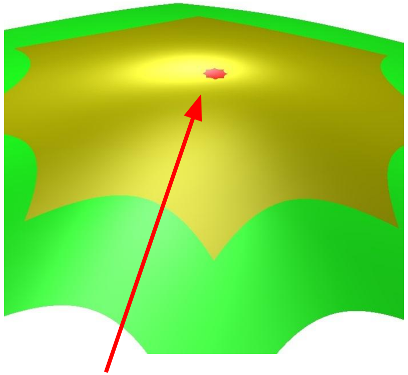
Improved Caps for Improved Subdivision Surfaces



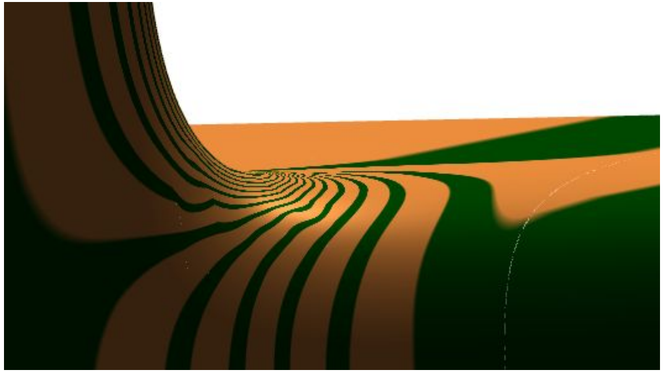
Caps can assume less fluctuation than general n-sided surface models

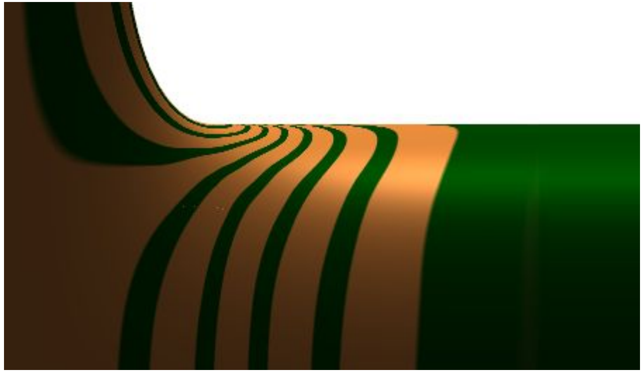
Kestutis Karciauskas and Jorg Peters

SPM 2023, Genoa

Reminder to Me

Show the Bezierview of the Problem



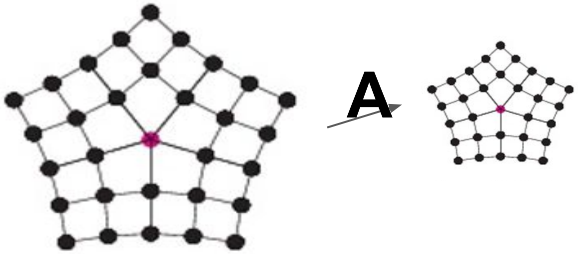


Overview

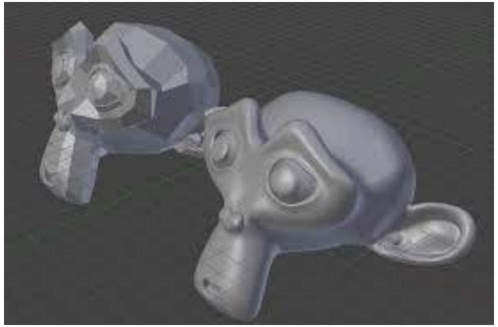
- Improved subdivision surfaces
  - PAS (Point Augmented Subdivision)
  - QAS (Quadratic-Attraction Subdivision)

- Improved caps

Subdivision Surfaces

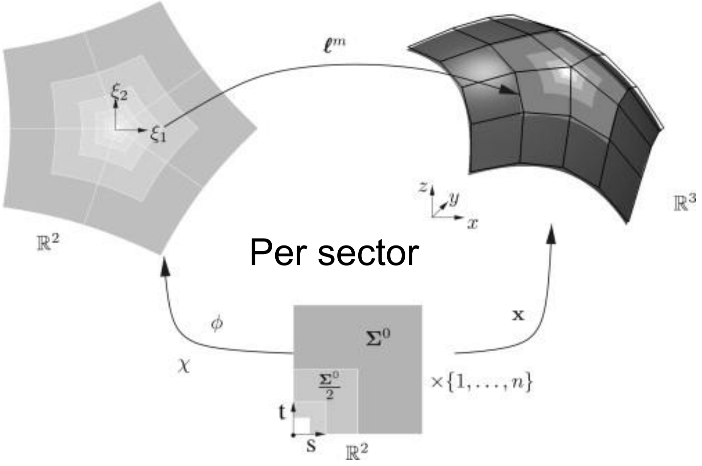


refined meshes with

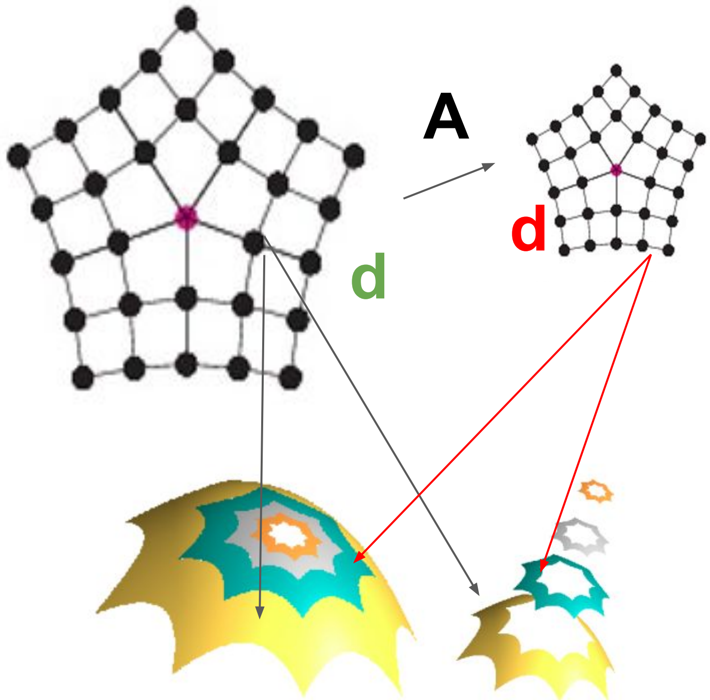


Blender: Suzanne

nested surface rings



Subdivision Surfaces

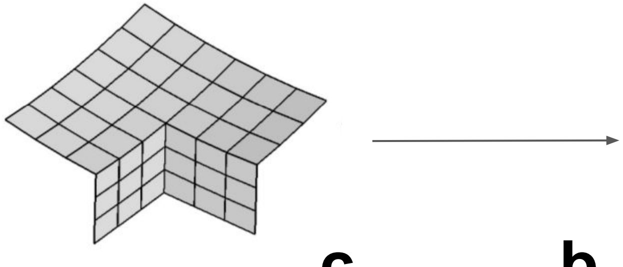


$$\mathbf{d} = A\ \mathbf{d}$$

New PAS, QAS subdivision:

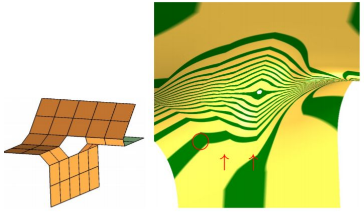
- ➢ Same structure and layout as Catmull-Clark subdivision,
- ➢ but better shape due to baked-in “guide”

Improved subdivision via guides



b

BB-coefficient of patches



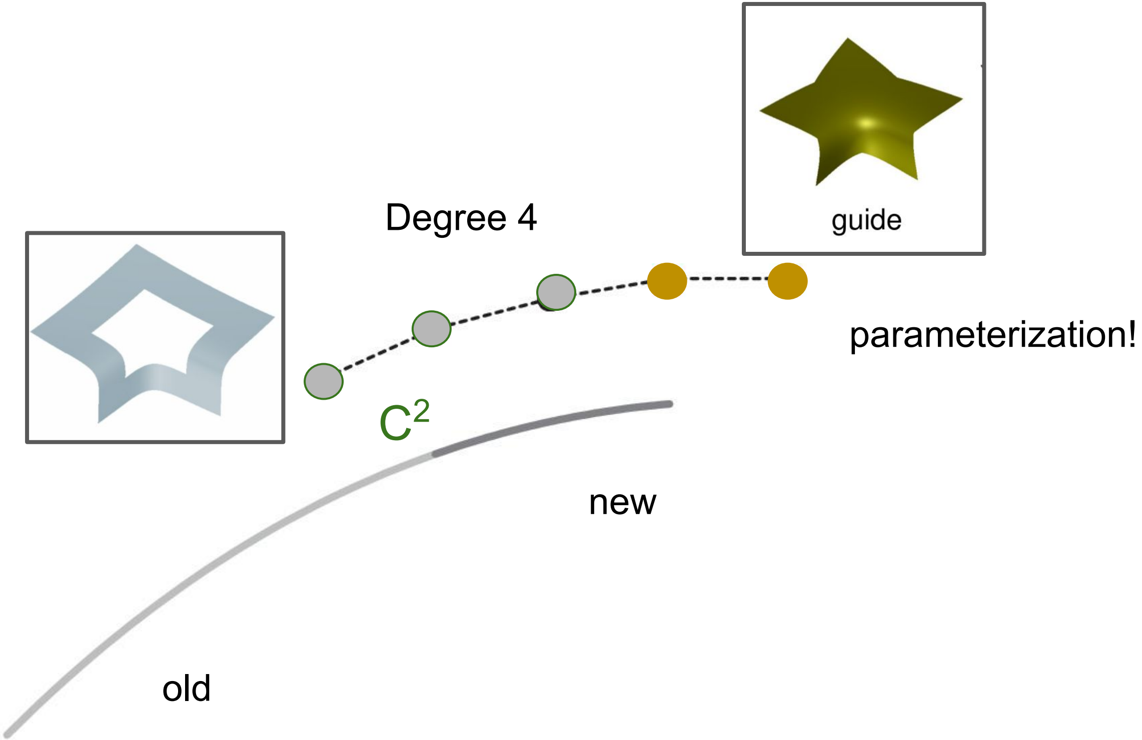
(a) Hexagonal face (b) Doo-Sabin

Guide-generation algorithm as (a complicated but) linear formula: **b** = G **c**

