyhedral-net Splines E. Gunpinar, K. Karčiauskas, J. Peters

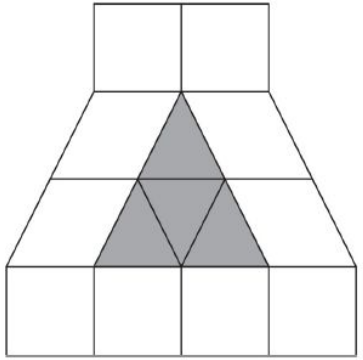




cascades



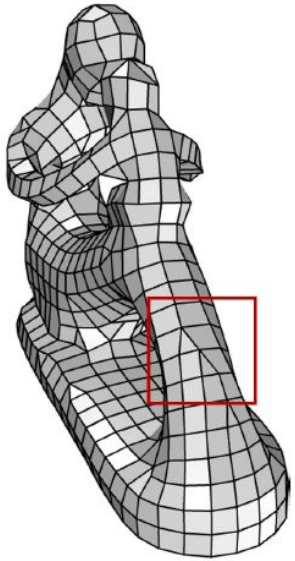
combined (T0 + T1)

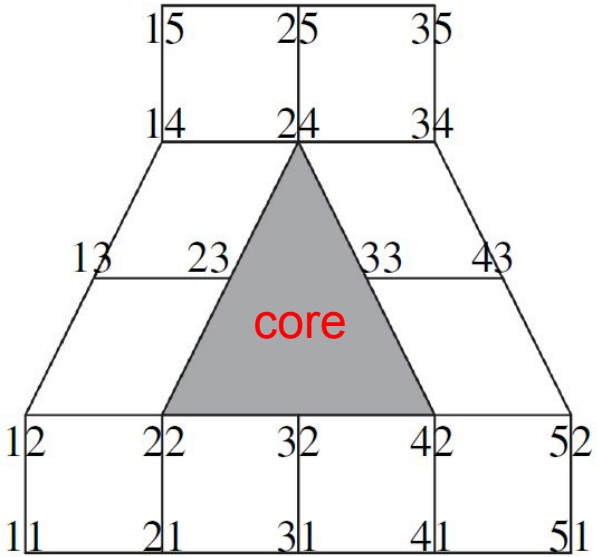


triangulated core

Fast-contracting polyhedral control nets

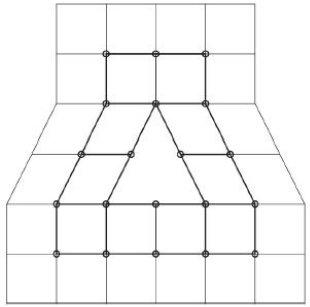
Fast-contracting polyhedral-net Splines E. Gunpinar, K. Karčiauskas, J. Peters



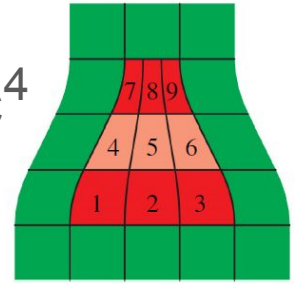


Our algorithm treats all cases uniformly regardless of the partition of the core

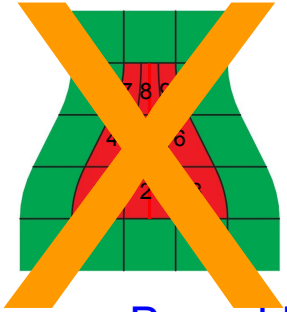
3 results in this paper



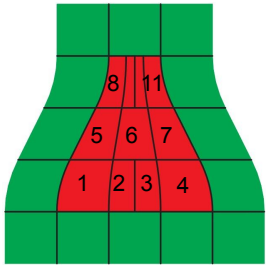
Extended Δ^2-net



FC^4



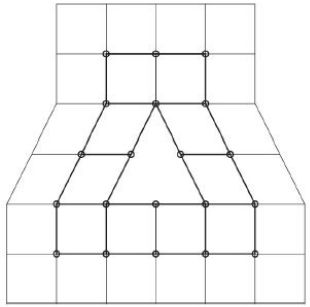
Lower Bound Proof:
no 3x3 bi-3 construction