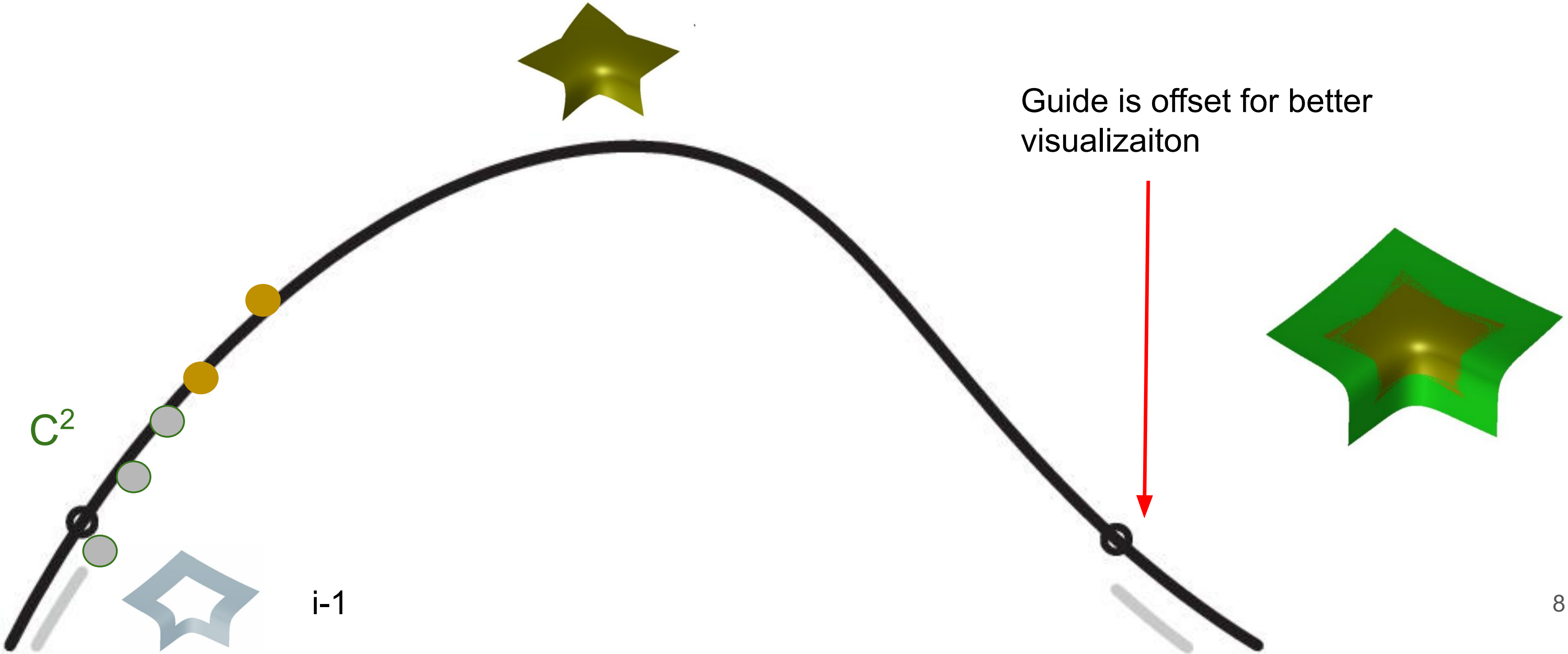
Tools to build a guide

- ➢ Sample total degree pre-guide
- ➢ Pre-solved G-constraints

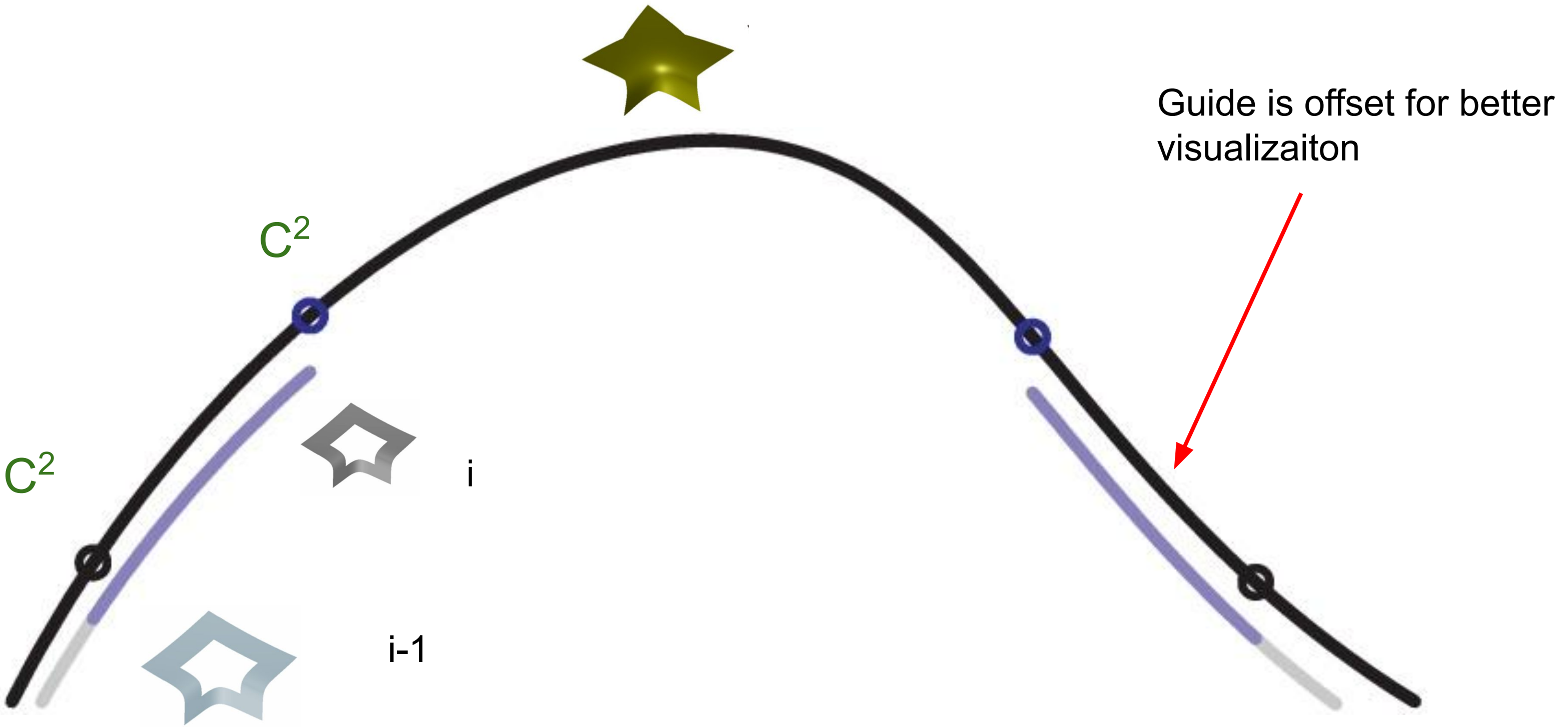
Guided subdivision = guide surface + prolongation



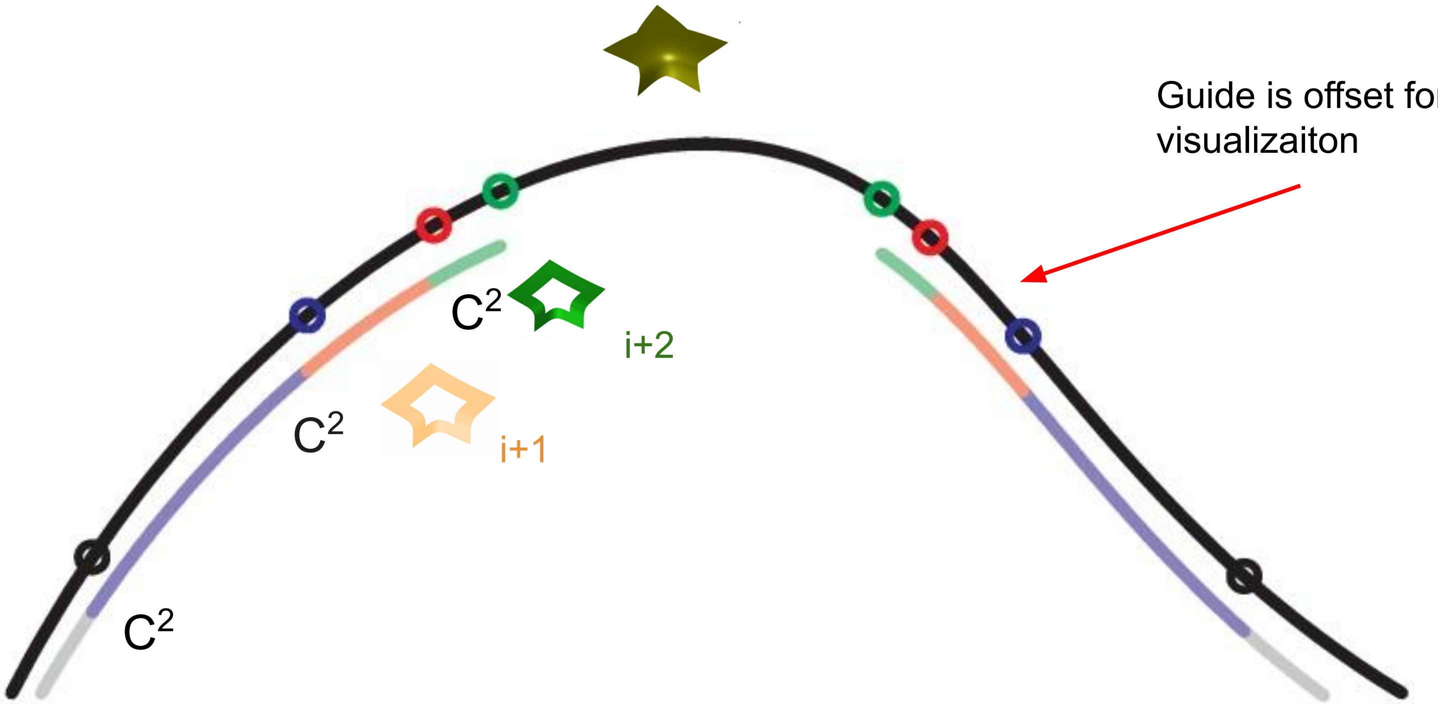
Guided curve subdivision = guide surface + prolongation



Guided curve subdivision = guide surface + prolongation

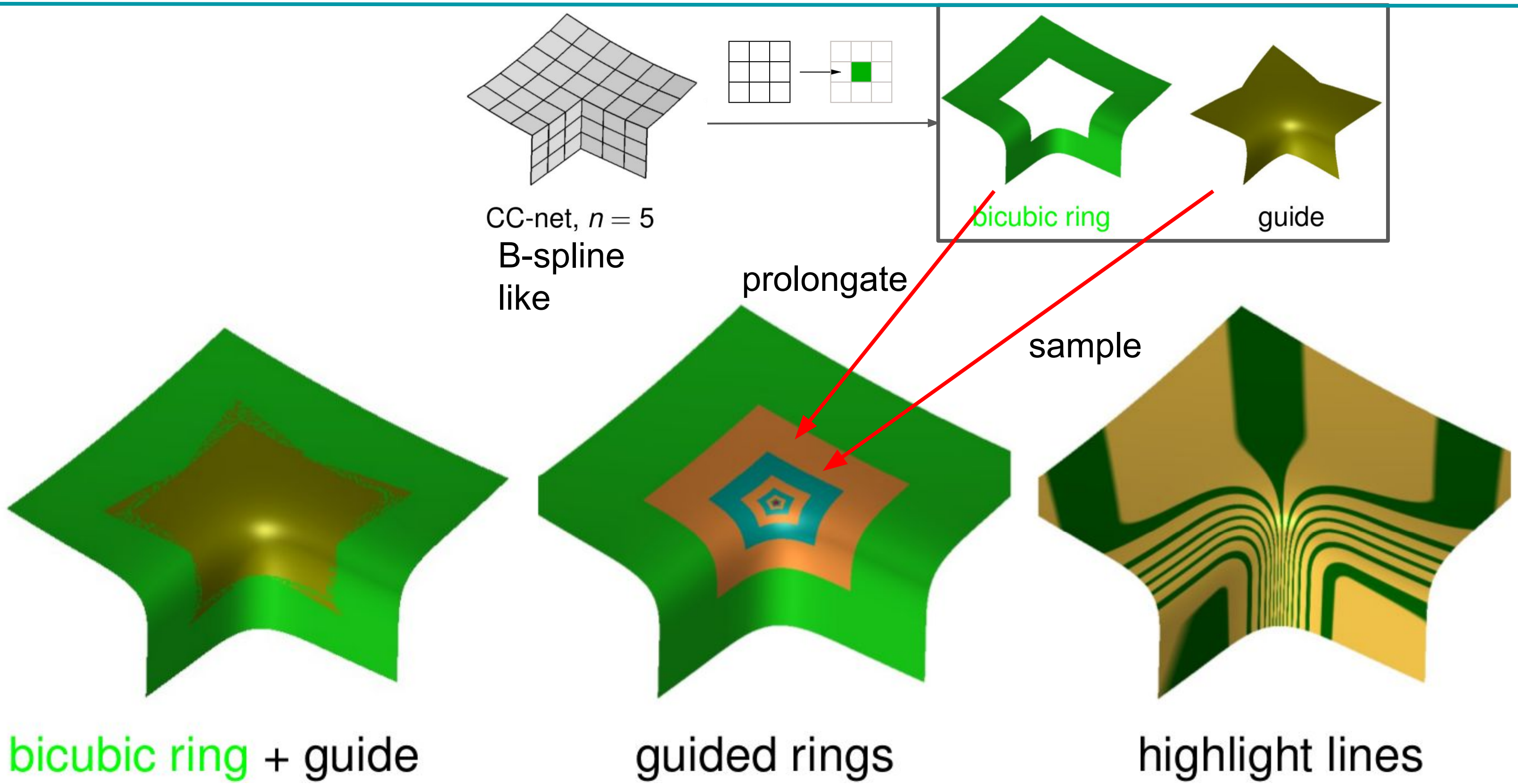


Guided subdivision = guide surface + prolongation

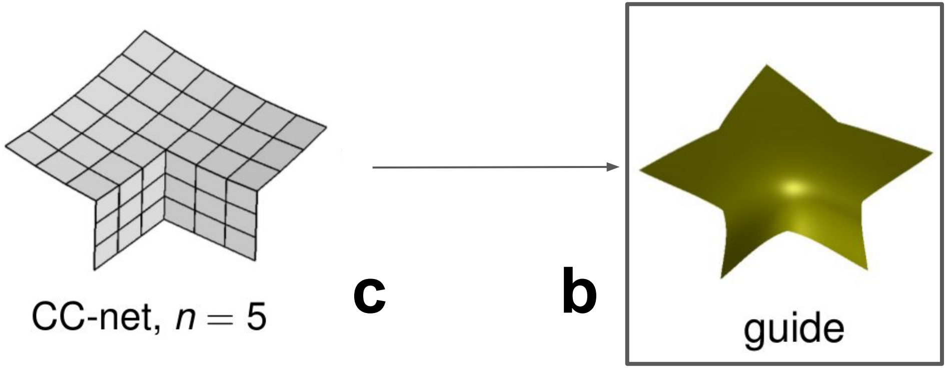


Guide is offset for better visualizaiton

Tools and tricks of the subdivision maker



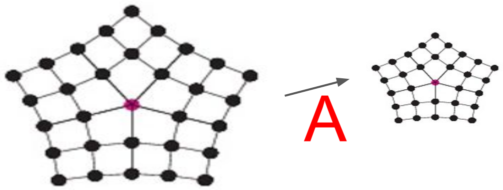
Tools and tricks of the subdivision maker



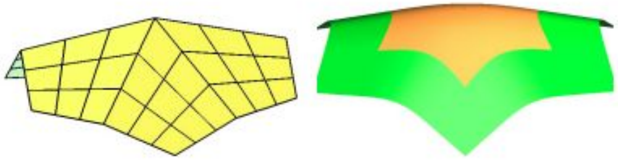
BB-coefficient of patches

Guide-generation algorithm as a (complicated but) linear formula: **b** = G **c**

Bake G into the subdivision matrix A



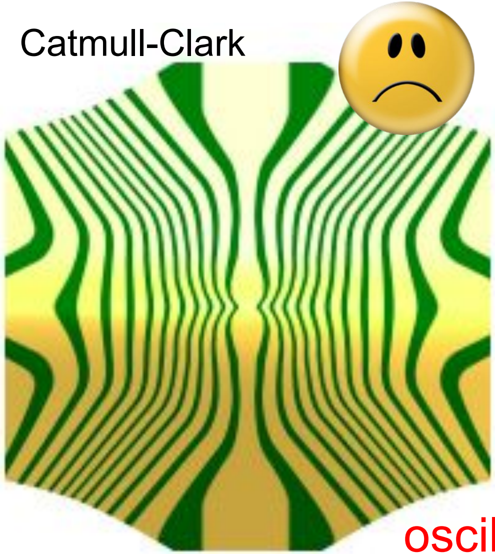
Improved Subdivision



(d) $n = 6$ (e) layout

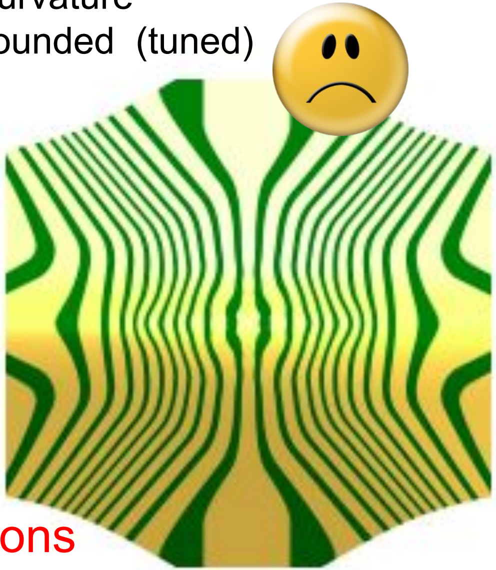
https://bitbucket.org/surflab/eg_refine

Catmull-Clark

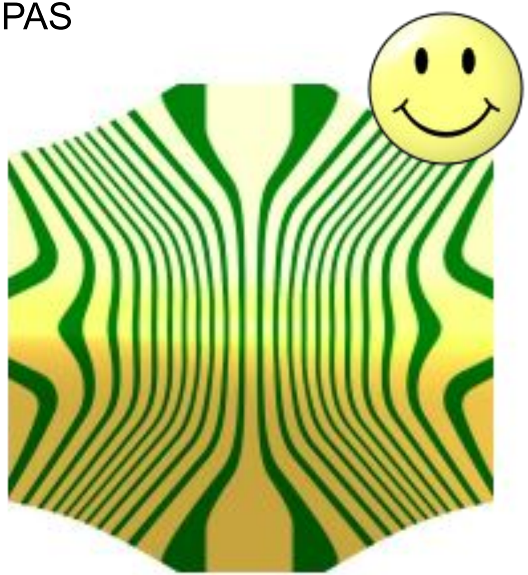


oscillations

Curvature
bounded (tuned)