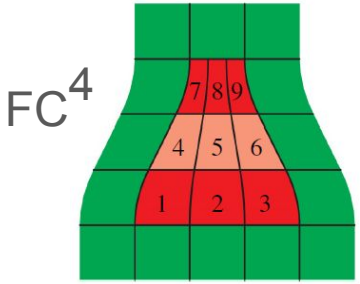
FC^3

3 results in this paper

Fast-contracting polyhedral-net Splines E. Gunpinar, K. Karčiauskas, J. Peters

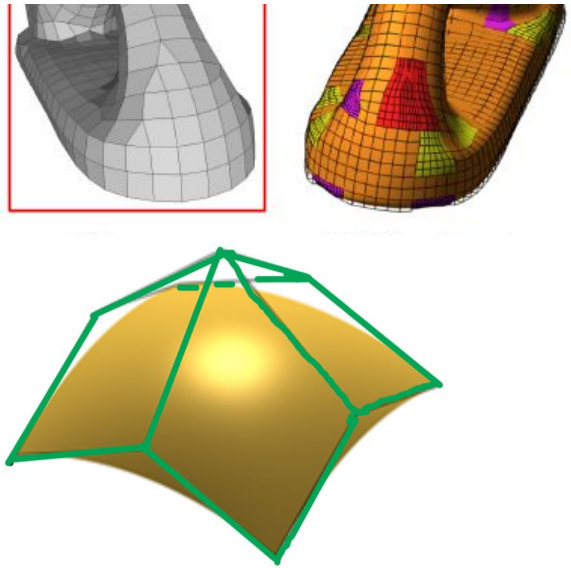


Extended Δ^2-net

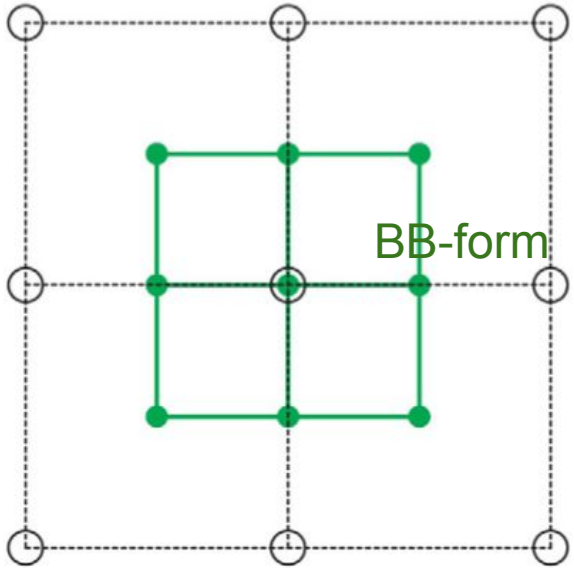


Recall: BB-form & B-spline

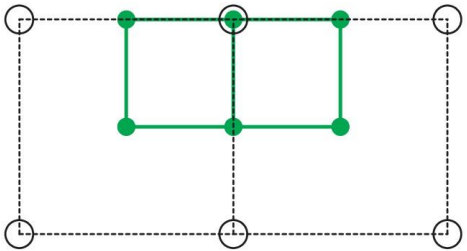
Fast-contracting polyhedral-net Splines E. Gunpinar, K. Karčiauskas, J. Peters