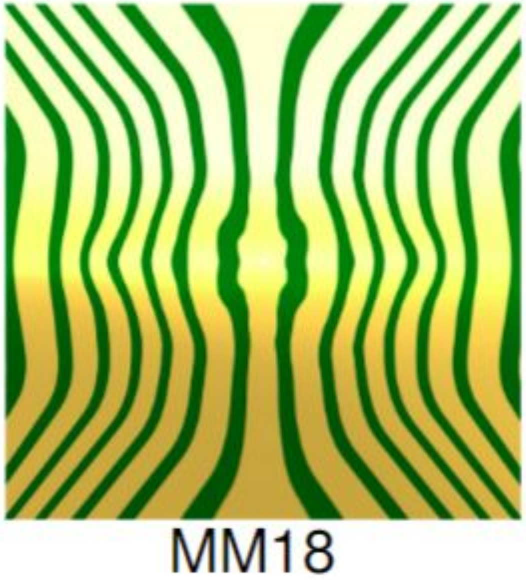


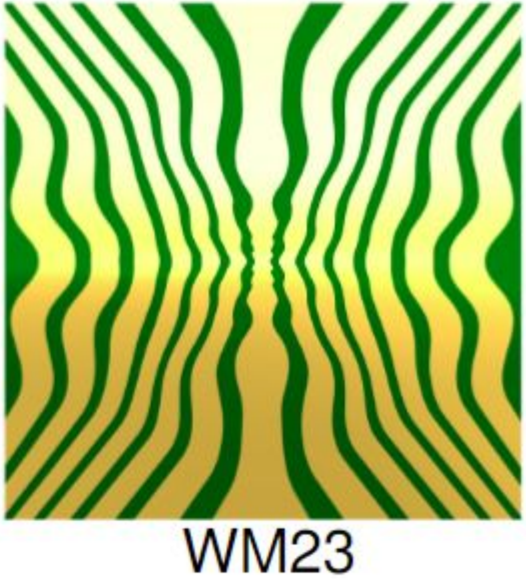
PAS



Quadratic-Attraction Subdivision

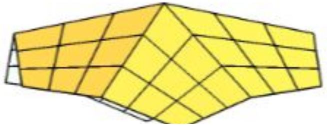
guide surface built into stencils



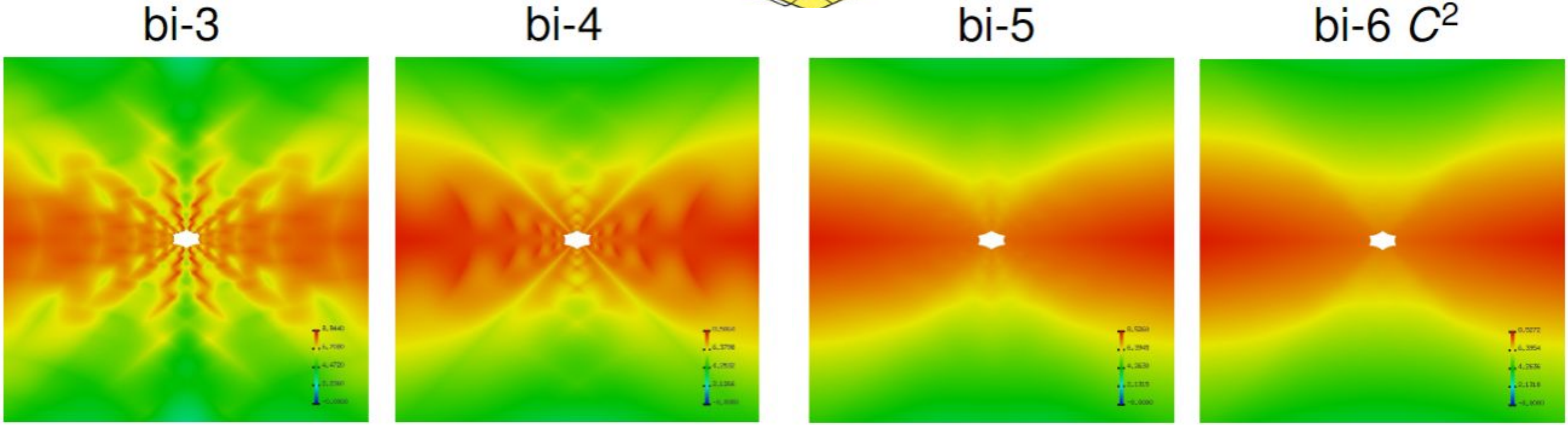


https://bitbucket.org/surflab/quadratic-attraction-subdivision

Quadratic-Attraction Subdivision (QAS)



Gauss curvature

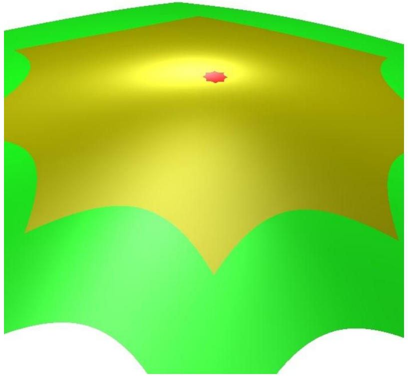


Bounded curvature

Interpretation of control points

Overview

- ➢ Improved subdivision surfaces
  - PAS (Point Augmented Subdivision)
  - QAS (Quadratic-Attraction Subdivision)
- ➢ Improved caps – one for each occasion



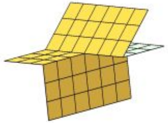
Possible Caps in the Literature

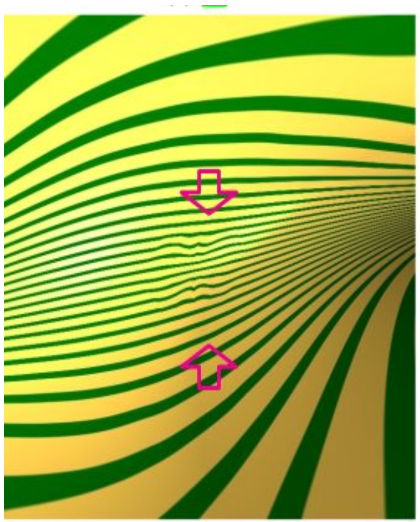
- Polynomial, C^2/G^2: [Loop, Schaefer, 2008] [KP Minimal bi-6 2016] …
- Multisided, rational: [Hettinga, Kosinka, 2020], [Vaitkus, Varady, Salvi 2021] …
- Rational corner singularity: [Gregory 1074], [Loop, Schaefer, Ni, Castano 2009] …
- Curved knot lines: [Sabin, Fellows, Kosinka 2022]...
- Manifold Splines: [Gu, He, Qin 2005]...
- Polynomial G1: [Kapl, Sangalli, Takacs 2017], [Blidia, Mourrain, Xu 2020], [Marsala, Mantzaflaris, Mourrain 2022], [Bonneau, Hahmann 2014]
- Polynomial almost G1: [KP 2015]
- …

Caps can assume less fluctuation than general n-sided surface models

Want same degree as subdivision rings

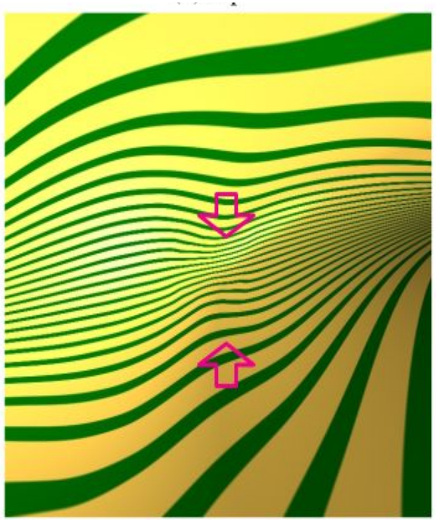
Why new caps for new subdivision?





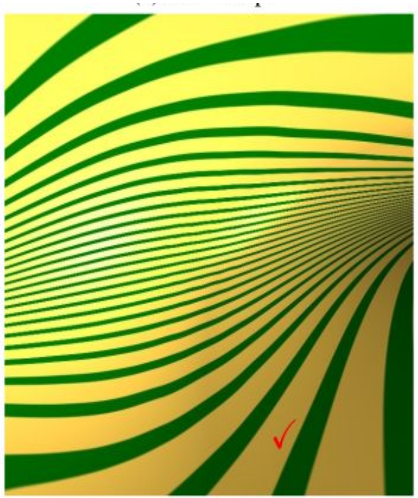
(g) AS^4 + GZ

New (good) subdivision + **old** cap



(h) CC^4 + cap^9

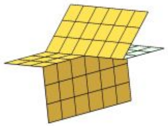
Old (poor) subdivision + **new** cap



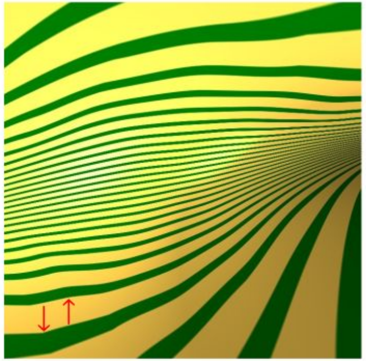
(i) AS^4 + cap^9

New subdivision + **new** cap

Cap without subdivision

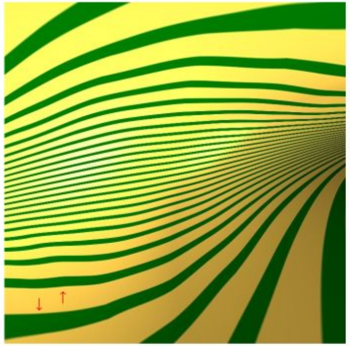


Unlike multi-sided surfaces, caps expect a few steps of subdivision → lower degree



(d) cap^9

bi-3

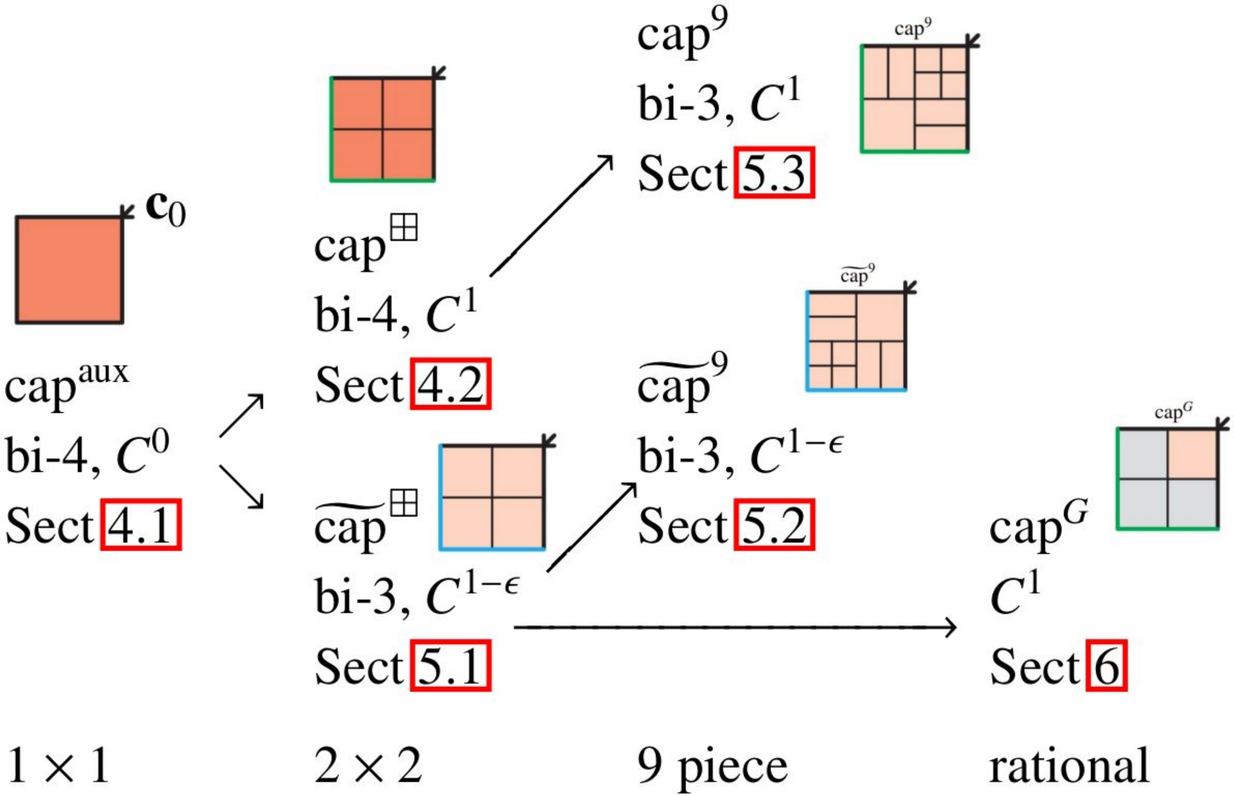


(e) $\text{AS}^1 + \text{cap}^9$

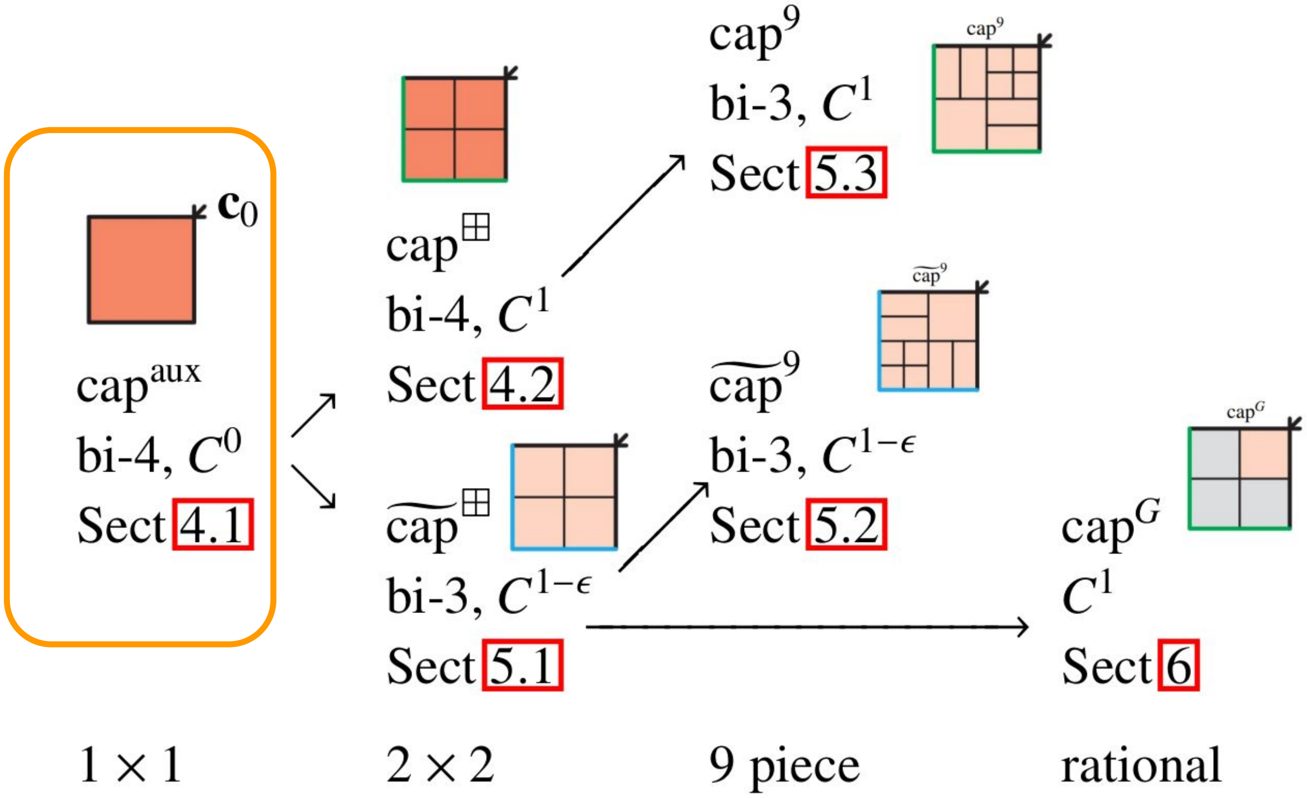
No subdiv step
New cap

One subdiv step
New cap

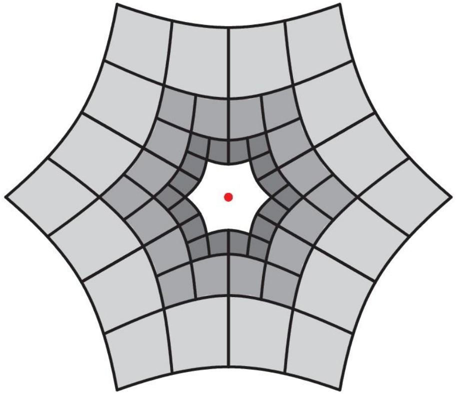
Genealogy: a hat for each occasion

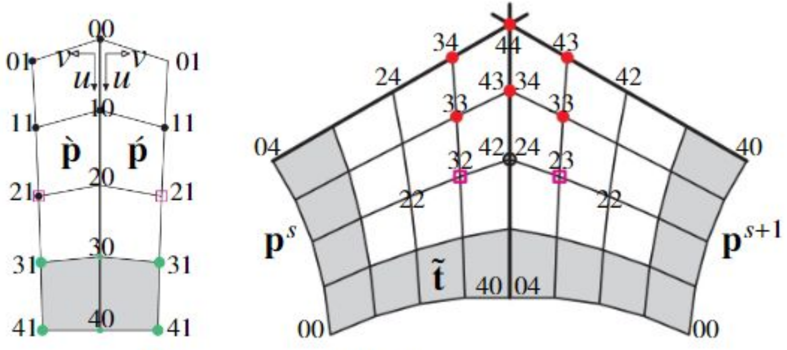


Cap-aux: auxiliary base cap

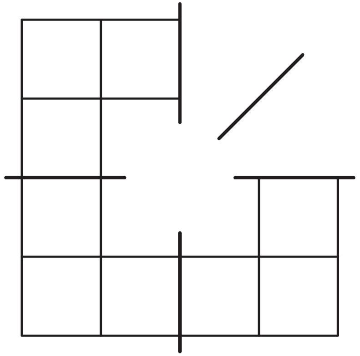


Cap-aux: auxiliary base cap

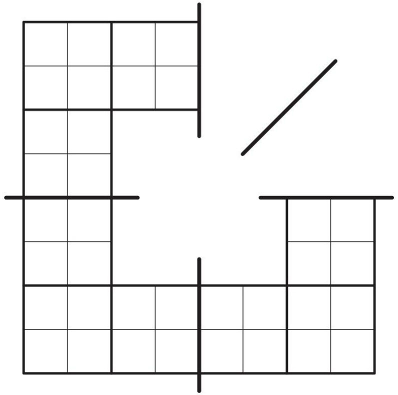




(a) G^1 (b) layout

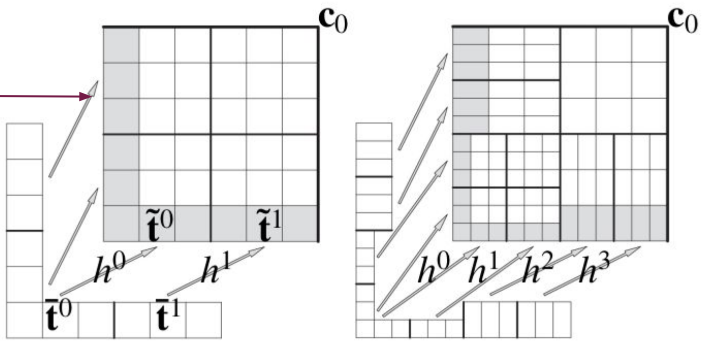


Bi-4, Bi-3

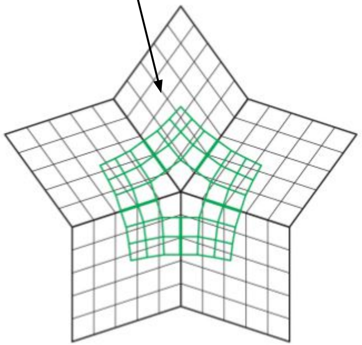