(a) BB-net and bi-2 patch



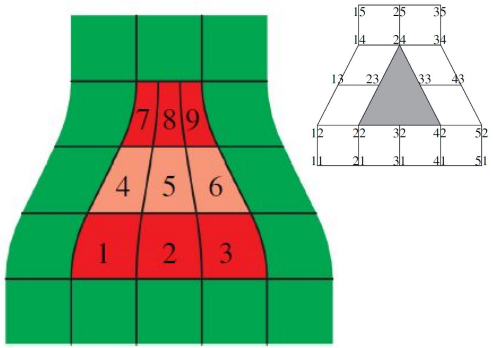
(b) bi-2 spline nets



(c) tensor-border

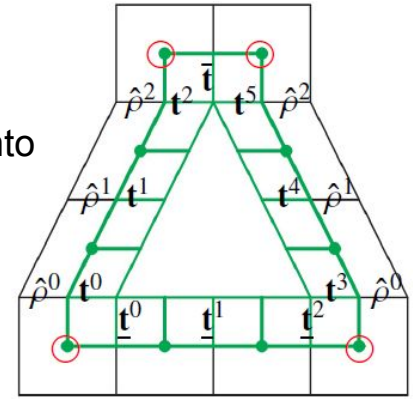
FC^4 construction

Fast-contracting polyhedral-net Splines E. Gunpinar, K. Karčiauskas, J. Peters



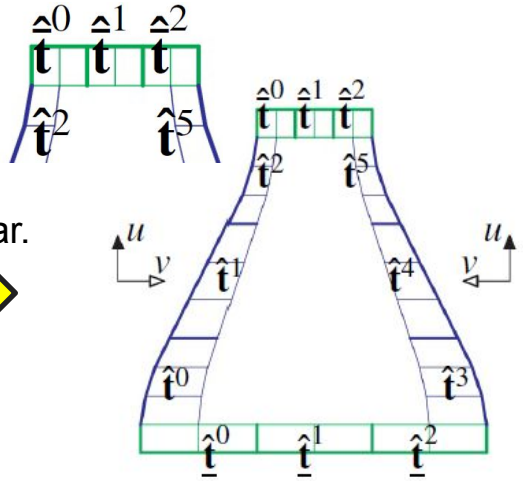
(a) Surface pieces

Convert Polyhedral-net into tensor-border



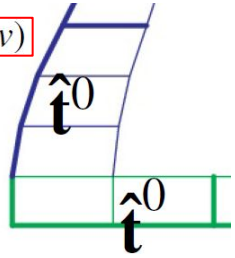
(b) Bi-2 tensor-border frame

Repar.



(c) Final frame

$$\rho^s(u,v) := (u, a^s(u)v)$$



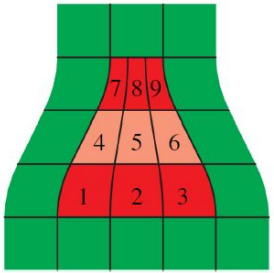
Reparametrize (change of variables)

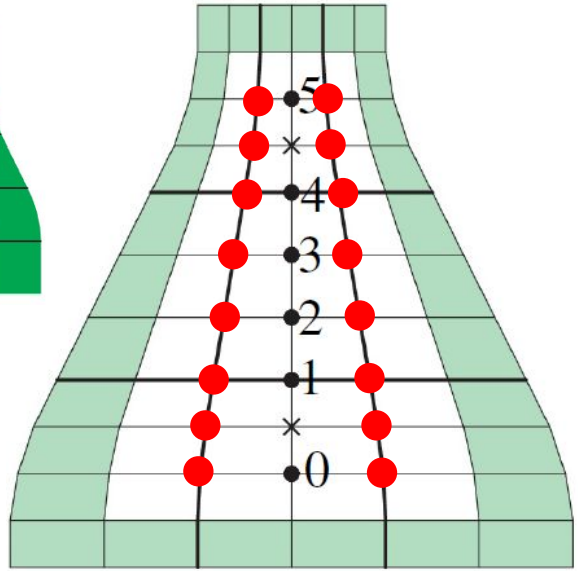
$$\mathbf{p}(u,v) := \mathbf{q} \circ \rho(u,v), \qquad \rho(u,v) := (u + b(u)v, a(u)v)$$

$$\partial_v \mathbf{q}(u,0) = a(u)\partial_v \mathbf{p}(u,0) + b(u)\partial_u \mathbf{p}(u,0), \qquad (u,v) \in [0..1]^2$$

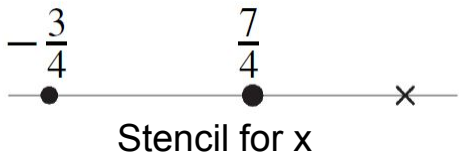
FC^4 construction

Fast-contracting polyhedral-net Splines E. Gunpinar, K. Karčiauskas, J. Peters

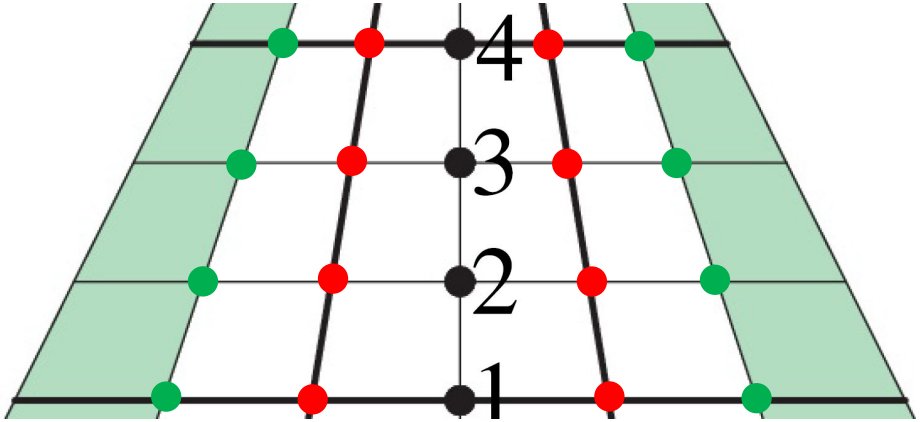




(d) 6 BB-coefficients ●



Stencil for x

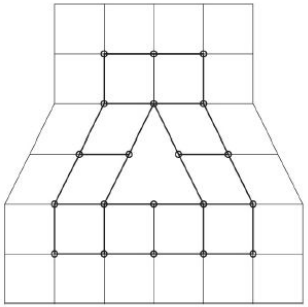


(e) 16 BB-coefficients ● = Average(●, ●)

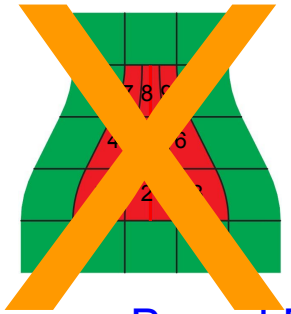
(f) Minimize $\mathcal{F}_4$ over 9 patches!

$$\mathcal{F}_4 := \int_0^1 \int_0^1 \sum_{i+j=4, i,j\geq 0} \frac{4!}{i!\,j!} (\partial_s^i \partial_t^j f(s,t))^2 \, ds \, dt$$

3 results in this paper

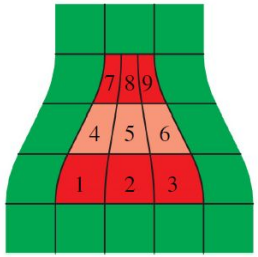


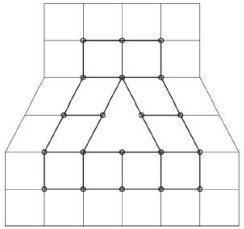
Extended Δ^2-net



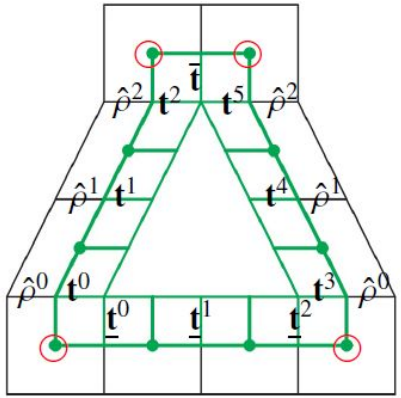
Lower Bound Proof:
no 3x3 bi-3 construction

No analogous 3x3 bi-3 surface construction

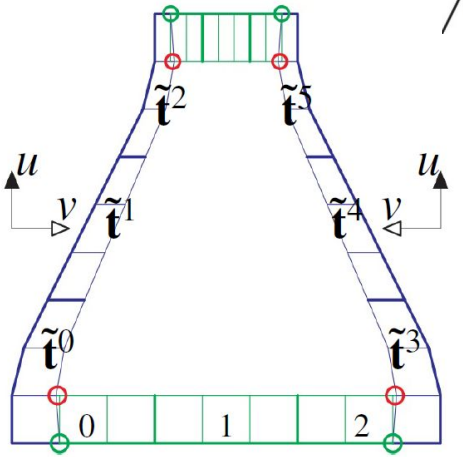




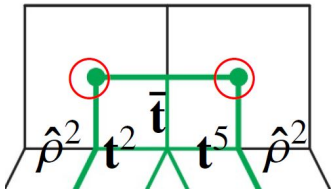
Surface pieces



(a) Bi-2 tensor-border frame

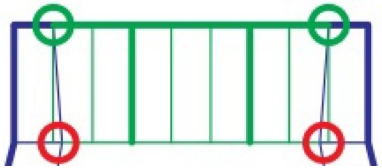


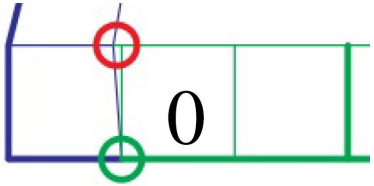
(b) Bi-3 frame



(c)

Split & degree raised





(d)