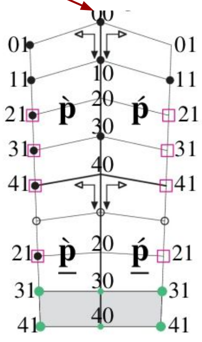
Tools and tricks of the cap maker

- Prolongation
- Splitting
- Pre-solved G-constraints
- Sampling

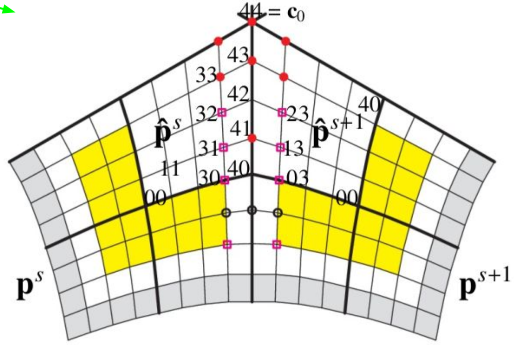


(a) 2 × 2 (b) 9 pieces



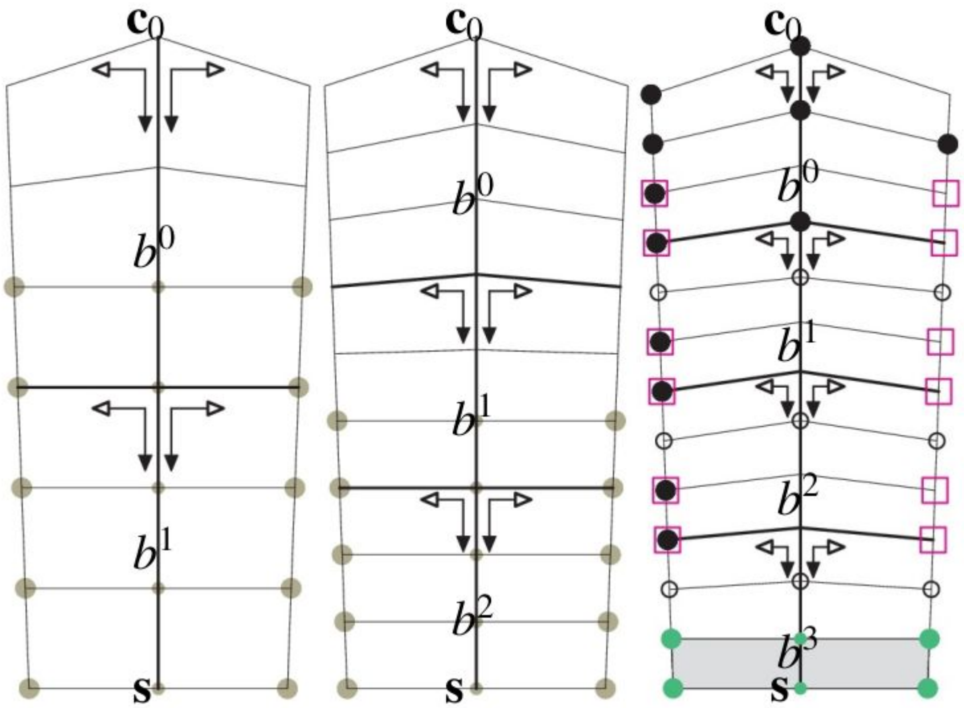


(b) G^1



(a) 2 × 2 layout, global indexing

Choices of reparameterization

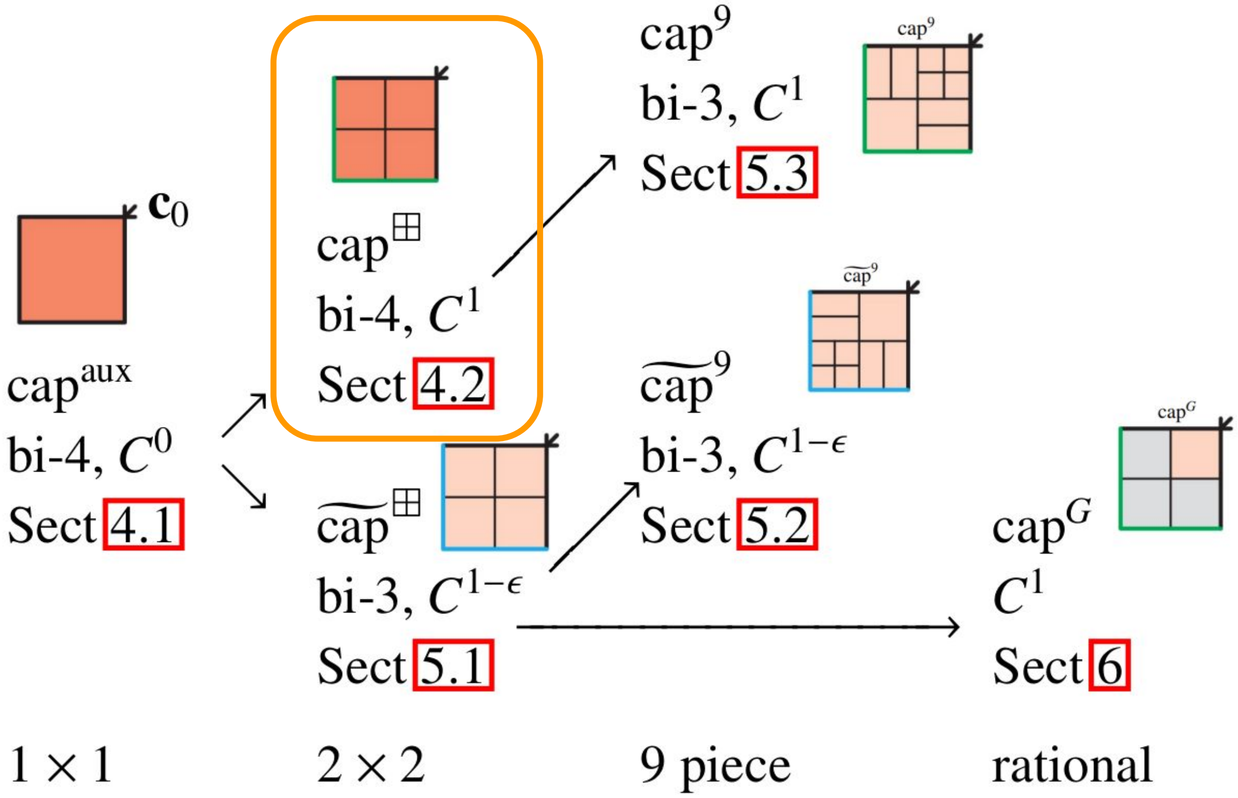


(a) [6] (b) [5] (c) cap9

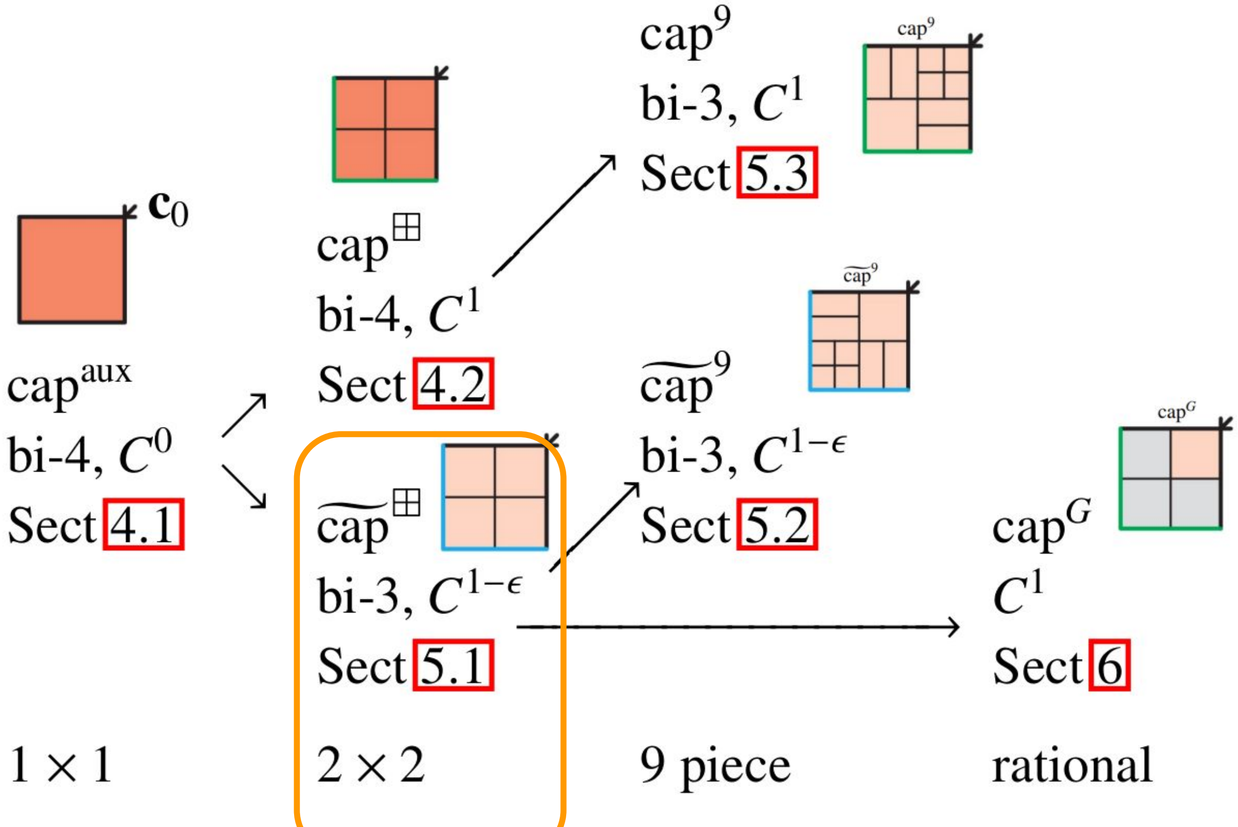
$$\partial_v \grave{\mathbf{p}} = a(u)\partial_v \acute{\mathbf{p}} + b(u)\partial_u \grave{\mathbf{p}},$$