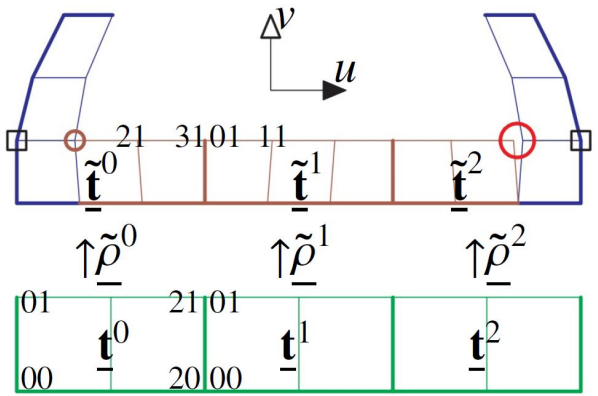
$$\rho^s(u,v) := (u, a^s(u)v)$$

No analogous 3x3 bi-3 surface construction

Fast-contracting polyhedral-net Splines E. Gunpinar, K. Karčiauskas, J. Peters

$$\tilde{\underline{\mathbf{t}}}^s := \underline{\mathbf{t}}^s \circ \tilde{\underline{\rho}}^s$$

$$\tilde{\underline{\rho}}^s := (u + b^s(u)v, v)$$



(e) Bottom tensor reparametrization

No 3x3 bi-3 construction exists!

$[b_0^2, b_1^2, b_2^2] = [0,1/9,0]$

left-right symmetry

contradiction!

$[b_0^0, b_1^0, b_2^0] = [0,-1/9,0]$

$[b_0^2, b_1^2, b_2^2] = [0,-1/9,0]$

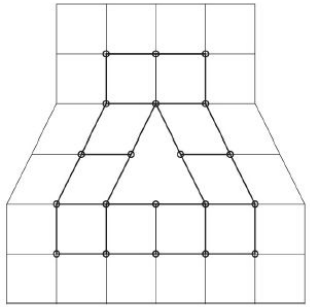
C^1 continuity

C^1 continuity

$[b_0^1, b_1^1, b_2^1] = [0,1/9,0]$

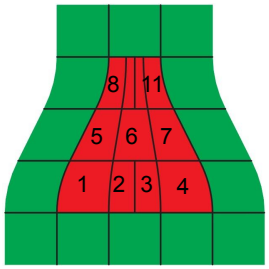
3 results in this paper

Fast-contracting polyhedral-net Splines E. Gunpinar, K. Karčiauskas, J. Peters



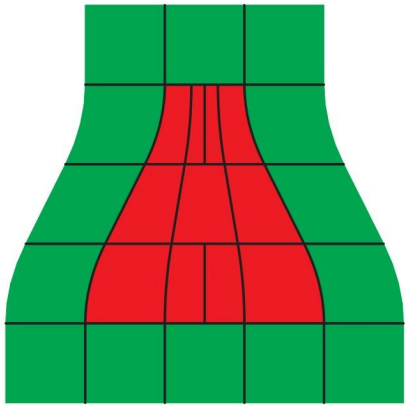
Extended Δ^2-net

FC^3

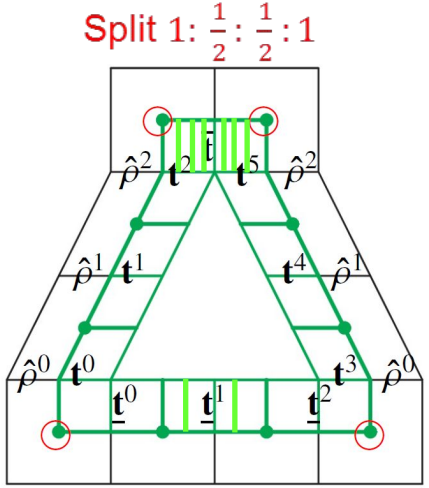


FC^3 construction

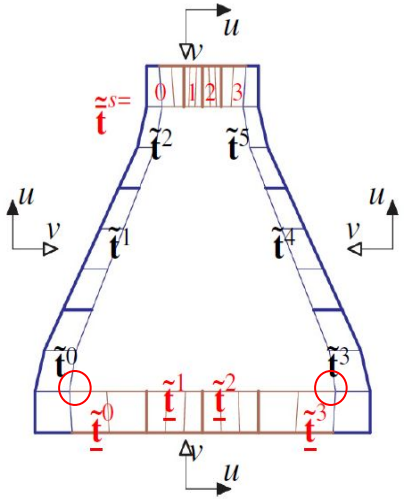
Fast-contracting polyhedral-net Splines E. Gunpinar, K. Karčiauskas, J. Peters



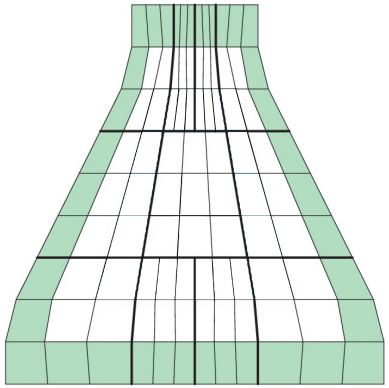
11 bi-3 patches



Split top and bottom
bi-2 tensor border frame



Bi-3 frame

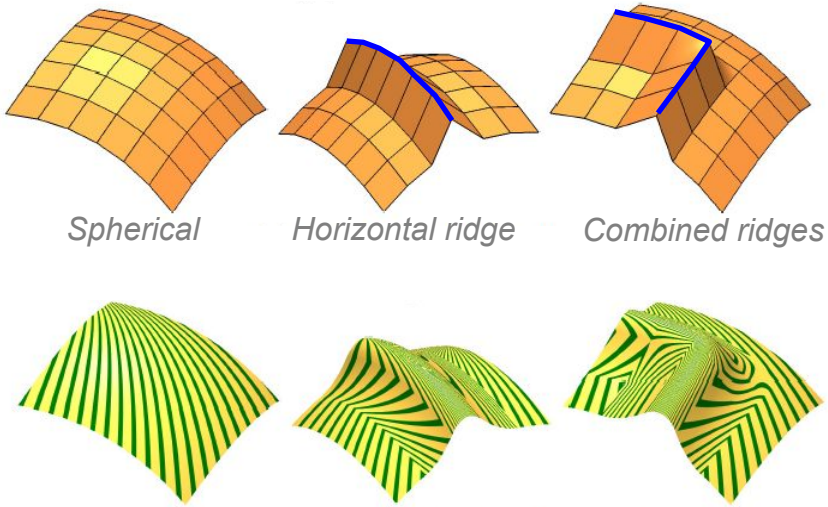


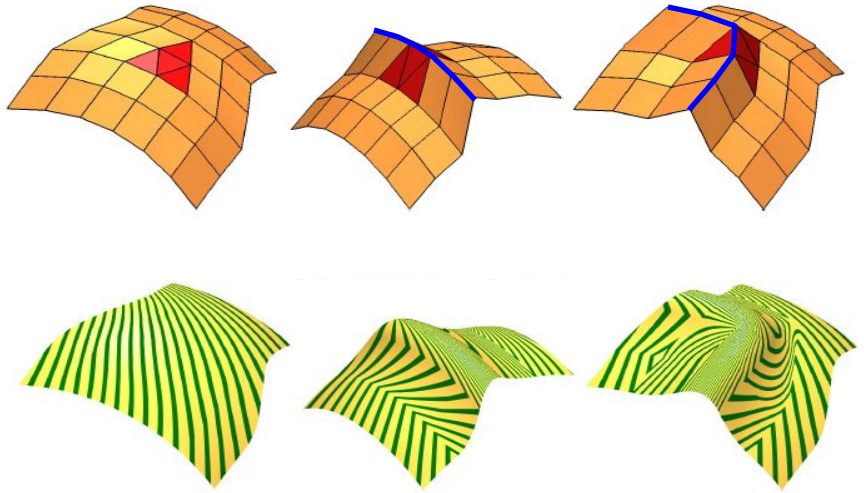
Final BB-net

$$\tilde{\tilde{\rho}}^s := (u + b^s(u)v, v)$$

Comparison: Ridges

Fast-contracting polyhedral-net Splines E. Gunpinar, K. Karčiauskas, J. Peters

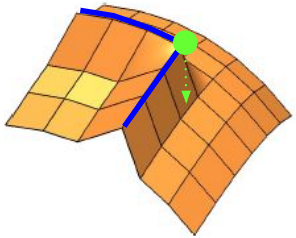


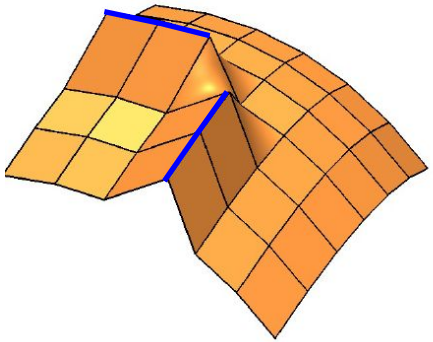


Bi-2 spline

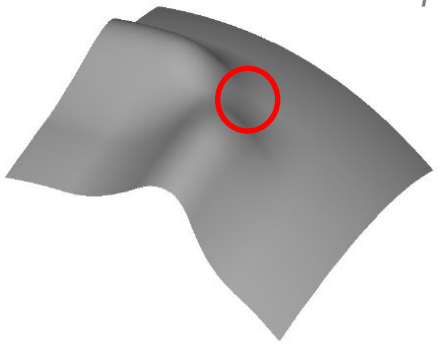
FC^4