$$\rho(u, v) := (u + b(u)v, a(u)v)$$

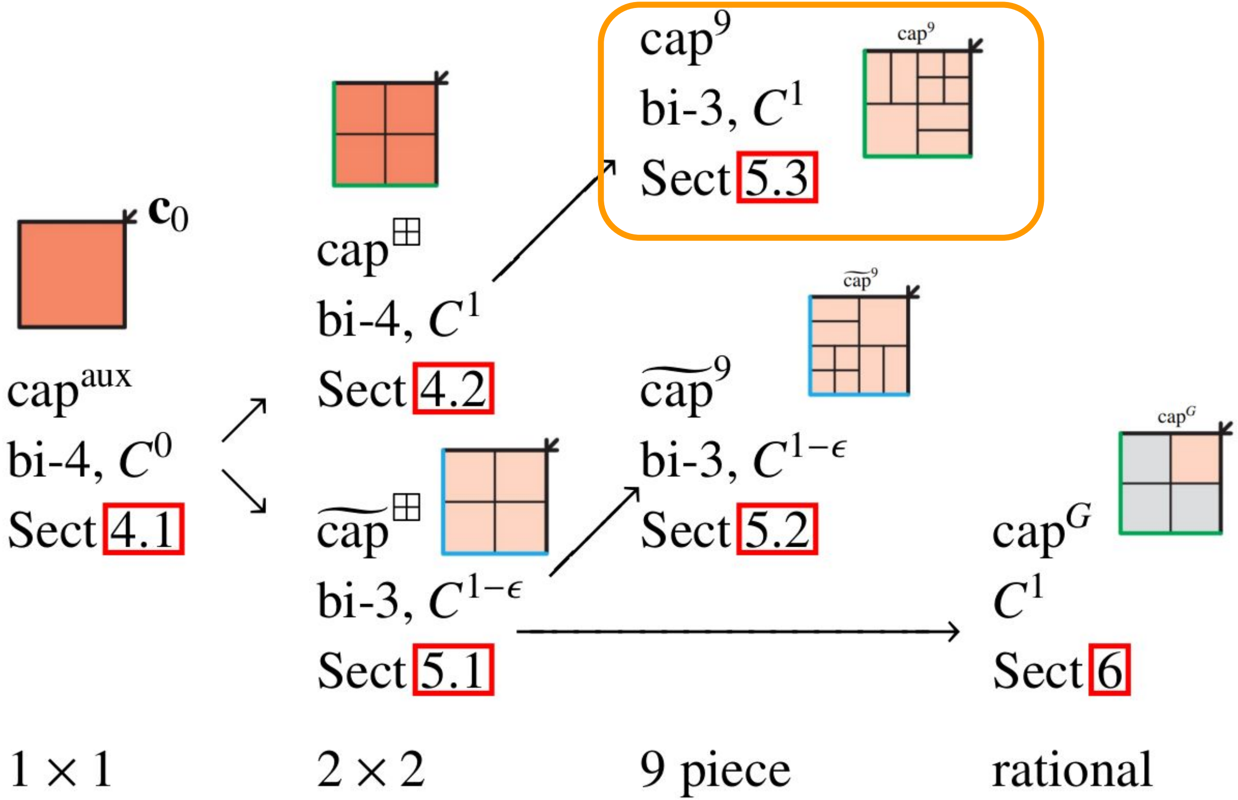
Cap-aux: auxiliary base cap



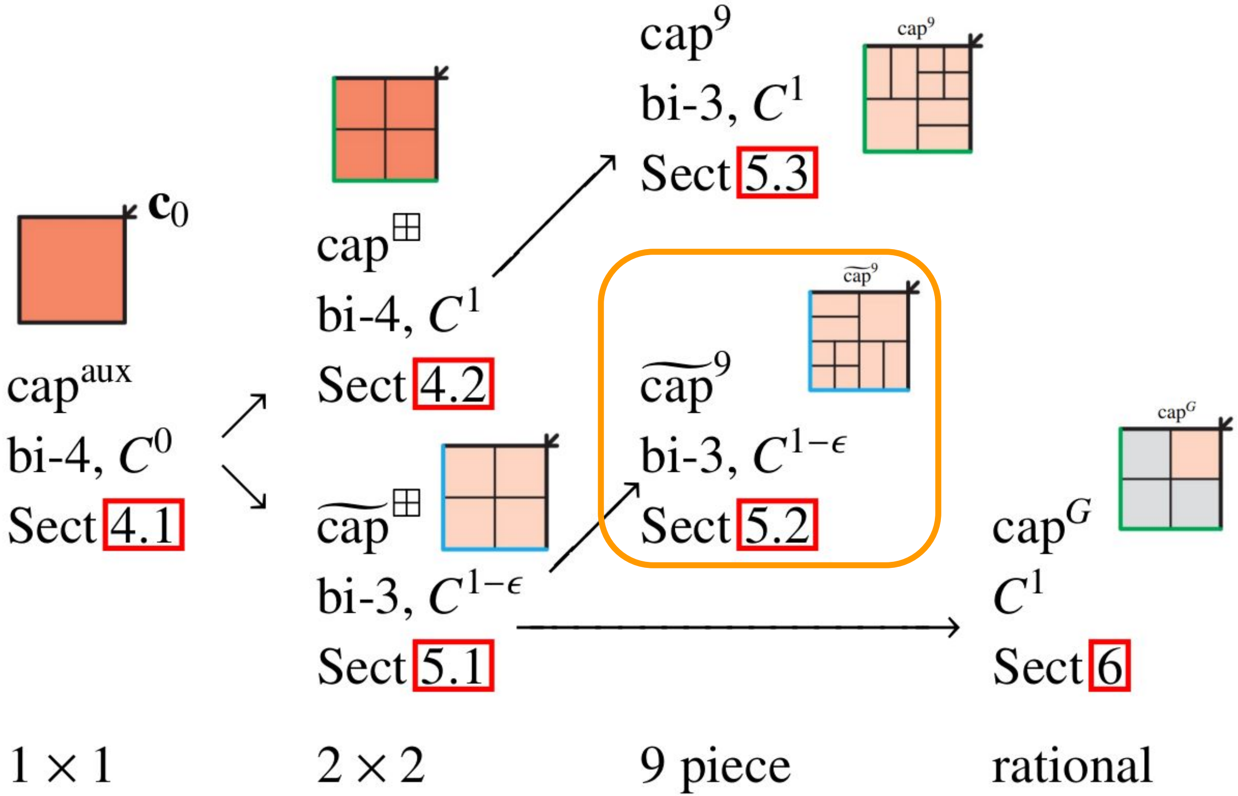
Cap-aux: auxiliary base cap



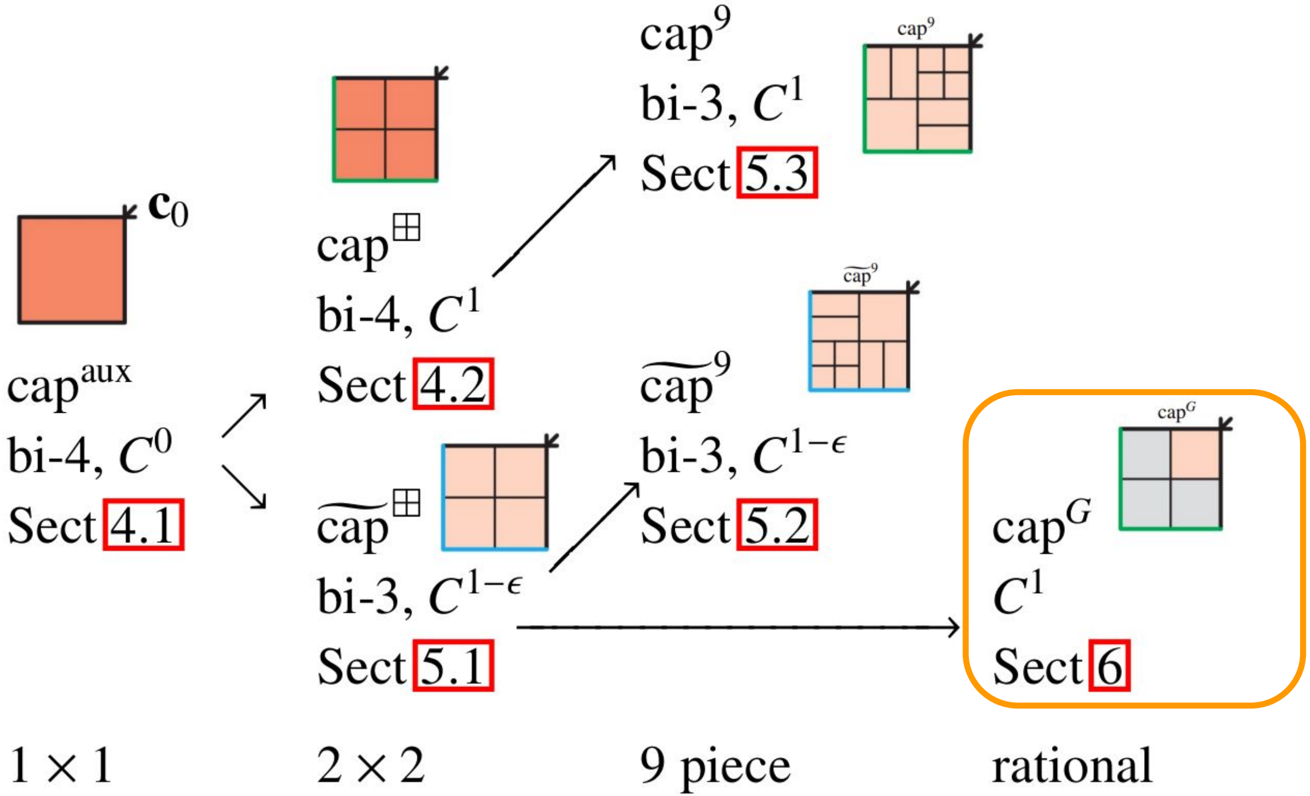
Cap-aux: auxiliary base cap



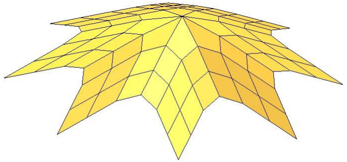
Cap-aux: auxiliary base cap

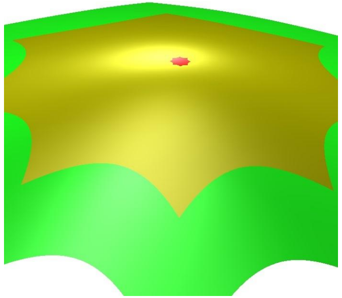


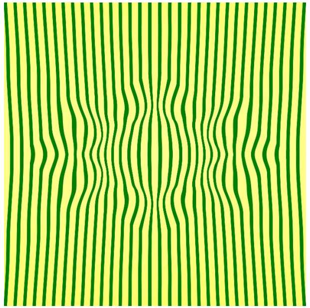
Cap-aux: auxiliary base cap



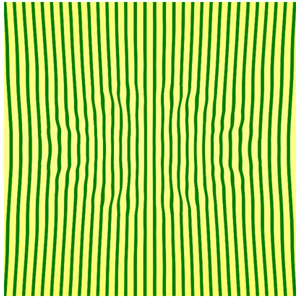
Comparisons



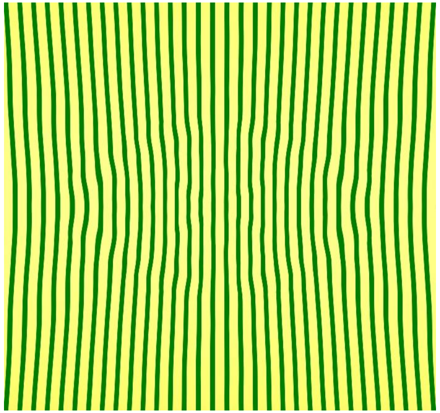




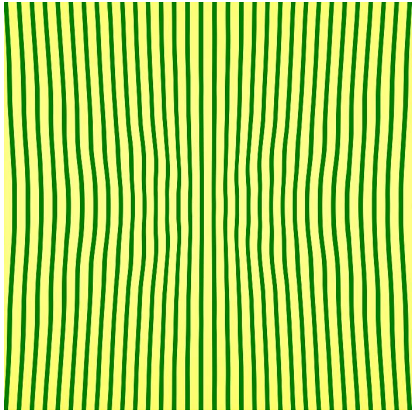
GZ



capG



cap9 bi-3



multi-sided surface
Bi5

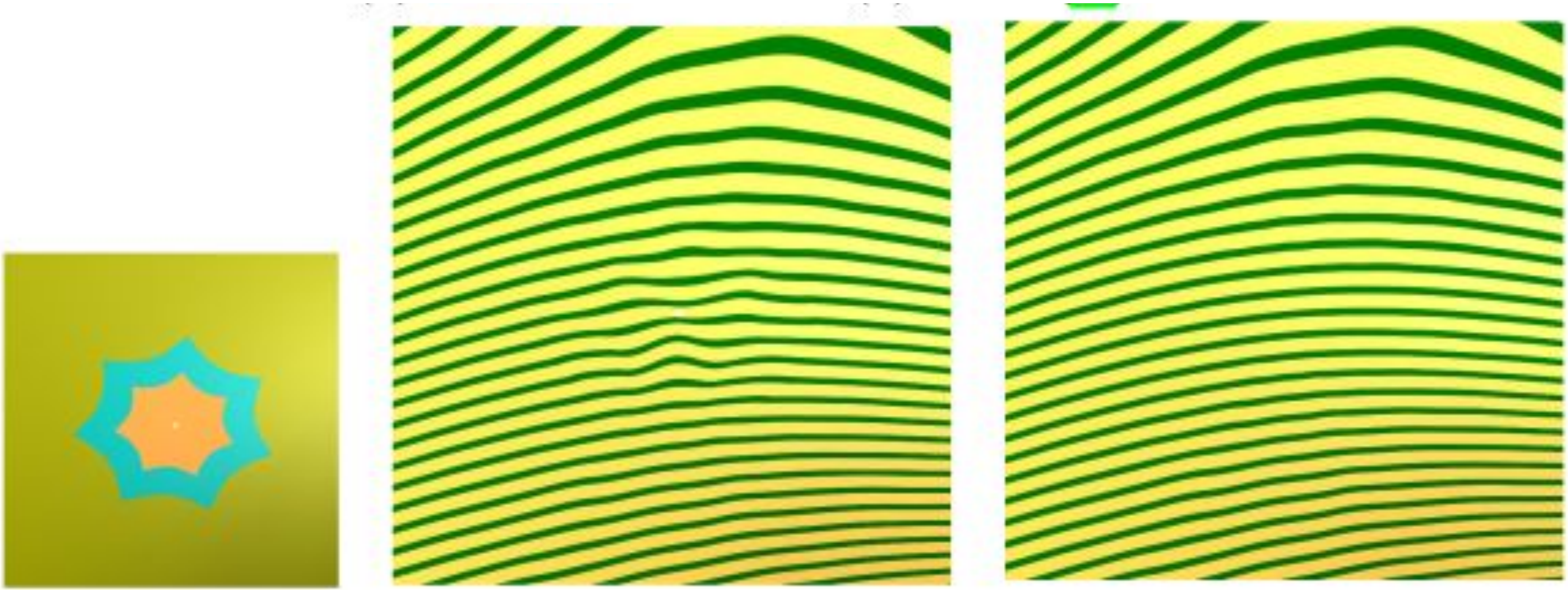
Many more comparisons in the paper

Effect of a transition ring to accommodate

(a) transition

(b) AS^6 + [7] = good multi-sided surface

Effect of a transition ring to accommodate



(c) transition   (d) AS^6 + cyan + AS^6 (e) AS^{13} + $cap^{⊞}$

Implementation via explicit Tables

9.2. Tables and assignments of $cap^{\boxplus}$

We define the BB-net $\hat{\mathbf{p}}_{rs}$ of the central piece attached to $\mathbf{c}_0$ in terms of the BB-coefficients $\mathbf{p}_{ij}$ of cap^{aux} and tabulated κ_{ij}^{rs} as

$$\hat{\mathbf{p}}_{rs} := \sum_{i=0}^{4}\sum_{j=0}^{4} \kappa_{ij}^{rs}\mathbf{p}_{ij}, \quad 0 \le r, s \le 2,$$

$$\kappa_{ij}^{rs} := \frac{1}{10^5}(K_{rs}^n)_{i+1,j+1}, \quad rs \in \{00, 10, 20, 11, 21, 22\},$$

$$K_{00}^6 := \begin{pmatrix} 520 & 1935 & 2698 & 1672 & 388 \\ 1935 & 7195 & 10032 & 6216 & 1444 \\ 2698 & 10032 & 13987 & 8667 & 2014 \\ 1672 & 6216 & 8667 & 5371 & 1248 \\ 388 & 1444 & 2014 & 1248 & 299 \end{pmatrix}, \quad K_{10}^6 := \begin{pmatrix} 48 & 1156 & 2971 & 2684 & 819 \\ 115 & 4057 & 10713 & 9772 & 2998 \\ 71 & 5320 & 14468 & 13334 & 4113 \\ -11 & 3088 & 8675 & 8082 & 2507 \\ -15 & 669 & 1948 & 1836 & 582 \end{pmatrix},$$