Comparison: rounding right angle feature

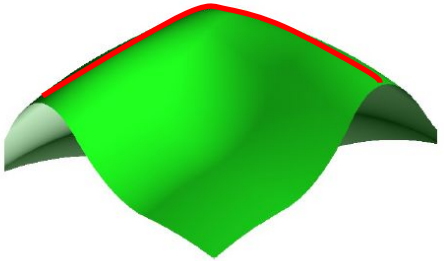




undesirable dip



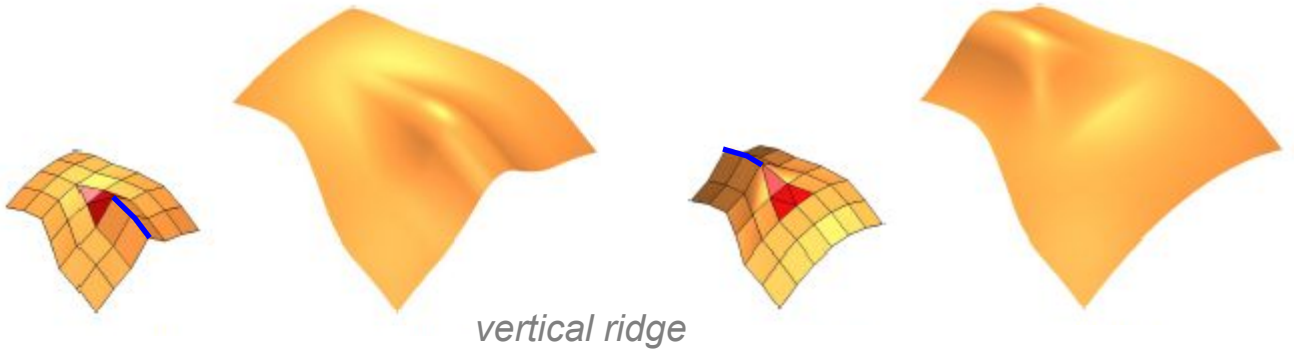
better



Bi-2 spline

FC^4

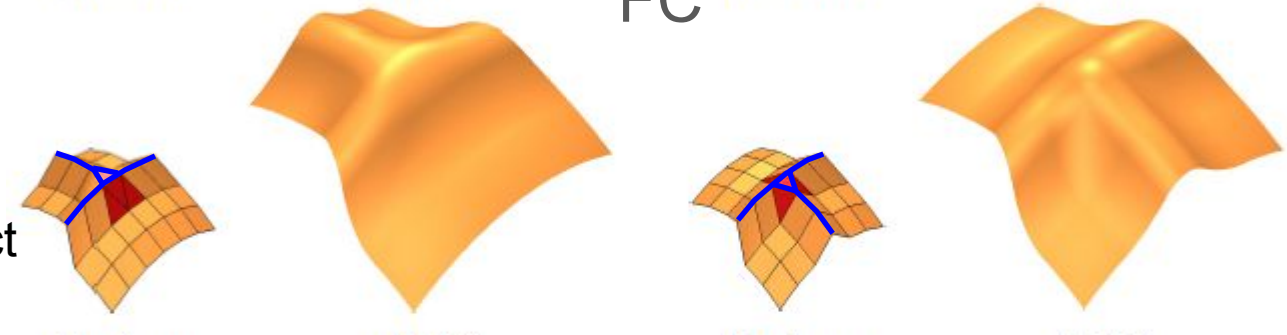
Comparison: directional features



vertical ridge

FC^4

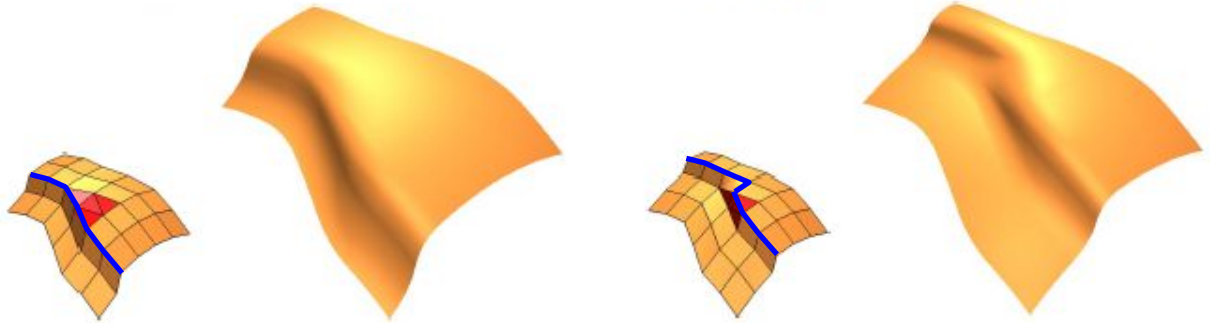
Limitation:
bulge
(same as
tensor-product
splines)



vertical + horizontal ridge

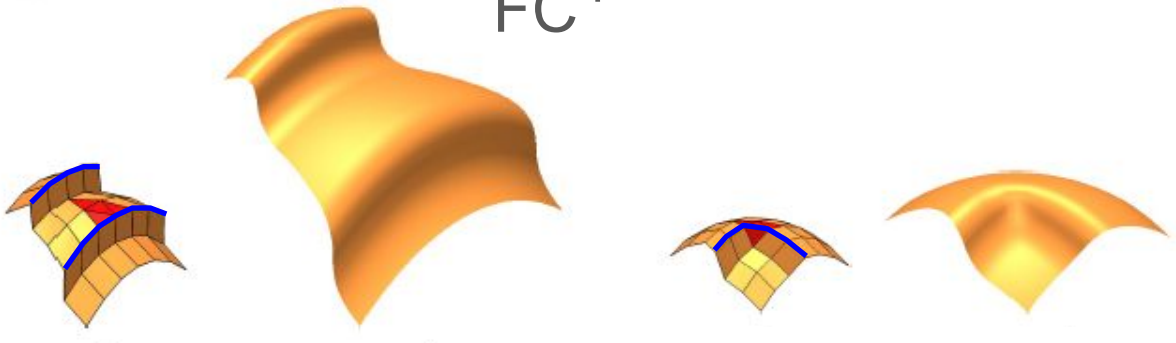
Comparison: directional features

Fast-contracting polyhedral-net Splines E. Gunpinar, K. Karčiauskas, J. Peters



ridge along the contracting direction

FC^4



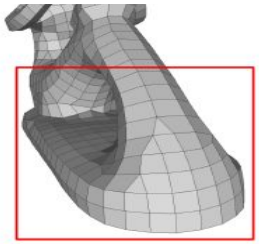
horizontal features

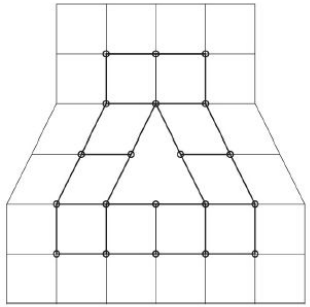
change of f. direction

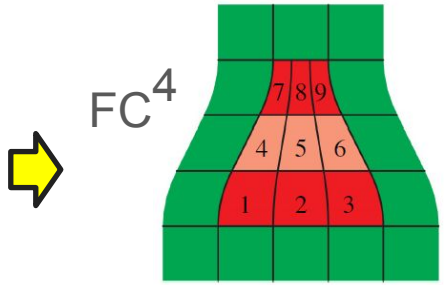
Limitation: core wants to be preserved

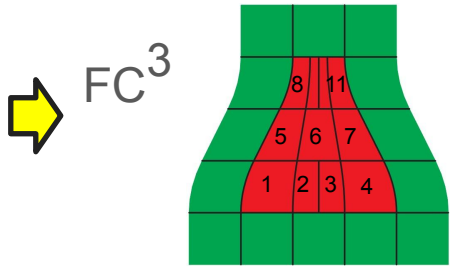
3 results in this paper

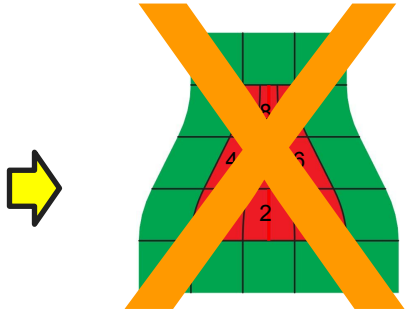
Fast-contracting polyhedral-net Splines E. Gunpinar, K. Karčiauskas, J. Peters











Lower Bound Proof:
no 3x3 bi-3 construction