$$K_{20}^6 := \begin{pmatrix} -12 & 77 & 2508 & 3928 & 1745 \\ -37 & 163 & 8276 & 13945 & 6267 \\ -34 & 69 & 10216 & 18456 & 8439 \\ -7 & 2 & -29 & 1142 & 2360 \end{pmatrix}, \quad K_{11}^6 := \begin{pmatrix} 17 & 122 & 158 & 34 & -20 \\ 122 & 2501 & 6001 & 5066 & 1441 \\ 158 & 6001 & 15442 & 13660 & 4056 \\ 34 & 5066 & 13660 & 12370 & 3739 \\ -20 & 1441 & 4056 & 3739 & 1156 \end{pmatrix},$$

$$K_{21}^6 := \begin{pmatrix} -3 & 28 & 276 & 252 & -110 \\ -34 & 181 & 5196 & 7798 & 2994 \\ -57 & 200 & 11712 & 19522 & 8520 \\ -26 & 5 & 9310 & 16842 & 7743 \\ 0 & -42 & 2498 & 4869 & 2326 \end{pmatrix}, \quad K_{22}^6 := \begin{pmatrix} 520 & 1935 & 2698 & 1672 & 388 \\ 1935 & 7195 & 10032 & 6216 & 1444 \\ 2698 & 10032 & 13987 & 8667 & 2014 \\ 1672 & 6216 & 8667 & 5371 & 1248 \\ 388 & 1444 & 2014 & 1248 & 299 \end{pmatrix}.$$

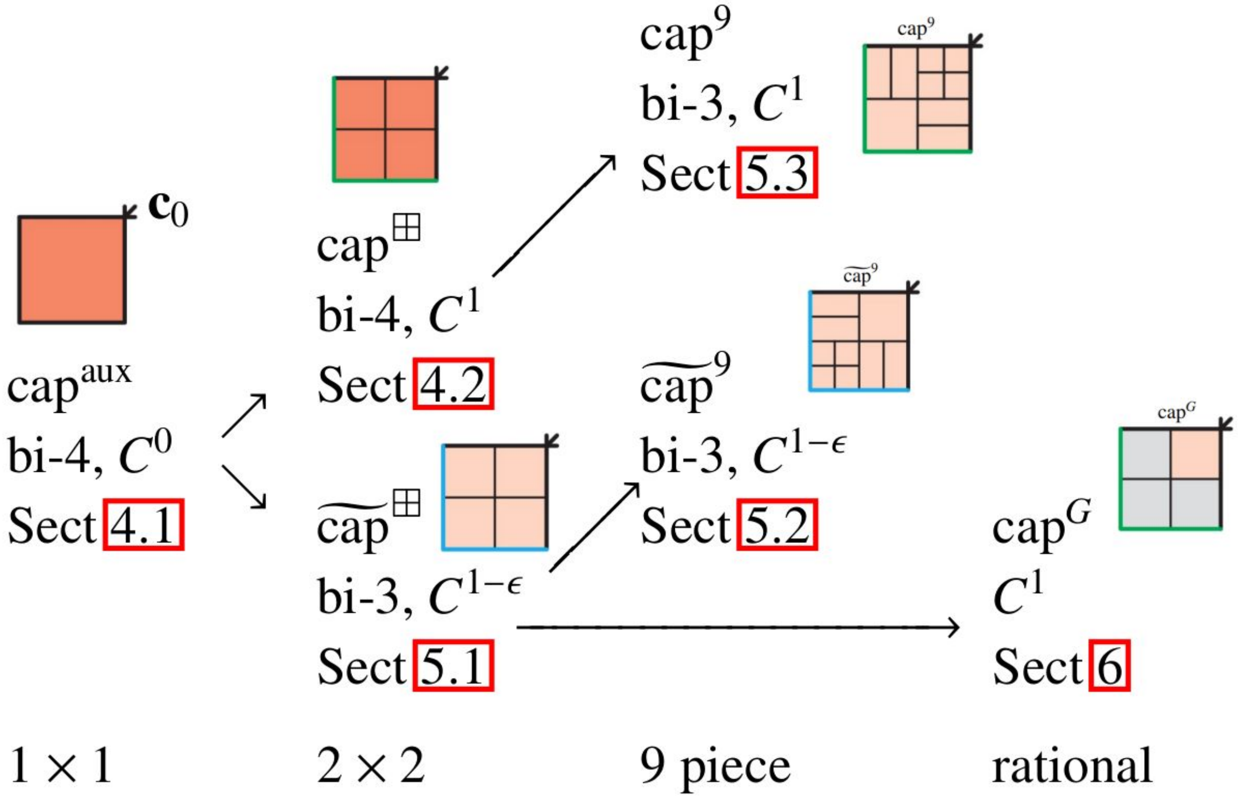
$$K_3^6 := \begin{pmatrix} 4971 & 16014 & 19346 & 10387 & 2091 & 3897 & 13537 & 17542 & 10056 & 2159 \\ 474 & 9864 & 21987 & 17212 & 4554 & 224 & 7434 & 18325 & 15519 & 4407 \\ -198 & 572 & 18471 & 26871 & 9912 & -131 & 162 & 13444 & 21737 & 9160 \\ 0 & 0 & 0 & 32641 & 24048 & 0 & 0 & 0 & 24048 & 19263 \end{pmatrix},$$

$$K_{41}^6 := \begin{pmatrix} 669 & 17942 & 40744 & 32115 & 8530 \end{pmatrix}, \quad K_{43}^6 := \begin{pmatrix} 0 & 0 & 0 & 57133 & 42867 \end{pmatrix}.$$

The tables K^n for $n \neq 6$ are listed in Section 9.5. Then

Summary


BASEBALL
FEDORA
NEWSBOY
COCKTAIL
PANAMA
BERET
TRAPPER
BOWLER
VIZOR
CLOCHE
BOATER
PILLBOX


$\mathbf{c}_0$
$\mathrm{cap}^{\mathrm{aux}}$
bi-4, C^0
Sect 4.1
$\mathrm{cap}^{\boxplus}$
bi-4, C^1
Sect 4.2
$\widetilde{\mathrm{cap}}^{\boxplus}$
bi-3, $C^{1-\epsilon}$
Sect 5.1
cap^9
bi-3, C^1
Sect 5.3
$\widetilde{\mathrm{cap}}^9$
bi-3, $C^{1-\epsilon}$
Sect 5.2
cap^G
C^1
Sect 6
1×1
2×2
9 piece
rational